1. Motivations

- Modeling texture through orientation and scale emerging modulations.
- Multiband feature extraction and narrowband image analysis.
- Counterweighting amplitude and frequency in channel selection.
- Extract robustly low-dimensional features for texture segmentation.

2. AM-FM modulations image model

- Any 2D narrowband signal can be expressed in terms of spatially varying amplitude and frequency modulations, as a superposition of locally smooth, non-stationary sinusoids:

\[ I(x,y) = \sum_{k=1}^{K} a_k(x,y) \cos(\Omega_k(x,y)) \]

- \( a_k(x,y) \) and \( \Omega_k(x,y) = \nabla \phi_k(x,y) \) are the Amplitude and Frequency Modulation signals modeling image contrast and locally emergent frequency variations respectively.
- Multiband analysis using Gabor filterbanks and demodulation of individual bandpass components.

3. Dominant Component Analysis

- Channel selection by maximizing a normalized demodulated-amplitude criterion on a pointwise basis:

\[ I_k(x,y) = \nabla I(x,y) + \hat{b}_k \nabla \phi_k(x,y) + f(x,y) \]

- \( \hat{b}_k \) and \( \hat{\phi}_k \) are the real and imaginary parts of a complex Gabor filter and \( \hat{H}_k(f) \) its spatial frequency response.
- Extraction of dominant Amplitude and Frequencies estimates.
- Forms a low dimensional texture descriptor.
- Unclear interpretation of decision results.
- No probabilistic interpretation of parameter estimation.

4. Modulation Energy & Separation


\[ \Psi(I) = |\nabla I|^2 - I \nabla I \]

- Applied to a 2D AM-FM signal \( I_k(x,y) = a_k(x,y) \cos(\Omega_k(x,y)) \)

\[ \Psi[I(x,y)] \approx [\frac{a_k(x,y)}{\sqrt{\sigma^2 + \|\nabla I\|^2}}]^2 \]

- Negligible approximation error, assuming smoothly varying instantaneous signals compared to the carriers.
- 2D Energy Separation Algorithm (ESA)

5. Energy Dominant Component

- Apply Teager Energy criterion to select Dominant channel
- Alternative to conventional DCA
- Captures Edges and sharp texture variation
- Exploits both channel amplitude and local spatial frequency information
- Compact energy measurement even in small scales
- Filterbank consisting of 40 Gabor filters, in 4 scales radially arranged in the spatial frequency domain.

6. Multiple Hypothesis Testing

- Channel Selection formulated as detecting a sinusoid with the frequency of the channel and unknown amplitude and phase.
- Multiple Hypothesis Testing is used to decide for the presence of one out of many sinusoids of frequency \( \Omega \) and white gaussian background noise (Null Hypothesis, \( \mathcal{H}_0 \)).
- Hypothesis \( \mathcal{H}_0 \) around point \( x = 0 \), the signal is a sinusoid of frequency \( \Omega \), unknown phase \( \phi \), amplitude \( a \), and DC value \( b \).

\[ \mathcal{H}_1 : I(x,y) = a(x,y) \cos( \Omega \phi(x,y) + \phi ) + b \]

- \( \phi \) is the observation vector of size \( N \), \( v(x,y) \) a white gaussian noise process of mean \( C \) and variance \( \sigma^2 \).
- Minimum Description Length (MDL) criterion: Choose hypothesis \( \mathcal{H}_1 \), maximizing

\[ \hat{\phi} = \arg \min_{\phi} \left[ \hat{g}(\mathcal{H}_1) + \hat{b}_k \right] \]

\[ \hat{g}(\mathcal{H}_1) = \sum_{k=1}^{K} \left[ \hat{b}_k \hat{g} + \sigma^2 \right] \]

- Energy pre-averaging and demodulation post-filtering (non-linear, linear).

7. DCA as Hypothesis Testing

- Locality in the decision process is introduced through a spatial weighting function, \( G(x,y) = \exp[-c ||x||^2] \).
- Under \( \mathcal{N}_0 \), the likelihood of the signal \( I \) at point \( x \) becomes:

\[ P(I(x,y) = a(x,y) \cos(\Omega \phi(x,y) + \phi) + b) \]

- Log Likelihood on a small image area

\[ P(I(x,y) = a(x,y) \cos(\Omega \phi(x,y) + \phi) + b) \]

- Maximum Likelihood (ML) Estimates and GLRT:

\[ \hat{b}_k = \frac{1}{\sum_{k=1}^{K} \hat{b}_k^2} \frac{\sum_{k=1}^{K} \hat{b}_k^2}{\sum_{k=1}^{K} \hat{b}_k} \]

- The bound \( P(I(x,y) \geq r) \)

\[ \approx \frac{1}{1 - \beta} \frac{C}{\sqrt{2\pi}} \]

where \( \beta \) is the power of the signal.
- The data \( I(x,y) \) is the likelihood of unknown-estimated parameter vector \( \theta \) of dimension \( n \).

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**Main References**