Coupled Geometric and Texture PDE-based Segmentation

Anastasia Sofou,
Georgios Evangelopoulos,
Petros Maragos

Computer Vision & Signal Processing Group,
School of Electrical and Computer Engineering,
National Technical University of Athens,
Greece

URL: *http://cvsp.cs.ntua.gr*

Summary

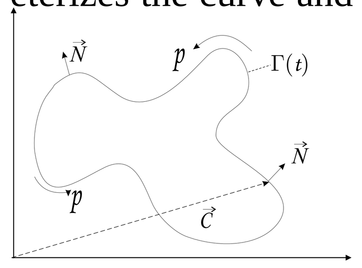
Along with recent trends in segmentation using multiple image cues, we examine the integration of modulation texture features, image contrast and region size for image decomposition in homogenous regions. First, we propose the use of a watershed type morphological PDE-based segmentation scheme, based on seeded region-growing and level curve evolution with contrast - size dependent speed. Second, we analyze object surface texture by modeling image variations as local spatial modulation components estimated via multi-frequency filtering and instantaneous energy-tracking operators. We exploit contrast and texture information, through multiscale image decomposition, and propose a PDE-based coupled segmentation method. Experimental results on various classes of images such as soilsections, aerial and natural scenes indicate that the combined effect of image decomposition and multi-cue segmentation improves the overall segmentation process.

1. Motivations and Objectives

- Application dependent nature of image segmentation introduces the necessity of multiple cues usage.
- Intensity contrast, region area and texture appear to be useful cues.
- Morphological watershed transform exploits intensity contrast and region size criteria.
- PDE modeling ensures better mathematical formulation, and approximation to continuous geometry of the problem.
- Texture modeling enables extraction of image features related to geometrical complexity, rate of change in local contrast variations and orientation.
- $U+V$ image decomposition incorporates contrast (Cartoon U) and texture information (V).

2. Generalized Watersheds and PDEs

- PDE modelling in watershed flooding process: each emanating wave's boundary is an evolving curve, with predefined speed.
- Uniform (height/volume) flooding: moving smooth closed curve (marker's boundary) $\vec{C}(p,t)$ where $p \in [0,1]$ parameterizes the curve and t is an artificial marching parameter.



$$\frac{\partial \vec{C}}{\partial t} = \frac{c}{A(t)\|\nabla I\|} \cdot \vec{N}$$

$A(t) = 1$ (height flooding), $A(t) = \mathrm{Area}(\vec{C})$ (volume flooding).

3. Texture Modulations

- Textures can be modeled by a superposition of 2D spatial non-stationary, locally smooth sinusoids:

$$I(x,y) = \sum_{k=1}^{K} a_k(x,y)\cos[\phi_k(x,y)]$$

- $a_k(x,y)$ and $\vec{\omega}_k(x,y) = \nabla\phi_k(x,y)$ are the *Amplitude and Frequency Modulation* (AM–FM) signals modeling image contrast and locally emergent frequency variations resp.
- Teager energy operator $\Psi(f) \triangleq \|\nabla f\|^2 - I\nabla^2 f$ applied to a 2D AM–FM component $f_k(x,y) = a_k(x,y)\cos[\phi_k(x,y)]$ yields the *texture modulation energy*:

$$\Psi[a_k\cos(\phi_k)] \approx a_k^2\|\vec{\omega}_k\|^2$$

4. Energy Dominant Components

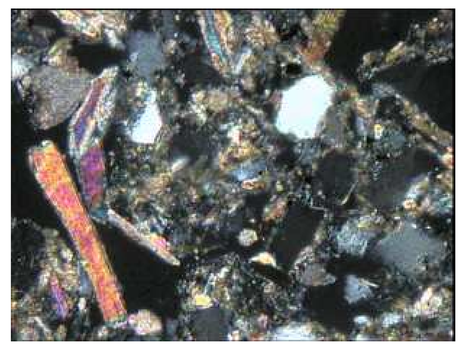
- Modulation models through multiband analysis. Gabor filterbank (40 filters in 4 scales, radially arranged).
- Dominant texture components by energy tracking. Apply Ψ to narrowband images and select dominant channel pixel-wise by max average value.
- *Maximum Average Teager Energy*

$$\Psi_{\text{mat}}(I(x,y)) = \max_k \Psi[((I * h_k) * h_a)(x,y)] \qquad (1)$$

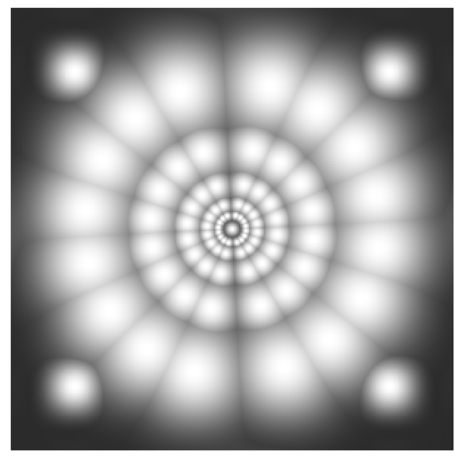
where h_k the response of k-th filter, h_a an averaging filter.

- Ψ_{mat} is a smooth indication of the texture modulation energy $\rightarrow$ texture detection cue.
- Instantaneous amplitude and frequency values may be given by demodulating the dominant channel [ICIP 04].

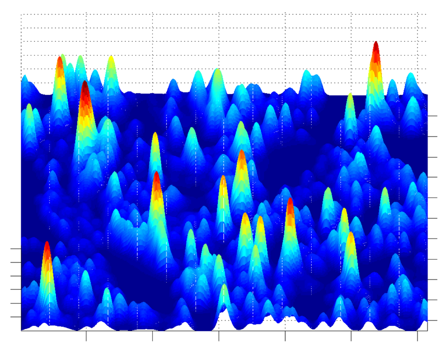
5. Texture Analysis



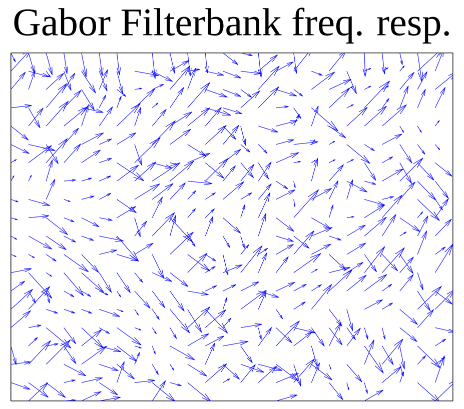
Soilsection Image



Gabor Filterbank freq. resp.



MAT Energy Ψ_{mat} (persp. view)



Dominant Freq. Orient. Vecs

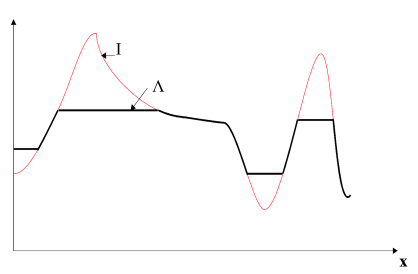
6. U+V Decomposition

- $U+V$ Image Decomposition: $I = U + V$.

  - U: Cartoon Component obtained as leveling of I

  - V: Texture Component $V = I - U = I - \Lambda(M|I)$

Levelings are nonlinear object-oriented filters that simplify an image I through a simultaneous use of locally expanding/shrinking an initial seed image, called the marker M, and a global constraining of the marker evolution by the reference image.



$$\lambda(F|I) = (\delta(F) \wedge I) \vee \varepsilon(F)$$
$$\Lambda(M|I) = \lim_{k\to\infty} F_k$$
$$F_k = \lambda(F_{k-1}|I), \; F_0 = M$$

7. Coupled Segmentation Scheme

- Generalized PDE Scheme

$$\frac{\partial \vec{C}}{\partial t} = \left(\frac{\lambda_1}{A\|\nabla f_1\|} + \lambda_2\Psi_{\text{mat}}(f_2) - \mu\,\kappa\right)\vec{N}$$

f_1, f_2 : image transformations w.r.t I.

- Level set formulation (the evolving planar curve is embeded as the zero-level curve of an evolving space-time function $\Phi(x,y,t)$)

$$\frac{\partial \Phi}{\partial t} = \left(\frac{\lambda_1}{A\|\nabla f_1\|} + \lambda_2\Psi_{\text{mat}}(f_2) - \mu\,\mathrm{curv}(\Phi)\right)\|\nabla\Phi\|$$

where $\mathrm{curv}(\Phi)$ is the curvature of the level sets of Φ.

- Resulting Scenarios
  (1) $f_1 = I, f_2 = I$
  (2) $f_1 = U, f_2 = V$

8. Method Evaluation

Quality Criteria	Segmentation Method			
	Coupled Scheme Cont.& Text.	Coupled Scheme Vol.& Text.	Watershed	K-Means
YLGC	0.25	0.09	0.6	7.7
MSF	3.25	3.30	3.40	3.69

- Mumford Shah energy minimization functional (MSF)

$$E(\Gamma) = \mu \int\int_R (g-I)^2 dxdy + \int\int_{R-\Gamma} \|\nabla g\| dxdy + \nu|\Gamma|$$

g: segmented mosaic image, Γ: segmentation boundary. μ and ν are constants.

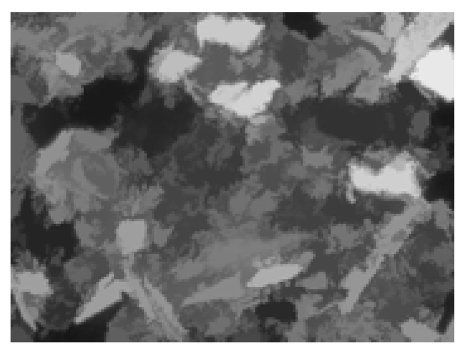
- Liu and Yang Global Cost function (LYGC)

$$F(I) = \sqrt{N} \times \sum_{i=1}^{N} \frac{e_i^2}{\sqrt{A_{R_i}}}$$

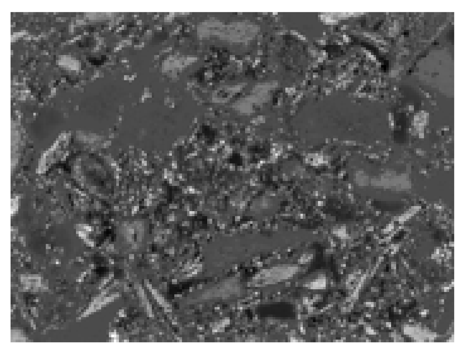
N: number of regions, e_i^2: color error over region i, and A_{R_i}: region area i (in pixels).

9. Soilsection Segmentation Results

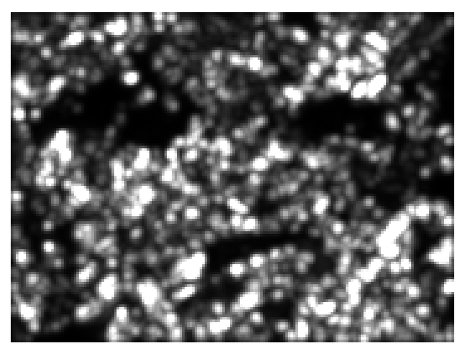
Soilsection



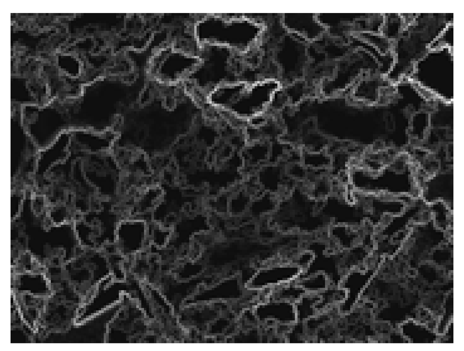
"Cartoon" U



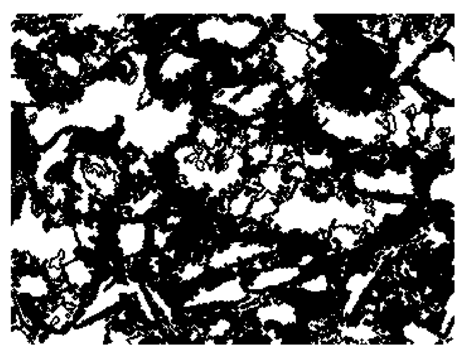
Texture V



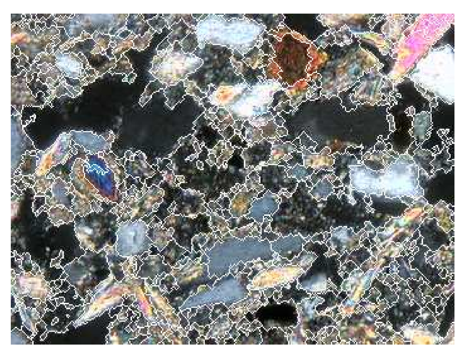
Texture energy $\Psi_{\text{mat}}(V)$



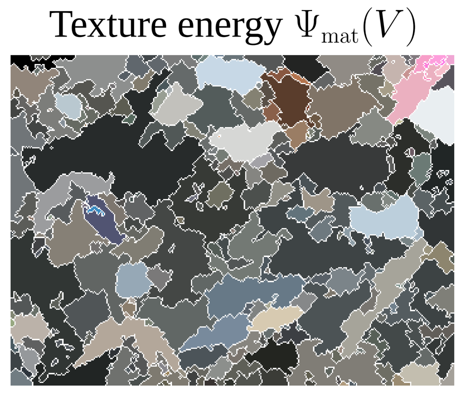
Gradient of U



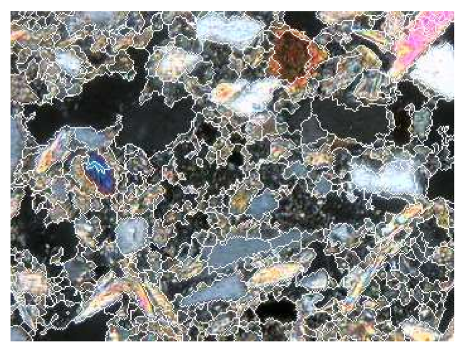
Markers



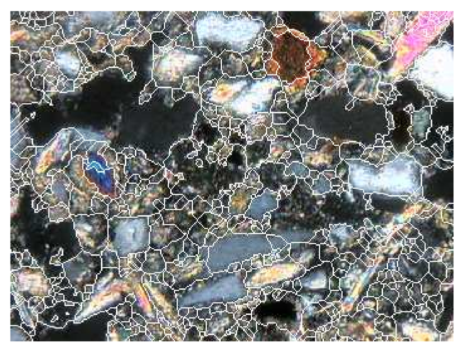
Coupled Contrast-Texture segmentation



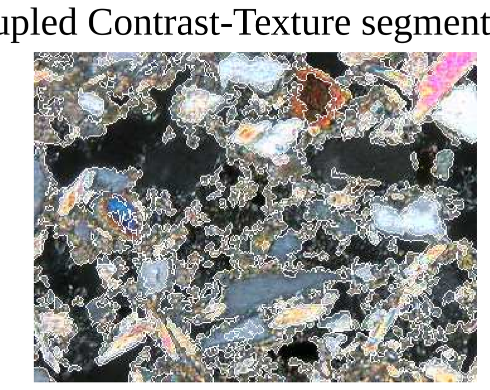
Refined regions



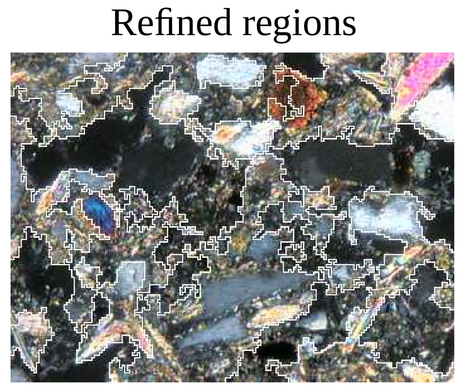
Coupled Volume-Texture segmentation



Watershed



K-means clustering



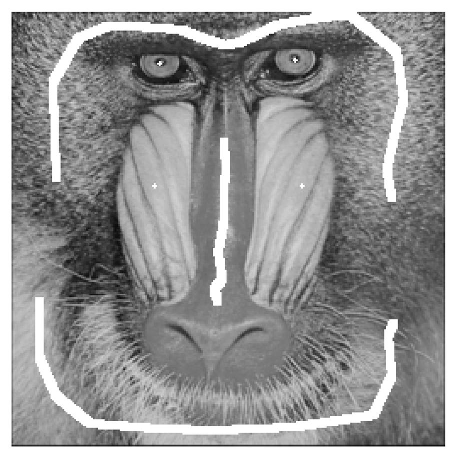
RSST segmentation

10. Applications

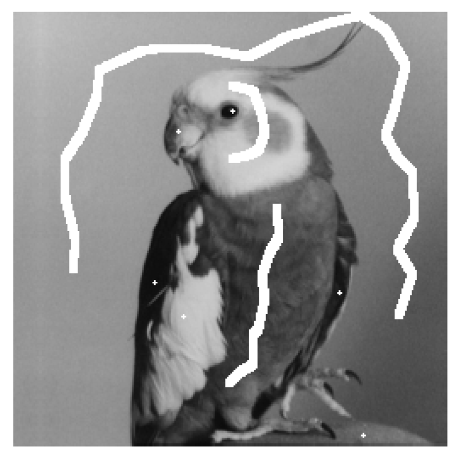
aerial image

biomed

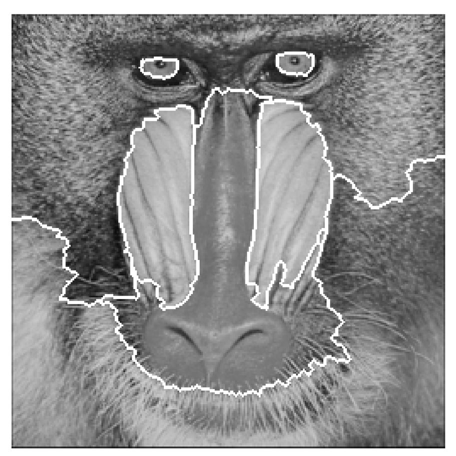
madrill



bird



s e g m e n t a t i o n   r e s u l t s



Main References

S.Beucher and F.Meyer,"The morphological approach to segmentation: The watershed transformation,"in *Math. Morphology in Im. Proc.*, E.R.Doughertty, Ed. Marcel Dekker, New York, 1993.

F.Meyer and P.Maragos,"Multiscale morphological segmentations based on watershed, flooding, and eikonal pde," in *Proc. of Scale-Space*:351-362, 1999, Springer.

P. Maragos and M. A. Butt, "Curve evolution, differential morphology, and distance transforms applied to multiscale and eikonal problems," *Fundamenta Informaticae*, 41:91-129, 2000.

A. Sofou and P. Maragos, "PDE-based modelling of image segmentation using volumic flooding", in *Proc. IEEE ICIP*, 2003.

S. Osher and J. Sethian, "Fronts propagating with curvature-dependent speed: Algorithms based on hamilton-jacobi formulations," *J. Comp. Physics*, 79:12-49, 1988. A. C. Bovik, N. Gopal, T. Emmoth, and A. Restrepo, "Localized measurement of emergent image frequencies by gabor wavelets," *IEEE Trans. Info. Theory*, 38:691-712, 1992.

P. Maragos and A. C. Bovik, "Image demodulation using multidimensional energy separation," *J. Opt. Soc. Amer. A*, 12(9):1867-1876, 1995.

I. Kokkinos, G. Evangelopoulos, and P. Maragos, "Advances in texture analysis: Energy dominant components and multiple hypothesis testing," in *Proc. IEEE ICIP*, 2004.

Y. Meyer, "Oscillating Patterns in Image Processing and Nonlinear Evolution Equations", University Lecture Series Vol. 22, AMS 2002.

L. Vese and S. Osher, "Modeling textures with total variation minimization and oscillating patterns in image processing," *J. Sci. Comp.*, 19:553-572, 2003.

F. Meyer and P. Maragos, "Nonlinear scale-space representation with morphological levelings," *J. Vis. Commun. & Im. Repr.*, 11:245-265, 2000.