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RESEARCH ARTICLE

The universal density of measurement

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Abstract The notion of measurement plays a central role in human cognition. We measure people's height, the weight of physical objects, the length of stretches of time, or the size of various collections of individuals. Measurements of height, weight, and the like are commonly thought of as mappings between objects and dense scales, while measurements of collections of individuals, as implemented for instance in counting, are assumed to involve discrete scales. It is also commonly assumed that natural language makes use of both types of scales and subsequently distinguishes between two types of measurements. This paper argues against the latter assumption. It argues that natural language semantics treats all measurements uniformly as mappings from objects (individuals or collections of individuals) to dense scales, hence the Universal Density of Measurement (UDM). If the arguments are successful, there are a variety of consequences for semantics and pragmatics, and more generally for the place of the linguistic system within an overall architecture of cognition.

Keywords Number · Degrees · Implicature · Exhaustivity · Comparatives · Negative Islands

1 Introduction

It is common to assume that the semantics of natural language makes reference to a notion of degree or quantity. For example, a standard formulation of the claim that Mary is taller than John involves reference to two degrees corresponding to Mary's height and

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to John's. Such a formulation motivates a domain of degrees that can be compared to each other to determine which is greater, i.e., a (totally) ordered domain, or a scale.

A great deal of literature in natural language syntax and semantics argues that this is the correct formulation and attempts to determine the properties of the relevant scales. The goal of this paper is to contribute to this literature by arguing that scales are always dense. More specifically, this paper has two goals. The first is to argue that scales of height, size, speed, and the like are dense. This, although not universally accepted, seems rather intuitive. If Mary is taller than Bill, there has to be a degree of height that is somewhere between Mary's height and Bill's. Degrees of height correspond to our spatial intuitions, which likely involve the notion of a continuous substance. Nevertheless, this assumption is not necessary. We will argue that various puzzles relating to the theory of exhaustivity, scalar implicatures, and the semantics of questions and definite descriptions can be resolved in a principled way under our intuitive claim that the relevant domains are in fact dense.

The second goal is to argue for a radical extension of this claim to *all* degree domains. In particular, we will defend the unintuitive claim that these domains are dense even when they are commonly thought to be discrete. In other words, when we say that John has 3 kids or that he has more kids than Mary, the presupposed scale (according to this unintuitive claim) is not the ordered set of natural numbers or anything like it. Instead, it is the same domain of measurements that is needed to capture our intuitions of space and time, something closer to the rational or real numbers.

The intuitive claim has already been discussed in the literature, in the context of the semantics of sentences such as *John is taller than Bill is*. Various proposals building on von Stechow (1984) assume that the semantics of this statement makes crucial appeal to the *maximal* degree of height that Bill possesses (see Heim, 2001). In particular, the sentence is claimed to assert that John's height is above that maximal degree. Pinkal (1989) challenges this assumption on the basis of the fact that sentences such as *John is taller than he has ever been before* are acceptable and have a coherent interpretation (see also Artstein, 1998). He argues that this is unexpected under von Stechow's view.[1] According to von Stechow, if the sentence is to be true, John's height at the present has to be greater than the maximal degree d such that John has been d tall at a point in time prior to the present. But there couldn't be such a maximal degree, at least not if John's growth is assumed to be continuous. Therefore, the sentence shouldn't receive a coherent interpretation.

Pinkal's argument is based on the assumption that the domain of degrees of height (and the domain of times) is dense. This assumption, while intuitive, is not independently supported in Pinkal (1989). Von Stechow might therefore simply respond to this challenge by denying the density assumption.[2] We will not engage in this particular debate. Our goal, instead, will be to provide a related, though independent, argument that the relevant domain of degrees is formally dense.[3]

[1] Pinkal argues therefore for an alternative analysis, one that involves universal quantification. See Heim (2003) for the comparison of various alternatives.

[2] Kai von Fintel has pointed out to us the potential relevance of a debate between Stalnaker and Lewis about the semantics of counterfactuals. A similar claim to Pinkal's was made by Lewis, and a similar response to the one that seems available for von Stechow is made by Stalnaker. For detailed discussion see Bennett (2003).

[3] Interestingly, the consequences for the von Stechow Pinkal debate will be inconclusive given the modularity claim that we will end up making. See Sect. 5.4.

Suppose that natural language has operators that take degree properties and return their maximal member (maximality operators). Pinkal expects that the system will fail when such operators combine with properties of the sort that he identified (e.g. *λd. John has never been d tall before*). This expectation—which leads to the conclusion that the comparative construction does not make use of a maximality operator—is justified only if the relevant degree domains are dense and Pinkal is right in assuming continuity of growth.

A more direct way to test the expectation will be available if we identify various constructions that can be argued to involve a maximality operator on independent grounds. This will be our strategy. We will start with properties that, like Pinkal's, apply to members of intuitively dense scales. We will construct the properties in a way that will ensure that they have no maximal member if and only if intuitively dense domains are in fact dense. The crucial observation will be that the combination of such properties with maximality operators yields systematic failure, an observation that will provide strong support, we hope, for our claim about intuitively dense domains. Second, we will look at parallel constructions that are traditionally analyzed as involving the non-dense domain of cardinalities. We will show that these constructions display a virtually identically pattern, which is explained under our radical extension of the density assumption, and is mysterious otherwise.

We will argue for our two claims in various ways. We will start in Sect. 2 with a puzzle pertaining to implicatures and the semantics of *only* in intuitively discrete domains and explain why the puzzle disappears once we think of the domains as dense. We discuss additional empirical consequences studying in tandem both the intuitively discrete and dense domains. We then move in Sect. 3 to a discussion of apparent constraints on *wh*-movements which, following Rullmann (1995) and von Stechow (1984), we analyze as syntactically well-formed but semantically incoherent (due to a violation of a maximality requirement). In contrast to these authors, however, we argue that incoherence is a result of density along the lines outlined above. The two sets of problems (from Sects. 2 and 3) are shown in Sect. 4 to have a common logical structure. This leads us to new predictions that we argue are corroborated in a variety of contexts. Finally, we draw various consequences from our proposal for the architecture of the linguistic system and the nature of interactions between semantics and pragmatics.

If our arguments are correct, they have potential consequences that go beyond the core linguistic system. In particular, a possible connection is suggested with work in cognitive science which aims to discover the origins of human reasoning about quantities and numerosities. One recurring hypothesis that arises in that context is that prior to the development of adult arithmetic there is a core system that allows the measurement (or at least the estimation) of quantities, but crucially does not have access to anything like the notion of a natural number. If this hypothesis is correct, it will be interesting to investigate whether the postulated core system could possibly be the system at the heart of the semantics of degree constructions. Needless to say, an investigation of this question goes beyond the scope of this paper.

2 Implicatures and Only

When the sentence in (1) is uttered, a belief that John doesn't have 4 children is normally attributed to the speaker.

(1) John has 3 children.

This attribution is standardly accounted for by a mechanism that generates pragmatic inferences, in this case the scalar implicature that (the speaker believes) that John doesn't have 4 children.

One might expect that a similar implicature will arise when the sentence in (2) is uttered, i.e. the scalar implicature that John doesn't have more than 4 children.

(2) John has more than 3 children.

However, this expectation is not borne out. As discussed in Krifka (1999), a sentence with a numeral determiner *n* generates a scalar implicature, but a sentence with the comparative determiner *more than n* does not.[4]

In responding to this puzzle one might hope to capitalize on the relative complexity of comparative constructions. Specifically, one might suggest that the use of a ''complex'' comparative invites the inference that the use of an alternative ''simpler'' construction was impossible (maxim of brevity).[5] In particular, the idea might be that (2) doesn't have the implicature that John has exactly 4 children because there is a simpler (briefer) way to convey that information. However, consider an utterance of the following sentences in a context in which it is known that Bill has exactly three children.

(3) a. John has more children than Bill does. (Context: Bill has 3 children.)
*Implicature: John has exactly four children.

b. John has two more children than Bill does.
Implicature: John has exactly 5 children. (Context: Bill has 3 children.)

By parity of reasoning (maxim of brevity), if it is known that Bill has exactly 3 children, the assertion in (3)b should not trigger the implicature that it in fact does. Similarly, although it is somewhat artificial to say that John has 2 more children than 3, to the extent that it is possible, it can convey the information that John has exactly 5 children.

[4] Krifka discusses another problem namely that the corresponding *at least* sentence (*John has at least 3 children.*) lacks an implicature as well. In fact, on initial inspection the following constructions seem to be quite similar:

(1) a. John has more than 3 children.
b. John has at least 4 children.
c. John has 4 or more children.
d. John has no fewer than 3 children.

Our account does not extend to (b, c), though with a modification it does extend to (d), see Fox (in progress). This is of course a challenge for the account. However, we think there are good arguments in favor of independent accounts of (b) and (c), see Fox (2004: class 5, in progress), and for additional arguments pertaining to (b) (see Geurts and Nouwen, 2005), as well as Hackl (2000). For an alternative perspective, see Spector (2006).

[5] Although see Matsumoto (1995) on the relative scarcity of inferences based on the maxim of brevity.

We therefore think that an alternative account is needed. We suggest that the account should follow from the principles that explain the contrast between (4) and (5).

(4) John has very few children. He only has 3_F (children).[6]

(5) *John has very few children. He only has more than 3_F (children).[7]

The sentence in (4) provides us with a particular paraphrase of the scalar implicature that the sentence in (1) has. This exemplifies a fairly general fact, pointed out in Fox (2004), building on earlier observations and proposals made by Groenendijk and Stokhof (1984) and Krifka (1995). It is generally true that the scalar implicature of a sentence can be stated explicitly with the use of a focus particle *only* that associates with the relevant scalar item:

(6) The *only* implicature generalization (OIG): Utterance of a sentence, *S*, as a default, licenses the inference/implicature that (the speaker believes) *only S′*, where *S′* is (a minimal modification of) *S* with focus on scalar items.

The sentence in (5) is a failed attempt to state a parallel implicature for the sentence in (2). We suggest that an account of the failure of (5) should yield an explanation of the missing implicature in (2).

But what might account for the unacceptability of (5)? A possible answer is suggested when we look at the sentences in (7). The sentence in (7)a, just like (1), generates a scalar implicature, in this case the implicature that John only weighs 120 pounds. The comparative construction in (7)b, just like (2), lacks a scalar implicature, and again, an attempt to describe the would-be-implicature with the focus particle *only* yields an unacceptable sentence, (7)c.

(7) a. John weighs 120 pounds.
b. John weighs more than 120 pounds.
c. *John only weighs more than 120_F pounds.

[6] F-subscript represents the presence of focus, which is, in turn, associated with prosodic prominence.

[7] All of the effects discussed in this paper do not arise when the context specifies a discrete set of *relevant* alternatives. For example:

(1) A: How many points did Iverson score last night?

B: I don't know.

A: Was it more than 10, more than 20, more than 30, or more than 40.

B: He (only) scored more than 20_F (points)

The version of B's final reply which contains *only* is acceptable (in contrast to (5)) and the version without *only* yields an implicature (in contrast to (2)). This is reminiscent of Kroch (1989), where it is demonstrated that such context specification (D-linking) circumvents negative islands. This will be directly relevant to the discussion in Sect. 3 and will be addressed in Sect. 5.5. The basic idea, as one might expect, will be that contextual specification allows for a richer syntactic representation in which a pronoun refers to a non-dense domain of alternatives. (Cf. Westersthal, 1984; von Fintel, 1994), and much subsequent work.

But in this case, the explanation of the facts is readily available: (7)c presupposes that John weighs more than 120 pounds, 120 + ε pounds;[8] John, therefore weighs more than 120 + $\varepsilon/2$ pounds, and, hence, there is a degree, *d*, greater than 120 such that John weighs more than *d* pounds. If this degree is relevant for the meaning of (7)c, if it is in the domain of quantification for *only*, the unacceptability of the sentence is expected. In other words, if we make the intuitive assumption that the set of degrees is dense in this case (from which it follows that 120 + $\varepsilon/2$ is a member of the set) the presupposition of (7)b ensures that the sentence will never be true, and this, we suggest, is a plausible explanation for unacceptability.[9]

It is also obvious why (7)b lacks an implicature. The implicature, had there been one, would have been the implicature that for any degree, *d*, greater than 120, the speaker believes that the claim that John weighs more than *d* pounds is false. In other words, (7)c would itself be the implicature of (7)a. However, the speaker asserted that John weighs more than 120 pounds, and this assertion, given the density of degrees, is incompatible with the implicature.

But how could we get this explanation to carry over to the facts in (5) and (2)? In (7), it is quite intuitive to assume that the set of degrees relevant for the evaluation of the sentence is dense. But a similar assumption seems quite radical for (5) and (2). Nevertheless, our goal is to argue that it is correct. I.e. we will argue for the following hypothesis about degree scales:

(8) The Universal Density of Measurements (UDM): measurement scales needed for natural language semantics are *always* dense.

Let us start with a discussion of the implications of the UDM for (5). Suppose that, despite appearances, the set of degrees that *only* quantifies over is dense even

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8 For convenience, we assume the treatment of *only* due to Horn (1969), who argues that a sentence with *only* presupposes the truth of the corresponding sentence without *only*. However, this assumption is not crucial for anything we're saying. If the presupposition is taken to be part of the assertive content, a similar type of incoherence is predicted. We also talk here as if *only* quantifies over degrees, rather than alternatives, though translating back and forth is quite trivial. See note 35. Furthermore, we are ignoring interesting questions about the semantics of *only*. In particular, the lexical-entries we are considering in this paper are all too strong for a variety of constructions, and it is important to ensure that the correct weakening will be consistent with our account. We think that there are a variety of possibilities, but since the issue is quite complex, we will not enter it here. The interested reader could verify that the following lexical entry from Spector (2006), based on Groenendijk and Stokhof (1984) and van Rooy and Schultz (2004), would be consistent with our account:

(1) $[[\text{Exh/only}]]\ (A_{\langle st,t\rangle})(p_{st}) \Leftrightarrow p(w)\ \&\ \neg\exists\ w' \in \mathrm{A}[p(w')\ \&\ (w' <_{\mathrm{A}} w)]$

$$w' <_A w \Leftrightarrow \{p \in A\colon p(w') = 1\} \subset \{p \in A\colon p(w) = 1\}$$

9 See Stalnaker (1974), and much subsequent work, for a model of communication that prohibits such conflicts.

It is a recurring hypothesis that syntactically well-formed sentences should be unacceptable when they are contradictory. However, since clearly not all contradictions are judged unacceptable, a semantic explanation of the unacceptability of sentences like (7)c that appeals to its contradictory nature requires also a way of distinguishing between contradictions that are ruled out by the grammar and those that are not. See Sect. 5 for further discussion drawing on Gajewski (2002, 2003), and Chierchia (1984).

in this case. (5), thus, asserts that for any degree *d* greater than 3, John doesn't have more than *d* children (see Hackl 2000). Without the UDM, there would be no problem. The set of degrees relevant for evaluation would be, as is standardly assumed, possible cardinalities of children (i.e., 1, 2, 3,…). The sentence would then assert that John doesn't have more than 4 children. Since it presupposes that John has more than 3 children, it would end up conveying that he has exactly 4. If density is assumed, however, we get exactly the same contradiction that we've seen in (7)c. The presupposition, of course, remains the presupposition that John has more than 3 children. However, the assertion would now not just exclude 4 as a degree exceeded by the number of John's children. It would also exclude any degree between 3 and 4. This is obviously in contradiction with the presupposition.

What about (2)? To account for the lack of an implicature, we will assume that the would-be implicature must be derived in the syntax with a covert exhaustive operator (*exh*) akin to *only* in its interpretation. (This operator is in principle optional and its presence yields implicatures.) What *only* and *exh* have in common, is the requirement that a particular proposition be stronger than a set of alternatives, and its employment would, thus, be impossible in (2) for the same reason that (5) is unacceptable.

The suggestion that implicatures are due to *exh* was advanced in Groenendijk and Stokhof (1984), Krifka (1995), and van Rooy and Schulz (2004). It has been defended in Fox (2004) based on evidence that the OIG (in (6)) is a better predictor of the distribution of scalar implicatures than the traditional/neo-Gricean account (but see note 39).

The data in (9)–(12) seem to provide additional support for our line of reasoning. In (9) and (10) (in contrast to (5) and (7)b) *only* is able to associate with the numeral of the comparative *more than n*.

(9) a. I can only say with certainty that John weighs more than 120_F pounds.
b. I can only say with certainty that John has more than 3_F children.

(10) a. I was only able to demonstrate that this refrigerator weighs more than 120_F pounds.
b. I was only able to demonstrate that this candidate received more than 500_F votes.

Similarly, in (11) and (12), in contrast to (2), the comparative *more than n* can generate an implicature.

(11) a. I can say with certainty that John weighs more than 120 pounds.
Implicature: I can only say with certainty that John weighs more than 120_F pounds.
b. I can say with certainty that John has more than 3 children.
Implicature: I can only say with certainty that John has more than 3_F children.

(12) a. I was able to demonstrate that this refrigerator weighs more than 120 pounds.
Implicature: I was only able to demonstrate that this refrigerator weighs more than 120_F pounds.

b. I was able to demonstrate that this candidate received more than 500 votes.
Implicature: I was only able to demonstrate that this candidate received more than 500_F votes.

All these facts are expected under the UDM given the presence of the modal operator. Specifically, even if *more than n*, and *only*, quantify over a dense domain, contradictions of the sort we've seen in (2), (5), and (7) do not arise in the present case.

To see this, let's focus on (9)a. One can presuppose it to have been demonstrated that x weighs more than 120 pounds, and subsequently assert consistently that there is no degree d, greater than 120, such that it has been demonstrated that x weighs more than d pounds. The reason for this is obvious: a demonstration that x weighs more than d pounds doesn't entail a demonstration that x weighs $d + \varepsilon$ pounds for some specific degree ε. In possible world semantics, this fact would be captured by assuming a dense set of worlds corresponding to the degrees. Once this is assumed, there could be for every ε a world consistent with the demonstration such that in that world John weighs less than $d + \varepsilon$ pounds. The fact that the (a) and the (b) sentences continue to behave identically lends further support to the universality of the UDM, i.e. to the claim that, contrary to traditional assumptions, the linguistic system treats the (a) and the (b) sentences on a par. However, there are further issues to discuss pertaining to the (b) cases to which we return in Sect. 5.

We've learned that the effect we are looking at cannot be attributed to a general inability of numerals inside comparatives to associate with *only* or to generate implicatures, since this effect can be ameliorated in the presence of a modal operator. However, it is also important to show that the effect is not ameliorated by any modal operator, but only by those operators that can eliminate the incoherence that is otherwise predicted under the UDM. An argument to this effect is provided by the following contrast:

(13) a. You're required to read more than 30 books.
Implicature: There is no degree greater than 30, d, s.t. you are required to read more than d books.
b. You're only required to read more than 30_F books.

(14) a. You're allowed to smoke more than 30 cigarettes.
*Implicature: There is no degree greater than 30, d, s.t. you are allowed to smoke more than d cigarettes.
b. *You're only allowed to smoke more than 30_F cigarettes.

In (13)a, the presence of a modal operator licenses an implicature and in (13)b it allows *only* to associate with the numeral 30. In (14), by contrast, an implicature is absent, (14)a, and association with *only* yields an incoherent interpretation, (14)b.[10] The difference between (13) and (14) can be traced to the difference between the

[10] The facts change for some speakers in a context in which it is presupposed that smoking a certain number of cigarettes is required, i.e., that there is a minimal amount that needs to be smoked. For the relevant speakers, (14)b can convey the information that 30 is the *minimal* number such that one is allowed to smoke *any* amount greater than that number. Such a reading is not in conflict with the UDM. See our discussion of (34) and (35).

semantic properties of the two modal operators *required* and *allowed*. If you are allowed to smoke more than 30 cigarettes, it follows that you're allowed to smoke 30 + ε cigarettes, for some degree ε. This consequence would contradict the potential implicature, which is equivalent to the sentence in (14)b. A similar consequence does not follow when you are required to read more than 30 books. If you are required to read more than 30 books, there need not be a degree ε, such that you are required to read 30 + ε books, and incoherence is avoided in exactly the manner we've discussed when we accounted for (9)–(12).

This intuitive difference can be traced to the fact that *required* is a universal modal and *allowed* is existential. If the presupposition of (14)b is met, i.e., if you are allowed to smoke more than 30 cigarettes, we can say that there is a world, w, (compatible with your requirements) and there is a degree ε, such that you smoke 30 + ε cigarettes in w. From this it follows (by the UDM) that in w, there is a degree greater than 30, 30 + $\varepsilon/2$, such that you smoke more than that degree of cigarettes in w. Two existential quantifiers can commute without affecting truth conditions. Hence there is a degree, d, greater than 30 such that you are allowed to smoke more than d cigarettes, and this is the negation of (14)b.

When you are required to read more than 30 books, we say that for *every* world, w, (compatible with your requirements) there is a degree ε, such that you read 30 + ε books in w. From this it follows (by the UDM) that in w, there is a degree greater than 30 (30 + $\varepsilon/2$) such that you read more than that degree of books in w. However, an existential and a universal quantifier cannot commute without effecting truth conditions. Hence, we cannot conclude that there is a degree, d, greater than 30 such that you are required to read more than d books. There could be a dense set of worlds corresponding to the density of the set of degrees (given the UDM), and the contradiction is avoided along the lines discussed above for the epistemic universal modals (*can-say-with-certainty, demonstrate*).

3 Negative islands: definite description and questions[11]

In the previous section, we have seen that the UDM accounts for the impossibility of embedding a comparative sentence under the operator *only* (when *only* associates with the degree expression). We have seen that the same reasoning can be extended to account for the lack of implicatures in comparatives under the assumption that implicatures are derived from constructions that contain a covert exhaustive operator. In this section, we would like to study the predictions of the UDM for additional operators with a similar interpretation.

3.1 Rullmann (1995)

Our starting point is the well-known puzzle that degree questions can't be formed by *wh*-movement across negation.[12] This is exemplified by the contrast between the question in (15) and in (16).

[11] This section is concerned with degree constructions. The new facts we report are problematic for existing accounts of the basic paradigm. However, it is not clear whether our account extends to other negative islands, and whether we are loosing important generalizations. In the future, we hope to investigate this in greater detail, in particular in comparison with Szabolcsi and Zwart (1993).

[12] Cf. Obenauer (1984), Rizzi (1990), etc.

(15) John didn't read many of these books?
Question: Which books did John not read?

(16) John doesn't weigh 190 pounds.
Question: *How much does John not weigh?[13]

Rullmann (1995) suggests a very interesting explanation for this contrast. He suggests that there is nothing in syntax that rules out the question in (16). The sentence is generated by the syntax but ruled out by the semantic component as a question that cannot have an answer.

To understand Rullmann's proposal consider a run-of-the-mill degree question, such as (17). If John weighs exactly 150 pounds, then for any degree d smaller than 150, it is true to say that John weighs at least d pounds. Nevertheless 150 pounds would be the only acceptable answer to (17).

(17) How much does John weigh?

One could imagine pragmatic accounts of this fact along Gricean lines. Rullmann suggests, instead, that it should be captured in the semantics. Specifically, he suggests that degree questions ask for the largest degree that satisfies a certain property:

(18) How much/many φ?
What is the maximal degree d st. $\varphi(d)$?

The question in (17), for example, asks for the maximal degree d such that John weighs (at least) d:

(17′) How much does John weigh?
What is the maximal degree d such that John weighs (at least) d pounds?

Under Rullmann's proposal, this question has only one true answer in the scenario considered above, namely 150 pounds.[14]

This, as Rullmann points out (developing a claim that von Stechow, 1984), has made in a different context), could be the source of the unacceptability of the question in (16). If John weighs exactly 150 pounds then any degree above 150 is

[13] As discussed in Kroch (1989) and indicated already in footnote 7, questions of this sort are acceptable when the context provides a discrete set of alternatives. See Sect. 5.5 for discussion.

[14] Obviously the unacceptability of less informative answers is related to emergence of scalar implicatures. Consequently, the decision to account for the former in the semantics is related to the decision to account for the latter within the grammar.

a degree *d* such that John doesn't weigh *d* pounds. Hence, as long as the set of degrees corresponding to weights has no upper bound, there can be no maximal degree *d* such that John doesn't weigh *d* pounds, and the question, therefore, cannot have an answer.[15] This, Rullmann suggests, is sufficient grounds for unacceptability.[16]

Our goal is to argue for a modification of Rullmann's account that depends on the UDM. But before we get there, we would like to point out the contrast in (19), which provides further support for the basic line of reasoning.

(19) a. *I have the amount of water that you don't.
cf. I have the amount of water that you do.
b. I have an amount of water that you don't.

The unacceptability of (19)a can receive the same explanation that Rullmann gives for the unacceptability of the question in (16). Definite descriptions presuppose that the predicate they combine with has a maximal element in its denotation (Link 1983):

(20) *The* φ is defined only if there is a maximal object x st. $\varphi(x)$.
When defined, *the* φ refers to the maximal object x st. $\varphi(x)$.

This presupposition is exactly identical to the requirement that needs to be satisfied for a degree question to have an answer. The presupposition of (19)a, thus, cannot be met for the same reason that the question in (16) cannot be answered. The fact that (19)b is acceptable suggests that the syntactic object is not itself the source of unacceptability. Extraction of a degree argument across negation yields unacceptability only when combined with a maximality requirement. Since an indefinite comes with no maximality requirement, (19)b is acceptable.

3.2 Beck and Rullmann's challenge

Rullmann's (1995) account for the fact in (16) is unique in predicting the contrast in (19) and is therefore empirically preferable to competing accounts. More importantly, perhaps, it is a principled account that does not rely on assumptions that are not needed on independent grounds. It relies on the meanings of lexical entries, but those need to be specified anyhow.

However, Beck and Rullmann (1999) made an observation that casts serious doubt on Rullmann's assumptions. In particular, Beck and Rullmann pointed out

[15] The assumption that the weight scale has no upper bound seems natural. However, it is not essential for the account. If there is an upper bound, Rullmann could say that a question such as the one in (16) cannot play the communicative role that questions are designed for; the answer is known in advance to be the upper bound, and the question, therefore, cannot serve to seek information. Note, however, that a little more would have to be said to account for the unacceptability of the definite description in (19)a.

[16] Of course, some questions that don't have an answer are, nevertheless, acceptable, e.g. *What is the largest natural number?* Rullman's account, as well as our modification, implies that the grammar distinguishes between two types of unanswerable questions (parallel to its distinction among two types of contradiction). See Sect. 5 for discussion as well as Appendix 3.

that if Rullmann's (1995) assumptions about the meaning of degree questions are maintained, (21), just like the question in (16), should be unacceptable.

(21) How much flour is sufficient to bake a cake?

If exactly 3 pounds of flour are sufficient to bake a cake then any degree *d* above 3 pounds will suffice as well. Hence, there cannot be a maximal degree *d* such that *d*-much flour is sufficient to bake a cake. This is true for precisely the same reason that there cannot be a maximal degree *d* such that John does not weigh *d* pounds. However, (21) is acceptable. Moreover, the correct answer to (21) would specify the minimal amount of flour sufficient to bake a cake, rather than the maximal.

Beck and Rullmann suggest the following characterization as a unifying statement of what constitutes a good answer to a (degree) question: A (degree) question of the form *How many/much d* $\varphi(d)$ should be answered by the most informative degree *d* that satisfies φ (where *d* is the most informative degree satisfying φ if *d* satisfies φ and for every d' that satisfies φ the proposition that *d* satisfies φ is more informative than the proposition that d' satisfies φ).

If, following Rullmann (1995), this statement is to be captured in the semantics of degree questions, we would have to modify (18), repeated in (22)a. One modification that suggests itself is given in (22)b.[17]

(22) How much/many φ?
  a. Rullmann (1995):
     What is the maximal degree *d* st. $\varphi(d)$?
  b. modification based on Beck and Rullmann (1999):
     What is the degree *d* that yields the most informative among the true propositions of the form $\varphi(d)$?[18]

The modification accounts for the meaning of (21). However, this comes at the expense of loosing the principled account of the contrast in (19) and of the unacceptability of the question in (16). For example, at the moment one would expect (16) to be an acceptable question requesting the identification of the minimal degree d such that John doesn't weigh *d* pounds.

In the next section, we will argue that once the consequences of the UDM are factored in, a principled account very much in the spirit of Rullmann (1995) is possible after all. But before we get there, we would like to point out an observation due to von Fintel, Fox, and Iatridou (in progress). This observation suggests that if such an account were possible for the question in (16), it would carry over to the definite description in (19).

Consider the definite description in (23). The standard semantics of the definite article due to Link (1983) predicts that this definite description would be unacceptable for the same reason that Rullmann predicts that (21) should be unacceptable.

[17] Another possible modification is already present in Groenendijk and Stokhof (1984). Such a modification, however, is not going to allow us to develop our Rullmann-like account of the constraints on degree questions we are concerned with.

[18] This modification would follow from Dayal's (1996) more general approach to the semantics of questions. See Appendix 3 for complications and a possible amendment.

(23) I have the amount of flour sufficient to bake a cake.

The definite description in (23) will always yield a presupposition failure since there could never be a maximal amount of flour sufficient to bake a cake. In order to deal with this and similar observations, von Fintel, Fox and Iatridou suggest a semantics for the definite article very much like the modification of Rullmann for questions that we presented in (22)b. From this semantics it follows that a definite description should have the following meaning:[19]

(24) *The* φ is defined only if there is a unique individual x such $\varphi(x)$ is a maximally informative proposition among the true propositions of the form $\varphi(x)$.
When defined *the* φ refers to the individual x st. $\varphi(x)$ is the maximally informative true proposition of the form $\varphi(x)$.

This semantics is of course in exactly the same predicament as the modification of Rullmann presented in (22)b. Both have the advantage of providing a unified account for the variation between a maximality and a minimality effect. This variation is determined in a predictable way from the overall semantics of the relevant property (in particular its monotonicity).[20] Both, however, render Rullmann's original explanation of the negative island effects in definites and questions [(16) and (19)a] unavailable.

The goal of the remainder of this section is to argue that the UDM allows us to capture Rullmann's original insight while incorporating the necessary changes in the semantics of questions and definite descriptions. We will do this in two stages (following the logic outlined in the introduction). We will start in 2.3 by focusing on domains where the account is fairly intuitive—because the domains are intuitively dense. We will then move to domains where the account is unintuitive in the sense that our account of the lack of an implicature in (2) and the unacceptability of (5) was initially unintuitive. Once again, we argue that the facts motivate the assumption that—despite initial appearances—all degree domains are formally dense (i.e. treated as dense by the grammar).

[19] The required lexical entry is the following:

$[[\text{the}]] = \lambda P_{\langle e,st\rangle}.\lambda w{:}\exists x[P(x)(w) = 1 \text{ and } \forall y \neq x\ (P(y)(w) = 1 \ \rightarrow\ \mathrm{P}(x) \text{ asymmetrically entails } P(y))].$
$(\iota x)[P(x)(w) = 1 \text{ and } \forall y \neq x\ (P(y)(w) = 1 \ \rightarrow\ P(x) \text{ asymmetrically entails } P(y))].$

Where $\lambda\chi{:}\psi(\chi).\phi$ is a function defined only for objects of which ψ is true (convention from Heim and Kratzer, 1998).

[20] If φ is a property of degrees or individuals that is upward monotone (in the following sense: $x > \mathrm{y}$ iff $\varphi(x)$ is more informative than $\varphi(y)$), then *the* φ according to (24) would pick up the maximal individual x such that $\varphi(x)$, when defined. Similarly, *how many/much* φ? would ask for the identity of this maximal individual. Conversely for downward monotone properties ($x < y$ iff $\varphi(x)$ is more informative than $\varphi(y)$): if φ is such a property, *the* φ would refer to the minimal individual x such that $\varphi(x)$, and *how many/much* φ? would ask for the identity of this minimal individual. Von Fintel, Fox, and Iatridou argue for their semantics based on paradigms in which the monotonicity of properties is varied. For these paradigms, Link's lexical entry and the alternative that we provided in footnote 19 make different predictions.

3.3 Negative islands and dense domains

Rullmann (1995) suggested that degree questions (and definite descriptions) are sensitive to negative islands because negation stands in the way of satisfying a maximality requirement. However, we have seen that the maximality requirement cannot be right in exactly the way stated by Rullmann. This appeared to be a problem for Rullmann's original account. Nevertheless, we will now see that, once the density of the relevant degree domains is taken into account, an alternative emerges. Also in this alternative maximality is the major player, but since the notion of maximality has changed, there will be quite a few empirical differences.

Consider again the question in (16) repeated below.

(25) *How much does John not weigh?

If the maximality requirement is modified as in (22)b, (25) should ask for the maximally informative proposition among the true propositions of the form *John does not weight d-pounds*. If John does not weigh 190 pounds then it follows that for any degree *d* bigger than 190, John does not weigh *d* pounds. In other words, propositions of the form *John does not weigh d-pounds* become more informative the smaller *d* is. Hence the most informative proposition of this form is the proposition that John does not weigh *d* pounds where *d* is the minimal degree *d* for which this proposition is true.

Given the discussion in Sect. 2, it is probably clear where we are going. There is no minimal degree that yields a true proposition of the relevant form, since the relevant set of degrees is dense. To see this, suppose that John weighs exactly 150 pounds. This means that for any degree *d* in the set of degrees greater than 150, John doesn't weigh *d* pounds. From density it follows that there is no minimal member in this set. Hence there is no most informative answer to the question, and Rullmann's account can be maintained.

Exactly the same reasoning accounts for the unacceptability of (19)a. However, there are further predictions. One useful way to think of these predictions is to compare our maximality account to an account that assumes a syntactic ban on extraction across negation. As we will see, there are cases where a question or a definite description meets the relevant maximality requirement despite the fact that negation is crossed by the *wh*-operator. These cases involve a negative island, [+NI], but do not violate the maximality requirement, [−MV]. Conversely, there are cases where the maximality requirement is violated despite the fact that negation is not crossed by the *wh*-operator, [−NI, + MV]. Because maximal informativeness is the driving force in our account, we predict that cases of the first kind should be acceptable (despite the fact that a *wh*-operator crosses negation) while cases of the second kind shouldn't be (even though a *wh*-operator does not cross negation). In short, we can distinguish our account from the syntactic alternative by the following predictions:

(26) a. Prediction 1: [+NI,−MV] cases should be acceptable.
b. Prediction 2: [−NI,+MV] cases should be unacceptable.

We will attempt to corroborate both predictions.

Let's start with prediction 1. We have already seen one piece of evidence that the prediction is correct. The indefinite in (19)b involves a negative island without a maximality violation and is acceptable as predicted. But we are now in a position to conduct additional tests.

Consider the data in (27) originally noted in Kuno and Takami (1997).[21] Despite the fact that a *wh*-operator is extracted over negation in each case ([+NI]), all examples are acceptable.[22]

(27) a. How much are you sure that this vessel won't weigh?
b. How much radiation are we not allowed to expose our workers to?
c. The amount of radiation that we are not allowed to expose our workers to is greater than we had thought.
d. The amount of money that you are sure that this stock will never sell for is quite high.
(Are you sure that your estimation is correct?)

Notice further that in all four examples it is the minimal degree that plays a prominent role; it is the referent of the definite descriptions in (27)c and d and the object whose identity is to be specified if the questions in (27)a or b are to be answered. Consider (27)a. With this question the addressee is asked to specify an upper bound on possible weights of the vessel. But among the various upper bounds that she could specify, which one does she have to specify in order to satisfy the speaker? Of course it is the *minimal* upper bound: the minimal degree d such that the addressee is sure that the vessel will not weigh d pounds.

These are all [−MV] cases even when density is assumed and their acceptability is therefore predicted. To see this, let's continue with our discussion of (27)a. Even if the domain of degrees is dense, as we are assuming, there could be a minimal degree d such that the addressee is sure that the vessel won't weigh *d* pounds. Even though there can be no minimal degree *d* such that the vessel doesn't weigh d pounds, there could be a minimal upper bound to possible weights.

We can make sense of this fact using possible world semantics as we did in Sect. 2. We take it to be necessary that there is a degree d such that the vessel weighs exactly *d* pounds. In other words, the set of degrees *d* such that the vessel weighs at least *d* pounds is necessarily a closed interval. Consequently, given density, the complement set is necessarily an open interval, which cannot have a minimal member. However, the set of degrees *d* such that there is *certainty* (on someone's part) that the vessel doesn't weigh at least *d* could be a closed interval with a minimal member. To see this, consider the fact that this set could also be described as the set of degrees *d* for which there is no possibility that the vessel weighs *d* pounds. This set is the complement set of the set of degrees for which there is a possibility that the vessel weighs *d* pounds and this set in turn could be an open interval. For example, it will be an open interval if there is a degree *d* such that the vessel can't weigh *d* pounds, but for any smaller degree *d′*, there

[21] Thanks to Maria Luisa Zubizarreta for reminding us of this paper.

[22] To see that (27)a is acceptable, imagine a context in which it is obvious that the addressee does not know how much the vessel will weigh but she can put an upper bound on possible weights. (Imagine for example a freighter with an amount of cargo that varies over time and cannot be totally determined in advance.) The question in (27)a could be presented to a logistics officer in order to figure out the cheapest way to protect the vessel from various hazards (assuming that the cost of protection is a function of weight).

is a world (in the modal base) in which the vessel weighs exactly d' pounds. We leave it to the reader to see that the same reasoning applies to (27)b–d.

Clearly the prediction we make for the examples in (27) depends on the introduction of a further operator in addition to negation, which allows the examples to be of the [−MV] type. But as we have seen in Sect. 2, not any operator can do the job. To see that the same holds here, consider the following pairs.

(28) a. How much radiation is the company not allowed to expose its workers to?
b. *How much food is the company not required to give its workers?[23]

(29) a. How much radiation is the company required not to expose its workers to?
b. *How much food is the company allowed not to give its workers?

We have already seen why the a-questions are [-MV] despite the density of the degree domains. We will now see that this is not the case for the corresponding b-questions.[24] Notice that (28)b and (29)b are equivalent. So it is sufficient to show that one of them cannot satisfy our maximality requirement. Suppose that there is an answer to (29)b. Let's say the answer is 3 *bowls of rice*. Assume that it is true that the company is allowed not to give its workers three bowls of rice. In other words, assume that there is an allowed world, *w*, in which the company doesn't give its workers 3 bowls of rice. Let's say that in *w* the company gives its workers exactly 3-ε bowls of rice ($0 < \varepsilon \leq 3$). But given density there is a more informative answer. In *w* the company doesn't give its workers 3-ε/2 bowls of rice, and the proposition that this is allowed is more informative. We see the same contrast with definite description and the account is of course the same.

(30) a. The amount of radiation that the company is not allowed to expose its workers to is very high.
b. *The amount of food that the company is not required to give its workers is quite high.

(31) a. ?The amount of radiation that the company is required not to expose its workers to…
b. * The amount of food that the company is allowed not to give its workers…

We can now move to test the second prediction in (26). If there are cases where a question or definite description doesn't meet the relevant maximality requirement despite the fact that negation is not crossed by the *wh*-operator ([−NI,+MV]), our

[23] The (b) sentences are acceptable on a reading which does not result from degree extraction across negation: *what is the amount of food, d, such that there is food x in amount d and the company is not required to give x to its workers?* To avoid this confound, we can compare the following:

(1) (When you enter the country) *How much money are you not allowed to have?*
(2) (When you enter the country) **How much money are you not required to have?*

[24] As we will see in Sect. 4, this contrast follows from exactly the same logic that was employed in our account of (13) and (14).

account makes a clear prediction: such cases should be unacceptable. We will show that one such case exists under a semantics that postulates a dense domain of time and that it is in fact unacceptable. Furthermore, we will employ a universal modal operator in the same way we did above to argue that density is indeed the crucial factor that determines the acceptability status of the relevant examples. Finally, we will use, once again, the definite/indefinite contrast to argue that maximality is the determining factor. In Sect. 4.2, we will discuss another ([−NI,+MV]) case, which directly relates to our discussion in Sect. 2.

Consider the fact in (32) noted in von Fintel and Iatridou (2003). One could imagine that there is a syntactic constraint that blocks extraction of a *before*-phrase. However, this will not be a negative island. Barry Schein (p.c.) suggests that the question is ruled out as a maximality violation and hence corroborates our prediction that [−NI,+MV] cases are unacceptable.[25]

(32) *Before when did John arrive?

If the domain of time is dense, just like the domain of degrees, and if *when*-questions require a similar semantics to *how-many*-questions, we have a straightforward explanation for the unacceptability of (32). To see this, assume that John arrived at 10 am.[26] It follows that every time after 10 am is a time *t* such that John arrived before *t*. The proposition that John arrived before 11 am is more informative than the proposition that John arrived before 11:30 am. Hence, under the assumption that maximal informativeness plays a role in the semantics of *when*-questions, the appropriate answer for (32) would specify the earliest time t such that John arrived before *t*. I.e. the earliest time after 10 am. However, if density is assumed, there is no such time.[27]

This account makes a further prediction, which is by now quite familiar. If we add a universal modal operator (in the appropriate place) to sentences such as (32), they will no longer constitute [+MV] cases. They will, therefore, be predicted to improve in status, a prediction verified in (33).

(33) Before when do you have to arrive?

To understand why there is no maximality violation in (33) it is sufficient to realize that even if the time line is dense, there could be a time *t* which is earliest in the set of times *t′* such that you *have* to arrive before *t′*. To see that there could be such a time, imagine that the rules which constrain your arrival time specify that it has to be before a certain time, say 10 am, but don't specify anything other than that. Under such a scenario, there is a most informative time such that you have to arrive before that time.[28]

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[25] Schein made his suggestion during the question period of a talk presented by Kai von Fintel and Sabine Iatridou at MIT. At the time, we were just beginning our work on the UDM.

[26] If John did not arrive at all, then of course the question does not have an answer.

[27] Notice that the expression we use in the text, *the earliest time after* 10 *am*, is not ill formed, although at first sight it should be, if density is assumed. We thank Philippe Schlenker for discussion of this issue, which we address in Sect. 5.4.

[28] In possible world semantics we would say that the set of worlds that conform to the rules is the set of worlds in which you arrive before 10 am. This is a set that corresponds to the dense set of times: for every time *t* before 10 am there will be a world *w* such that you arrive at (exactly) *t* in *w*.

The question in (34), which employs an existential modal in place of a universal, appears to be problematic for our generalization that existential modals are not capable of obviating a maximality violation. Consider (34), when addressed to a person who is staying at a Youth Hostel which is locked up over night, say at 10 pm. To get in, one has to arrive before 10 pm. (34) seems to be a natural way to ask for the time at which the entrance doors are locked.

(34) Before when are you allowed to arrive?

On closer examination, however, this is not problematic for our proposal. The presence of a modal operator makes a richer contribution to the semantics of (34) than that of simple existential quantification over worlds. Once the full contribution is taken into account, it turns out that an obviation of a maximality violation is predicted after all.

To see what is involved, consider the true answer to (34):

(35) You are allowed to arrive before 10 pm

This answer is much stronger than the statement that there is a world (compatible with the relevant rules) in which there is a time before 10 at which you arrive. The later statement would be true if you were allowed to arrive at 7 and no later (from which it follows that there is a time before 10 at which you are allowed to arrive). (35), however, would not be true in such a situation. (35) requires the rules to allow you to arrive at *any* time before 10 pm. The source of this ‘‘free choice’’ interpretation has been debated in the literature quite extensively. However, it is most likely not pertinent for our purposes. What is important is that the existence of the free choice interpretation allows the question to receive a (most informative) answer.

In our example, 9 pm is a time *t* such that you can arrive any time before *t*. 10 pm is such a time as well, and the proposition that this is the case is more informative. However, for any time *t* after 10 pm it is no longer true that you are allowed to arrive (any time) before it. 10 pm is therefore the latest such time and would furnish the maximally informative answer to (34).

Finally, the definite/indefinite contrast that we saw with degree extraction, (19), emerges in this case, as well, thereby providing additional support for our claim that the unacceptable cases with *before* are instantiations of [−NI,+MV]:[29]

[29] We can use definites with resumptive pronouns to constructs examples with differentials, similar to those discussed in Sect. 2:

(1) a. *I can predict that you will arrive at the time such that John will arrive before it.
b. I can predict that you will arrive at the time such that John will arrive exactly 20 sec. before it.

There are further tests that we might conduct, and quite a few questions that arise when density is assumed in the domain of time, which we hope to investigate in the future. Our goal here is to understand the consequences of the density assumption for the domain of degrees (the UDM). The domain of time serves a limited function from this perspective: it lends plausibility to the maximality account of negative islands, which constitutes one of our central arguments for the UDM.

(36) a. *I arrived at the time before noon.
b. ?I arrived at a time before noon.

3.4 On the universality of the UDM

In 2.2, we have seen evidence supporting our claim that intuitively dense domains are in fact dense. Specifically, the claim enabled us to account for what initially seemed to be a syntactic constraint against extraction over a negative expression. The constraint turned out to be an artifact of a maximality requirement along the lines of Rullmann (1995). However, the source of the maximality violation for us was very different. This allowed us to derive a host of new predictions concerning the types of modal expressions that should obviate the effect.

In this section, we would like to argue that this line of reasoning should be extended to all degree domains, even those that seem at first sight to be based on the non-dense notion of cardinality. This will serve as an additional argument (to the one presented in Sect. 2) in favor of the universality of the UDM.

It is well-known that the problem of extracting a degree *wh*-phrase across negation is not limited to intuitively dense domains. This can be illustrated by the unacceptability of (37).[30]

(37) *How many kids do you not have?

If we are right in our account of the parallel fact in dense domains, we are forced either to claim that the two types of facts have a different source or that (despite initial appearances) all degree domains are dense. Just as we did in Sect. 2, we argue for the second alternative based on the observation that the two types of constructions (corresponding to the intuitively dense and the intuitively non-dense domains) behave identically on a variety of tests, and that this otherwise mysterious behavior is explained under the UDM.

The facts in (38)–(41) are a direct consequence of the maximality requirement and density if the UDM is assumed, but are quite mysterious otherwise. The examples in (38) show that the constraint against extraction across negation exemplified in (37) is obviated when certain modals are introduced in the appropriate location, specifically the combination of modals and locations that can circumvent a maximality violation.

(38) a. If you live in China, how many children are you not allowed to have?
b. How many days a week are you not allowed to work (according to the union regulations)?
c. How many soldiers is it (absolutely) certain that the enemy doesn't have?

[30] We use the definiteness effect in possessive *have* constructions to insure that a degree expression is indeed extracted out of the scope of negation, i.e., in order to exclude a representation in which the degree variable starts out outside the scope of negation and an individual variable is present in the theta position (see footnote 23, as well as Heim, 1987; Frampton, 1991). In the French examples below we use a similar technique based on a particular construction that the language makes available.

Under the UDM, this receives an identical explanation to the one we've given for parallel examples involving intuitively dense domains, e.g., (27), (28)a, and (29)a.[31] Parallel facts in French involving the much discussed split *combien* construction are provided in (39) and (40). (See Obenauer 1984 and much subsequent work.)

(39) a. ???Combien Jean n'a-t-il pas lu de livres?
How many John n'has-he not read of books
b. ?Combien peux-tu me dire avec (absolue) certitude que Jean n'a pas lu de livres?
How many can-you me tell with (absolute) certainty that John has not read of books (Benjamin Spector, p.c.)

(40)	a.	*Combien	Jean	n'a-t-il	(pas)	d'enfants?
		How many	John	n'has-he	not	of children
	b.	?Combien	les chinois	ne peuvent ils	(pas)	avoir d'enfants?
		How many	the chineese	n'allowed-them	not	have of-children?

(Valentine Hacquard, p.c.)

Once again, the overall logic of the account is corroborated by the observation that only the modals that can circumvent a maximality violation will allow the relevant degree operator to cross negation:

(41) a. *If you live in Sweden, how many children are you not required to have?
b. *How many days a week are you not required to work (even according to the company's regulations)?
c. *How many soldiers is it possible that the enemy doesn't have?

4 Exhaustivity and density: a unified account[32]

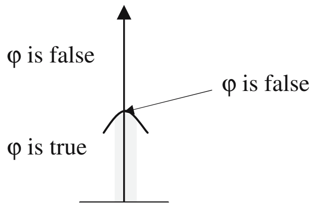
In the previous two sections we have seen 4 pieces of evidence for the UDM coming from the study of implicatures, *only*, questions and definite descriptions. In this section, we will see that there is a level of description under which all of these exemplify the same generalization. Once we see that, we will be able to draw additional predictions.

Let's start with a more formal characterization of the results from Sect. 2. Let φ be an upward monotone property of degrees (of type $\langle d,st\rangle$), where φ is upward monotone if for every d_1,d_2: $d_1>d_2$ iff $\varphi(d_1)$ is more informative than $\varphi(d_2)$ (asymmetrically entails it). Assume also that φ necessarily describes an open interval, i.e.

[31] The reader might have noticed a problem pertaining to the precise interpretation of (38)c. We will argue in Sect. 5, that this problem is identical to the one alluded to earlier pertaining to the truth conditions of (11)b and (12)b.

[32] We thank an anonymous reviewer for useful suggestions on how to make this section more reader friendly.

for every world w, $\lambda d.\varphi(d)(w)$ is an open interval. (There is a degree d such that $\varphi(d)(w)$=0 but for every degree d' smaller than d, $\varphi(d')(w)$=1.)[33] The following drawing represents what holds of φ in every world:



For every world w, it will be false to say that there is some degree d such that it is the maximal degree such that φ is true of that degree in w. Equivalently (given upward monotonicity), it will be false to say that there is a degree d such that $\varphi(d)(w)$ is true and $\varphi(d)$ is more informative than $\varphi(d')$ for every d' such that $\varphi(d')(w)$ is true.

These logical facts are at the heart of the account of our observations in Sect. 2. A simple property of degrees based on comparatives, e.g. λd. *John has more than d children*, is upward monotone. Assuming the UDM it also has to describe an open interval. Hence, it follows that a statement that there is a maximal (or maximally informative) degree that satisfies it would be infelicitous (whether it results from *only* or a covert exhaustive operator).

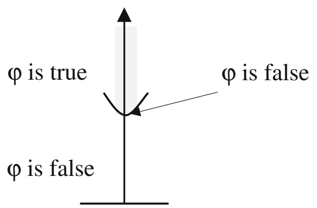
Furthermore, we can observe that if we append a universal modal operator, the modified property, $\lambda d.\ \Box\varphi(d)$, would no longer necessarily describe an open interval. The universal modal could quantify over a dense set of worlds that corresponds to the set of degrees with the result that the modified property describes a closed interval. For example, given a constant a, the modal base could consist of all worlds in which φ is true of some degree bigger than a, i.e. MB = $\{w: \exists d>a\ [\varphi(d)(w) = 1]\}$. [A useful aside: MB is precisely the deontic modal base that we would have in a situation in which the only requirement is that $\varphi(a)$ holds. Suppose that $w \in$ MB. Given upward monotonicity, $\varphi(a)(w)$=1. Conversely, suppose $\varphi(a)(w) = 1$. Since φ necessarily describes an open interval, a can't be the maximal element in its extension. Hence, there is some $d > a$ such that $\varphi(d)(w)$=1. Hence $w \in$ MB.]

Under MB, $\lambda d.\Box\varphi(d)$ will describe the closed interval I = $[0, a]$. To see this, suppose first that d' is in I. Let w $\in$ MB. Hence, there is a $d>a$ such that $\varphi(d)(w)$=1. Since $d>d'$ and φ is upward monotone, $\varphi(d')(w) = 1$. This is true for every $w \in$ MB. Hence, $\Box\varphi(d') = 1$. Suppose next that d' is not in I (i.e. $d' = a + \varepsilon$). Since $\varphi(d')$ asymmetrically entails $\varphi(a+\varepsilon/2)$, there is a world, w, in MB, s.t. $\varphi(d')(w) = 0$. Hence $\Box\varphi(d') = 0$. This means that complex predicates over degrees such as 'λd. you are required to read more than d books' do not necessarily describe an open interval. And this, we suggest, accounts for the fact that such predicates can be the source of an implicature or a parallel sentence with *only*.

[33] For the sake of discussion, we assume that degrees are points on a scale rather than intervals. If degrees were intervals (as assumed in Kennedy, 2001), our predicate φ would describe an open interval of intervals. We also ignore the "pathological" case where φ is an empty interval in some world.

Appending an existential modal operator, on the other hand, is of no help: the property $\lambda d.\Diamond\varphi(d)$ necessarily describes an open interval. To see this, assume that $\lambda d. \varphi(d)$ describes a closed interval (in a world w) and that d' is its maximal member. Of course in w $\Diamond\varphi(d')$=1. This means that there is a world w' in the modal base such that $\varphi(d')(w')$=1. But since φ necessarily describes an open interval, there is a degree d'' bigger than d′ such that $\varphi(d'')(w')$=1. Hence, in w, $\Diamond\varphi(d'')$ =1 and d' is not the maximal member. This, of course, explains the fact that complex predicates over degrees such as 'λd. you are allowed to smoke more than d cigarettes' cannot support an implicature and are unacceptable with *only*.

Let's now move to a similar characterization of the results from Sect. 3. Let φ now be a downward monotone property of degrees (of type $\langle d,st\rangle$), where φ is downward monotone if for every d_1,d_2: $d_1>d_2$ iff $\varphi(d_2)$ is more informative than $\varphi(d_1)$. Assume also that φ necessarily describes an open interval, i.e., for every world w there is a degree d such that $\varphi(d)(w)$=0 but for every degree d' bigger than d $\varphi(d')(w)$=1. The following drawing represents what holds of φ in every world:



For every world w it will be false to say that there is some degree d such that it is the minimal degree such that φ is true of that degree in w. Equivalently (given downward monotonicity), it will be false to say that there is a degree d such that $\varphi(d)(w)$ is true and $\varphi(d)$ is more informative than $\varphi(d')$ for every d' such that $\varphi(d')(w)$ is true.

These logical facts are once again at the heart of our account. In Sect. 3, we talked about downward monotone predicates of degrees that (given the UDM) necessarily describe open intervals such as λd. *I don't have d children*. We pointed out that a statement that there is a minimal (or maximally informative) degree that satisfies such a property would be infelicitous. This time, the source of the infelicity was a question or a definite article.

Once again, we can observe that if we append a universal modal operator, the modified property, $\lambda d.\Box\varphi(d)$, would no longer necessarily describe an open interval. The universal modal could quantify over a dense set of worlds that corresponds to the set of degrees with the result that the modified property describes a closed interval.

The example we use to show this is basically the mirror image of the one we constructed for upward monotone predicates. Given a constant a, we define the modal base as the set of worlds in which φ is true of some degree smaller than a, i.e. MB = $\{w: \exists d<a\ [\varphi(d)(w)=1]\}$. [Again, a useful aside: MB is precisely the deontic modal base that we would have in a situation in which the only requirement is that $\varphi(a)$ holds. Suppose that $w \in$ MB. Given downward monotonicity, $\varphi(a)(w)$=1. Conversely, suppose $\varphi(a)(w)$=1. Since φ necessarily describes an open interval, a can't be the minimal element in its extension. Hence, there is some $d < a$ such that $\varphi(d)(w)$=1. Hence $w \in$ MB.]

Under MB, $\lambda d.\Box\varphi(d)$ will describe the closed interval $I = [a, \infty)$. To see this, suppose first that d' is in I. Let $w \in$ MB. Hence, there is a $d < a$ such that $\varphi(d)(w)$=1. Since $d<d'$ and φ is downward monotone $\varphi(d')(w)$=1. This is true for every $w \in$ MB. Hence, $\Box\varphi(d') = 1$. Suppose next that d' is not in I (i.e. $d' = a - \varepsilon$). Since $\varphi(d')$ asymmetrically entails $\varphi(a\text{-}\varepsilon/2)$, there is a world, w, in MB, such that $\varphi(d')(w) = 0$. Hence $\Box\varphi(d') = 0$. This means that complex predicates over degrees such as 'λd. you are required not to have d children' do not necessarily describe an open interval. And this, we suggest, accounts for the fact that such predicates can serve as arguments of question operators or definite articles.

Appending an existential modal operator is again of no help: the property $\lambda d. \Diamond\varphi(d)$ necessarily describes an open interval. To see this, assume that $\lambda d.\Diamond\varphi(d)$ describes a closed interval (in a world w) and that d' is its minimal member. Of course, in w, $\Diamond\varphi(d')$=1. This means that there is a world w' in the modal base such that $\varphi(d')(w')$=1. But since φ necessarily describes an open interval, there is a degree d'' smaller than d' such that $\varphi(d'')(w)$=1. Hence, $\Diamond\varphi(d'')$ =1 and d' is not the minimal member. This will then explain the fact that complex predicates over degrees such as 'λd. you are allowed not to work d days' which is equal to 'λd. you are not required to work d days' cannot serve as arguments of definite articles or question operators.

Of course there is a generalization here that pertains to all monotone properties of degrees (whether they are upward or downward monotone). When these properties describe open intervals in every possible world, we will call them necessarily open properties, or in short N-open properties. If a property is N-open (in the relevant direction: top for upward monotone and bottom for downward monotone, i.e. always at the more informative end), maximization should be impossible. This follows if maximization always makes use of a primitive, MAX_{inf}, which selects the most informative member of an interval.

(42) Constraint on Interval Maximization (CIM): N-open monotone properties cannot be maximized by $\text{MAX}_{\text{inf.}}$

(43) $\text{MAX}_{\text{inf.}}(\varphi_{\langle\alpha,st\rangle})(w) = (\iota x)[\varphi(x)(w)=1 \text{ and } \forall y(\varphi(y)(w)=1 \rightarrow \varphi(x)$ entails $\varphi(y))]$.

We take the CIM to be self-evident. Our paper uses it to argue for specific claims about the nature of degree domains in natural language. The fact that the CIM plays an explanatory role in accounting for the status of expressions in natural language can be taken as evidence that natural language has N-open properties, i.e. for our first claim that certain degree domains in natural language are dense. The fact that the CIM seems to be at work in all degree constructions (even those that putatively make reference to cardinality) constitutes our argument for the universality of the UDM.

But we can now see additional predictions. We have isolated four operators that make use of MAX_{inf}: the definite article, the question operator *how many/much*, *only* and the exhaustivity operator, *exh* (responsible for implicatures):[34]

[34] Later on in the paper, we talk about *only* and *exh* as focus sensitive operators that take a proposition p and a set of alternatives A (usually the focus value of its sister). This way of talking translates automatically to (44)a and b as long as $\exists x[x\in D_\alpha \;\&\; p=\varphi(x) \;\&\; \forall q[q\in A \rightarrow \exists y[y\in D_\alpha \;\&\; q=\varphi(y)]]$. Under such circumstances, $\text{only}_{\text{focus-sensitive}}(A)(p) = \text{only}(\varphi)(x)$. Both ways of talking will, of course, require room for further domain restriction. See Appendix 1 for discussion of domain restriction under a syntax that assumes the focus sensitive *only*.

(44) a. $[[\text{exh}]]\ (\varphi)(d)(w) \iff d = \text{MAX}_{\text{inf}}(\varphi)(w)$.
b. $[[\text{only}]]\ (\varphi)(d)(w) \iff d = \text{MAX}_{\text{inf}}(\varphi)(w)$, when defined.
c. $[[?]]\ \varphi = \lambda w{:}\exists d[d = \text{MAX}_{\text{inf}}(\varphi)(w)].\{\varphi(x)\text{: x} \in D_\alpha\}$.
d. $[[\text{the}]]\ (\varphi)(w) = \text{MAX}_{\text{inf}}(\varphi)(w)$, when defined.

These operators allow us to state four specific consequences of the CIM for three different environments:

(45) Basic Consequence:
If φ expresses an N-open monotone property of degrees, then the following should be unacceptable[35]
a. *exh $\varphi(d)$
b. *only $\varphi(d_F)$
c. *wh$_d$ $\varphi(d)$
d. *the $\lambda d.\varphi(d)$

(46) Consequence for universal modals:
A universal modal can close an interval, hence even if φ is an N-open monotone property of degrees, the following should be acceptable
a. exh $\Box\varphi(d)$
b. only $\Box\varphi(d_F)$
c. wh$_d$ $\Box\varphi(d)$
d. the $\lambda d.\Box\varphi(d)$

(47) Consequence for existential modals:
An existential modal cannot close an interval. Hence, if φ is an N-open monotone property of degrees, then the following should be unacceptable
a. *exh $\Diamond\varphi(d)$
b. *only $\Diamond\varphi(d_F)$
c. *wh$_d$ $\Diamond\varphi(d)$
d. *the $\lambda d.\Diamond\varphi(d)$

In Sect. 2 we saw evidence for the UDM under the assumption that a and b hold for upward monotone degree properties. In Sect. 3, we saw parallel evidence under the assumption that c and d hold for downward monotone properties. But we can now search for additional predictions. Specifically, we might find cases in which a and b hold for downward monotone properties or conversely cases in which c and d hold for upward monotone properties.

4.1 Implicatures, *only*, and negation

It is well known that scalar implicatures are predicted to "reverse" in downward entailing environments. (see Horn, 1972; Fauconnier, 1975; Levinson, 2000; Chierchia, 2004, among others). To see this, consider the scalar implicatures of the sentences in (48). (48)a, with the existential quantifier *some*, has the implicature that the corresponding sentence with the universal quantifier *all* is false. Similarly, (48)b,

[35] These are of course sloppy descriptions of syntactic representations which we hope are nevertheless useful.

which contains disjunction, has the implicature that the corresponding sentence with conjunction is false.

(48) a. John did some of the homework.
Implicature: John didn't do all of the homework
b. John talked to Bill or Sue.
Implicature: John didn't talk to both Bill and Sue.

In downward monotone environments, we get what appears to be the opposite pattern. (49)a, with the universal quantifier *all*, has the implicature that the corresponding sentence with the existential quantifier *any* is false.[36] Similarly, (49)b, which contains conjunction, has the implicature that the corresponding sentence with disjunction is false.

(49) a. John didn't do all of the homework.
Implicature: John did some of the homework.
b. John didn't talk to both Bill and Sue.
Implicature: John talked to one of the two.

This reversal is expected under traditional Neo-Gricean accounts, where an implicatures is a denial of a stronger alternative that the speaker could have uttered. For us, the reversal is derived on the basis of a covert *exh* which makes use of $\mathrm{MAX}_{\mathrm{inf}}$. *Exh* will be generated with matrix scope and will take the audible sentence as its complement (the prejacent). As mentioned, this operator asserts that the prejacent is the most informative true member of a set of alternatives. Assuming that a sentence with a corresponding scalar alternative is a member of the set, the implicature that this alternative sentence is false follows, as long as it is not weaker than the prejacent.

Furthermore, since an implicature involves the exclusion of strong alternatives, an alternative that is too weak to be excluded in an upward monotone environment will yield a strong (hence excludable) alternative in a downward monotone environment. From this perspective, it is somewhat surprising that the sentences in (50)a and b do not give rise to parallel implicatures.[37]

(50) a. John didn't smoke 30 cigarettes.
*Implicature: John smoked 29 cigarettes.
b. John didn't read 30 books.
*Implicature: John read 29 books.

We would like to argue for an explanation of this fact based on the UDM. Of course, this fact could also be explained if the crucial alternatives are somehow disqualified form the alternative set. More specifically, if it could be ensured that the scalar word 29 is not considered as an alternative to 30 (when computing the ''strong meaning'' of (50)), the implicature would not be predicted (thanks to Polly Jacobson, p.c.). However we don't see an obvious way to justify this move. It seems to us

[36] The corresponding sentence with the positive polarity item *some* is not grammatical.

[37] The observations in this section grew out of attempts to grapple with facts noted by Spector (2004).

that if 29 is excluded when the strong meaning of (50) is computed, the same should hold for 31 when computing the strong meaning of *John smoked* 30 *cigarettes*. In other words, we will not understand why *John smoked* 30 *cigarettes* has the implicature that it does, namely that John smoked no more than 30 cigarettes.[38]

The UDM provides an immediate account for our puzzle. For the sentences in (50) to have an implicature, *exh* would have to be employed. The resulting interpretation would be equivalent to the conjunction of the standard semantics of the prejacent and the assertion that all stronger alternatives are false. For example (50)a would make the assertion in (51).

(51) John didn't smoke 30 cigarettes and for all degrees d smaller than 30 John smoked d cigarettes.

(51) is of course contradictory if the UDM is assumed. As is by now familiar, if John didn't smoke 30 cigarettes, then the exact degree, d, of cigarettes that he smoked is below 30. By the UDM, there has to be a degree d' between 30 and d. Since John didn't smoke d' cigarettes, (51) cannot be true.

Under the UDM, this is just an instantiation of (45)a for downward monotone properties. The UDM also makes a prediction based on (46)a, namely that the problem with adding the exhaustive operator should disappear with the introduction of a universal modal. Consider (52) and (53).

(52) John is not allowed to smoke 30 cigarettes.
Implicature: John is allowed to smoke 29 cigarettes.

(53) John is required not to smoke 30 cigarettes.
Implicature: John is allowed to smoke 29 cigarettes.

These sentences are equivalent given the duality of *allow* and *require*, and, as is transparent in (53), they involve appending a universal modal to a downward monotone N-open property. Since the universal modal can close an interval, maximization is predicted to be possible and the implicature is predicted to be acceptable. These cases serve another purpose, namely they indicate that the scalar word 29 can be an alternative to 30 in a negative context, contrary to the competing proposal that we sketched above.

Finally, we make the prediction based on (47) that an existential modal will not obviate the CIM:

(54) John is not required to read 30 books.
*Implicature: John is required to read 29 books.

(55) John is allowed not to read 30 books.
*Implicature: John is required to read 29 books.

[38] For some reason that is not completely clear to us, the facts seem to change for some speakers when a non-round numeral is used. We don't understand this phenomenon, but hope that it can be made consistent with our proposal once the pragmatic considerations that enter into choosing a level of precision (e.g. Krifka, 2002) are taken into account—perhaps along the lines discussed in Sect. 5.5.

Parallel facts are predicted for *only* (instantiating the b cases in (45)–(47)). The predictions seem to go in the right direction.[39] In (56) we see that *only* can associate with scalar items across negation. These examples are not perfect (hence our marking %, which reflects considerable disagreement among speakers), but contrast markedly with the sentences in (57), which violate the CIM under the UDM. This contrast corroborates the predictions of the UDM based on (45)b.

(56) a. %John has only not done all_F of the homework.
b. %John has only not spoken to both_F Bill and Sue.

(57) a. *John has only not smoked 30_F cigarettes.
b. *John has only not read 30_F books.
c. *John only does not weigh 190_F pounds.

The addition of a universal modal, (59), and the move to the equivalent construction with the existential modal, (58), result in sentences that are quite good, by contrast.

(58) a. John is only required not to smoke 30_F cigarettes.
b. John is only required not to weigh 190_F pounds.

(59) a. %John is only not allowed to smoke 30_F cigarettes.
b. %John is only not allowed to weigh 190_F pounds.

Finally, the addition of an existential modal, (60), and the move to the equivalent dual, (61), are of no help.

(60) *John is only allowed not to read 30_F books.

(61) *John is only not required to read 30_F books.

4.2 Questions, definites, and comparatives

We now want to check the predictions of the UDM for upward monotone properties given (45) (c) and (d), specifically, the prediction that properties that are N-open under the UDM cannot be maximized by a question operator or a definite article. Furthermore we want to check that the effect will be obviated by the universal modal operator, (46), but not by the existential modal operator, (47).

[39] Not all the data are as clean as we would like them to be. We think that this might be due to an independent syntactic constraint on the positioning of *only*. This constraint, we think, disfavors the placement of *only* immediately to the left of negation, and is thus responsible for the status of (56) and (59). See, however, Spector (2004) for an argument that this constraint might capture additional facts that we derive from the UDM.

As we've seen the exhaustivity operator cannot be restricted by the same constraints. This is a counter-example to the OIG as it is stated in (6), but not to the account that it motivates, which appeals to an exhaustivity operator. The two operators (although similar) need not share *all* syntactic properties.

Our test for this prediction is unfortunately far from perfect, given independent constraints on the formation of the relevant questions and definite descriptions. Nevertheless, the facts seem to us to go in the right direction.

(62) a. *More than how many books did John read?
b. ??More than how many books does John have to read?
b. ??/*More than how many cigarettes is John allowed to smoke?

(63) a. *How many feet is John under?
b. ?How many feet do you have to be under to take this ride? (Steve Yablo, pc)

(64) a. *The amount of food such that you ate more than that (is greater than I had thought).
b. ?The amount of food such that we are required to eat more than that (is greater than we had thought).
c. *The amount of candy such that we are allowed to eat more than that (is greater than we had thought).

It is worthwhile to investigate whether there are languages in which the interfering constraints are not at work, for example, due to the availability of appropriate resumptive pronouns. If there are, the contrasts ought to be sharper. Hagit Borer (p.c.) suggested the following, as corroborating data:

(65) a. * ?axalti ?et kamut ha-?oxel Se ?ata ?axalta yoter mi-mena.
ate-I ACC amount the-food that you ate more than-from-it
I ate the amount of food, d, such that you ate more than d.
b. ?axalti ?et kamut ha-?oxel Se carix le-?exol yoter mi-mena.
ate-I ACC amount the-food that necessary to-eat more than-from-it
I ate the amount of food, d, such that it is necessary to eat more than d.
c. * ?iSanti ?et kamut ha-tabak Se mutar le-?aSen yoter mi-mena.
smoked-I ACC amount the-tobacco that allowed to-smoke more than-from-it
I smoked the amount of tobacco, d, such that it is allowed to smoke more than d.

5. Arguments for a deductive system

In the previous sections we have learned that the UDM predicts that certain properties of degrees will necessarily describe open intervals, and that this, in turn, accounts—given the CIM—for various constraints on the formation of questions, definite descriptions, sentences with *only*, and corresponding implicatures. In this section, we would like to show that our account can be maintained only if the CIM is assumed to apply within a formal system which is encapsulated from various pieces of information that enter into the determination of the truth-conditions of a sentence. We will call this system the *deductive system,* DS (following Fox, 2000) and we will provide additional evidence for the postulation of such a system drawing on Chierchia (1984) and, in particular, Gajewski (2002, 2003).

5.1 Cardinality as a level of granularity

Assume that the UDM is correct. This means that the measurement scales that are part of the *formal* apparatus involved in the interpretation of linguistic expressions are always dense. However, it is clear that various considerations enter into the interpretation of a sentence that are not determined by the formal apparatus. It is therefore consistent with the UDM that integers will end up being relevant for interpretive purposes after all.

There is a more general issue here. It is well known that sentences involving degree expressions are evaluated differently in different contexts. The claim that the distance between Amsterdam and Vienna is 1,000 km will be taken to be true in casual conversation about travel plans. The fact that the exact distance is closer to 965 km will be irrelevant. However, the evaluation might be different when the statement is written on an official sign-post in which other distances are listed with a more fine grained level of precision.[40] Such contextual dependency illustrates the fact that non-formal considerations enter into the determination of meaning. This fact can be captured within two dimensional semantics (Kaplan, Stalnaker, and much subsequent work), under the assumption that among the contextual parameters that are relevant for the evaluation of the truth conditions of a linguistic expression is a parameter, G (for granularity), that determines the relevant level of precision.

If the UDM is correct, dense scales (or the rules that characterize them) are part of the formal component of natural language semantics, but discrete scales are not. However, the interpretation of a linguistic expression is ultimately determined by factors that are outside the formal component, and among these factors are the contextual considerations that determine the value of G. It is therefore possible that a discrete measurement scale—even if it is not part of the formal apparatus—will end up being relevant to the evaluation of the truth conditions of a linguistic expression.

To see how this could come about, we need to be a little bit more specific about the contextual parameter, G, and how it might determine a level of precision. Consider the sentence in (66).

(66) John is (exactly) 15 years old.

In many contexts, this sentence would be considered true as long as John's age turns out to be somewhere between 15 and 16. To capture this fact, we might think of G as an equivalence relation on the members of the dense measurement scale:

(67) xGy iff there is a natural number n, s.t. $x \in [n, n+1)$ and $y \in [n, n+1)$.

This would get us the right result if we work out our semantics so that the truth conditions in (68) are derived for the sentence in (66), where the contextual parameter C includes information about various aspects of the context, among them the value of the granularity level G_C:

[40] The example is taken from Krifka (2002) in which it is pointed out that the form of the degree expression affects the relevant level of precision. See Lasersohn (1999) for a different point of view.

(68) $[[\text{John is exactly 15 years old}]]^{C,w}$ = 1 iff John's age (in years) in w, $a_{J,w}$, stands in the G_C relation to 15.
= 1 iff $a_{J,w} \in [15, 16)$ (given (67))

The definition of the level of precision/granularity in (67) involves quantification over the set of natural numbers, and, if the UDM is correct, cannot be part of the formal apparatus relevant for the semantics of degree constructions. Instead, such quantification must rely on our extra-linguistic knowledge of arithmetic, and can affect truth conditions only in so far as other aspects of our (contextual) knowledge can.

5.2 First argument for a deductive system

Consider from this perspective the way that the UDM is supposed to explain the unacceptability of sentences such as (5) exemplified again below (with a minor change to which we will return).

(69) *John is only more than 15_F years old.

We've claimed that this sentence is unacceptable because its truth conditions can never be met. For concreteness, assume that the sentence receives the logical form in (70)a, where the variable A refers to the focus value of the prejacent *John is more than* 15_F *years old* (see note 34). The resulting truth conditions that we assumed (ignoring the difference between assertion and presupposition) are given in (70)b.

(70) a. Only[A][John is more than 15_F years old]
b. $[[\text{Only[A][John is more than } 15_F \text{ years old]]]}^{w} =$ 1 iff

1. John is more than 15 years old and
2. for every proposition, φ, in A, if $\varphi(w)=1$, then the proposition that John is more than 15 years old entails φ.

The truth conditions can never be met since the set of propositions in A is the dense set of alternatives that corresponds to the dense set of degrees. However, this line of reasoning ignores contextual parameters and in particular the granularity parameter G.

It turns out that once G is taken into account, it is no longer obvious that the truth-conditions of (69) are contradictory. This, on the face of it, is problematic for our account of the unacceptability of the sentence. In order to deal with this problem, we will argue that sentences such as (69) are ruled out in a formal system that does not take contextual knowledge into account. This, in turn, will allow us to solve certain problems that we have left unresolved in earlier sections. Furthermore, the argument will be reinforced in Sect. 5.4 by a consideration of other facts of a similar sort reported in the literature.

To understand the problem, consider how we might modify (70)b once we introduce the contextually given level of granularity:

(71) $[[\text{Only}[A][\text{John is more than } 15_F \text{ years old}]]]^{C,w} = 1$ iff
a. $[[\text{John is more than 15 years old}]]^{C,w} = 1$ and
b. for every member of A, φ, if $\varphi(C)(w) = 1$, then λw. $[[\text{John is more than 15 years old}]]^{C,w}$entails $\varphi(C)$.

Are the truth-conditions in (71) still contradictory? In order to determine this, we have to know what proposition the sentence *John is more than 15 years old* expresses under C and what the members of A are. The answer to the first question should fall out from a general perspective on the role played by granularity in the interpretation of comparatives. It seems that we should derive the following under the granularity in (67):

(72) $[[\text{John is more than 15 years old}]]^{C,w} = 1$ iff John is at least 16 years old in w.

In Appendix 1, we show that this follows from basic assumptions about the semantics of comparatives and context dependency. More generally, we get the following for every ε such that $0 \leq \varepsilon < 1$:

(73) $[[\text{John is more than } n + \varepsilon \text{ years old}]]^{C,w} = 1$ iff John is at least $n + 1$ years old in w.

The important thing to observe is that the truth conditions given in (73) are the same for every $\varepsilon \in [0,1)$ (given a choice of n). This means that the set of alternative propositions that we get given the focus structure of the prejacent is not going to be dense. Rather, we will get the following answer to our second question about the set of alternatives, A, in (71):

(74) $[[\text{A}]]^{C,w} = \{\lambda \text{w. John is at least n years old in w: n} \in \text{N} - \{0\}\}$

(72) and (74) allow us to elaborate (71) in the following way:

(75) $[[\text{Only}[A][\text{John is more than } 15_F \text{ years old}]]]^{C,w} = 1$ iff
1. John is at least 16 years old in w and
2. for every $n \in N$, if $n > 16$ John is not n years old.

These truth conditions are of course not contradictory. Specifically, they will be met as long as John's age in years is in [16, 17). Since this will eliminate the account of the facts described in the preceding sections, we are forced to conclude that G doesn't enter the picture at the level at which the CIM is evaluated. Instead we have to assume that it becomes relevant only at a "later" stage when the properties of a particular context are consulted in order to determine the truth conditions of a sentence. We thus conclude that the CIM is evaluated in an informationally encapsulated system that does not take into account the (contextual) knowledge that is appealed to when determining a level of granularity. If we wanted to have a semantic characterization of the workings of this system, we could say that it evaluates sentences under the most stringent granularity relation, namely identity.

Under this granularity, a sentence such as (69) can never be true and is therefore ruled out.[41]

What emerges from these considerations is that the cognitive system contains an informationally encapsulated deductive system, DS, (Fox, 2000) in which sentences are evaluated and ruled out if they can be proven to be contradictory.[42] Once a sentence passes DS, it is evaluated in a particular context, where a level of granularity may affect the interpretation. The dividing line between the rules of DS and the principles that enter into the derivation of truth conditions in a particular context is close to the traditional divide between syntax/semantics and pragmatics. If sentences are to be ruled out because they are contradictory, it seems natural to assume that this is done based on the properties of a formal level such as DS rather than on the basis of all contextually available information. This assumption, as we will see in Sect. 5.4, is supported on independent grounds.

5.3 Back to universal modals

In previous sections we have explained why sentences such as those in (9) repeated in (76) (with a minor variation in (76)b to which we will return shortly) are not contradictory despite the UDM.

(76) a. I can only say with certainty that John weighs more than 120_F pounds.
b. I can only say with certainty that John is more than 15_F years old.

We provided a general proof for this and illustrated it for (76)a. Specifically, (76)a is true as long as, on the one hand, the speaker is certain that John weighs more than 120 pounds, while, on the other hand, for every ε, the speaker is not certain that John weighs more than $120+\varepsilon$ pounds. A problem, however, arises when we consider (76)b. We are able to derive the fact that this sentence is acceptable. However it is not obvious that we derive the correct truth conditions. In certain contexts (76)b would be equivalent to (77) and thus would not entail that there is no ε in the dense set of degrees such that the speaker is certain that John is more than $15+\varepsilon$ years old.

(77) I can only say with certainty that John is 16_F years old.

It seems that (in the relevant contexts) the set of degrees that is relevant for the evaluation of (76)b is the set of integers. However, in order to account for the unacceptability of the corresponding sentence without a universal modal operator, it is crucial to make a different assumption, namely the UDM.

This virtual contradiction is resolved by the division of labor between syntax/semantics and pragmatics that we postulated in the last section. Specifically, the sentence without the universal operator is ruled out by DS as a contradiction whereas the sentence in (76)b is not. (76)b thus has a chance to be evaluated in a particular context while the sentence without a universal modal does not. In the

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[41] A different perspective, perhaps more natural, is that *G* is an optional contextual parameter, which, when present, leads to a weakening of the semantic interpretation. The inferences of DS are sound relative to a context independent (*G*-less) interpretation.

[42] Fox's evidence for DS is also based on "Information encapsulation". See Fox (2000), pp 66–74.

relevant contexts, the level of granularity would be the one given in (67) which will result in the truth conditions in (78) (as we will show in the appendix).

(78) [[Only[A][I can say with certainty that John is more than 15_F years old]]]^{C,w} = 1 iff

1. The speaker in C can say in w with certainty that John is at least 16 years old and
2. for every $n \in N$, if $n > 16$ it is not the case that the speaker in C can say in w with certainty that John is n years old.

We now move to discuss our original examples in (5) and (9)b repeated below.

(79) a. *John only has more than 3_F children.
b. I can only say with certainty that John has more than 3_F children.

(79)a will be ruled out within DS, while (79)b will not. (79)b, however, presents a problem similar to the one discussed in the context of (76)b: it does not entail that for every ε the speaker is not certain that John has more than 3+ε children. Rather, it seems to entail that the speaker is certain that John has 4 children. This, again, can be dealt with by postulating a level of granularity. The following seems to us to be a reasonable assumption about the granularity of measurement for collections of objects that are indivisible (based on world knowledge), such as children.

(80) **Granularity for the measurement of collections of indivisible objects**[43]
xGy iff there is a natural number n, s.t. $x \in (n, n+1]$ and $y \in (n, n+1]$.

Under this granularity, we get the right results. Specifically, we get the following truth conditions:

(81) [[Only[A][I can say with certainty that John has more than 3_F children]]]^{C,w} = 1 iff

1. The speaker in C can say in w with certainty that John has more than 3 children and
2. for every $n \in N$, if $n>4$ it is not the case that the speaker in C can say in w with certainty that John has more than n children.[44]

These are of course the correct truth-conditions. The assumption that levels of granularity are contextually specified and that DS is insensitive to this specification

[43] To talk about "context dependency" is a bit misleading for these cases. A better term would be "extra-linguistic dependency" indicating that the extra-linguistic information that enters into the determination of the truth-conditions might not vary across contexts.

[44] There are other levels of granularity that would produce the same results, e.g. the one in (67). (80) seems more natural to us since under this level of granularity the sentence *John has n+1/2 children* and the sentence *John has n+1 children* express the same propositions. An alternative approach to (79)b might appeal to a contextual notion of entailment.

provides us with the means to account for the contrast in acceptability in (79) as well as the truth-conditions of (79)b.[45]

5.4 Further evidence for a deductive system

In the previous sections we have seen that the pattern of acceptability of syntactic representations of the sort we have been looking at can be derived under the UDM. However, this can only be achieved if levels of granularity, which are needed to derive adequate truth-conditions, are ignored. This, in turn, would follow if we assume a formal system, DS, that rules out contradictions independently of a level of granularity. In this section, we will review another argument for DS.

The argument will be stated, once again, in the context of evidence that certain contradictions ought to be ruled out by the linguistic system. There is quite a bit of evidence, which comes from a variety of empirical domains distinct from the ones discussed in this paper. However, this evidence is in conflict with the well-known fact that the linguistic system is able to express many contradictory propositions. To resolve this conflict, we suggest, following Chierchia (1984) and Gajewski (2002), that the linguistic system rules out contradictions on the basis of formal considerations alone. If this claim is correct, it will provide further evidence for DS.

The idea that sentences might be ruled out because they express contradictory propositions has been advanced in the context of the analysis of a variety of constructions. (See Ladusaw, 1986 for a discussion of a few of the original proposals.) Take for example the contrast in (82). Von Fintel (1993) suggests that this contrast should be explained on the basis of the semantic import of *but*, specifically, that the lexical entry needed to account for the truth conditions of (82)a ensures that (82)b expresses a logical contradiction.

(82) a. Every man but John arrived.
b. *A man but John arrived.

The sentence in (82)a entails, on the one hand, that it is not the case that every man arrived and, on the other hand, that it is the case that every man who is not John arrived, (i.e. that {John} is the minimal set X that yields a true proposition of the form *every man who is not a member of X arrived*). The sentence in (82)b—if it were acceptable—would entail, on the one hand, that it is not the case that a man arrived and, on the other hand, that it is the case that a man who is not John arrived. These two entailments are of course contradictory, which, under von Fintel's proposal, accounts for the unacceptability of the sentence. Von Fintel shows that this type of reasoning is a good predictor of the distribution of exceptive *but* based on a variety of empirical considerations, and Gajewski (2002) shows that, under a particular implementation, it could be extended to account for certain previously unresolved puzzles.

[45] Similarly, we can account for the answers available for questions such as *How many children are you certain that John does not have?* The acceptability of these questions is explained by the fact that universal modals can close intervals. However, these questions are naturally answered by expressions such as *n children*, which should be ruled out by more informative answers of the form, e.g. *n-1/2 children.* This conflict is resolved by the observation that,—once G is taken into account—all answers of the form n-ε (where $0 \leq \varepsilon < 1$) express the same proposition.

Similarly, Dowty (1979) argues that the unacceptability of sentences such as (83) is to be explained on the basis of the logical properties of the verb phrase *accomplished his mission* and the adverb *for an hour*. The properties of the adverb yield the result that, if the sentence were acceptable, it would have expressed the proposition that there is a time interval in the past, T, lasting for an hour, such that $\forall t \subseteq T$, John accomplished his mission in t. The properties of the verb phrase ensure that this would never hold.

(83) *John accomplished his mission for an hour.
There is a time interval in the past T s.t. Length(T) = one hour and $\forall t \subseteq \mathrm{T}$ John accomplished his mission in t.

Further examples are discussed in Chierchia (2004, 2005), Dayal (1998), Gajewski (2002) drawing on Barwise & Cooper (1981), Ladusaw (1986), and Menendez-Benito (2005).[46] This line of reasoning thus seems to have some support. However, it doesn't appear to be universally applicable, as there are many contradictions that do not yield unacceptability. Some examples are given in (84).

(84) a. This table is both red and not red.
b. He's an idiot and he isn't.
c. I have a female (for a) father.
d. No student is sick and yet some student is sick.

One might suggest that the sentences in (84) are acceptable because the contradictions can be eliminated by virtue of a dynamic interpretation of contextual parameters. One might suggest that in (84)a, for example, the context provides a standard for redness which is different in the first and second conjunct, thereby yielding a contingent proposition. (See, among others, Chierchia & McConnell-Ginet, 1990: chap 8 Sect. 5, and Kamp & Partee, 1995.[47])

This, however, will not be consistent with what is needed to maintain the proposals we've made in the previous sections. As we have seen, there is a way to fix the contextual granularity parameter, G, that would eliminate the contradictions that are responsible (under our account) for the unacceptability of certain sentences. We would therefore like to adopt an alternative account of the contrast between sentences such as those in (82) and (83), on the one hand, and (84) on the other, namely a version of the proposal made in Gajewski (2002).

But before we get there, we would like to mention that there are cases where sentences are clearly interpreted as contradictions, i.e., cases for which postulating a contradictory interpretation is needed to account for the communicative function:

(85) What you are saying is clearly wrong. It leads to the conclusion that
a. some student (other than John) arrived and yet that no student arrived.
b. this table is both red and not red.
...

[46] Some of these examples require the assumption that tautologies and contradictions are treated on a par.

[47] Thanks to Jon Gajewski for pointing out this possibility.

More importantly, the sentences that are ruled out because they are contradictory can never serve a similar function, even in cases in which they are predicted to express exactly the same proposition.

(86) What you are saying is clearly contradictory. *It leads to the conclusion that some student but John arrived.

The difference between (85) and (86) suggests that there is a qualitative distinction between different types of contradictions, as suggested by Gajewski (2002).

Gajewski proposes to account for the facts described above under the assumption that there is a formal principle of grammar that rules out contradictory sentences and that this principle applies at a particular level of representation (LF′) which encodes information about the content of functional/logical words but not about the content of lexical categories.[48] More specifically, Gajewski suggests that we think of LF′ as if it were derived from a traditional logical form by the replacement of every occurrence of a non-functional word with a new variable.[49]

For example, the sentences in (84) would have the following representations at LF′ on the basis of which a contradiction cannot be derived.

(87) a. This NP_1 is both AP_1 and not AP_2.
b. DP_1 is a NP_1 and not a NP_2.
c. I have a AP for a NP.
d. No NP_1 is AP_1 and yet some NP_2 is AP_2.

By contrast, the sentence in (82)b will have the following representation at LF′, which is sufficient to determine its contradictory status under the assumption that the system can access various axioms that characterize the meaning of the indefinite quantifier and the exceptive morpheme *but*.

(88) A NP but DP VPed.[50,51]

To summarize, we have presented two arguments for DS. One argument was based on our account of the properties of certain degree expressions: definite descriptions of degrees, degree questions, sentences with *only* and corresponding implicatures. This account—in terms of the UDM and the CIM—can be maintained only if we assume a division of labor between DS and context-dependent interpretation, that is, between a formal system that rules out contradictory expressions

[48] A suggestion along similar lines was made by Kai von Fintel in a class presentation in 1999. We thank Kai for helpful discussions of all the issues examined in this section.

An obvious question to ask is what characterizes the set of functional/logical words. Gajewski suggests that every member of the set is permutation-invariant in Tarski's sense (Sher, 1991), building specifically on a formulation due to Van Benthem (1989). See also von Fintel (1994).

[49] If proposals such as that of Marantz (1997) and Borer (2005) are correct, LF' could be identified with LF.

[50] See Gajewski (2002) for further details of the nature of the structure.

[51] For (83) one might assume a more refined syntactic structure in which telic predicates are built from two independent morphemes one of which is a *cause* operator, or alternatively one might assume that all primitive predicates of time are (axiomatically) telic, and that non-telic predicate are syntactically complex.

independent of levels of granularity, and a later system responsible for the context dependent computation of truth-conditions (based, among others, on the setting of the granularity parameter). This allowed us to explain why certain expressions are ruled out by DS despite the fact that a plausible level of granularity could give them a non-contradictory interpretation. DS is a formal level whose inferences are independent of contextual interpretations: the inferences are sound relative to a semantics without a level of granularity (or relative to G=identity) and not relative to a semantics that considers contextual weakenings.

Our second argument was based on the observation that proposals due to Chierchia, Dowty, von Fintel, Gajewski and others rely on a similar distinction. Since not all contradictions are judged unacceptable, but only those that follow from certain formal considerations, a formal system, DS, needs to be postulated. This system determines for each expression whether it is contradictory based on the relevant formal properties of the expression.[52]

Both arguments are based on a similar logic. They both teach us that the overall considerations that determine the truth conditions of a sentence (semantics+pragmatics) yield different results from those needed to derive the relevant acceptability judgments. Our first argument teaches us that we need to postulate a system that derives contradictions, which would be eliminated if contextual parameters were introduced (a system that is not sound relative to semantics+pragmatics). Our second argument teaches us that the required system should not be able derive all intuitive contradictions (i.e., that it cannot be complete relative to semantics+pragmatics).

If we are correct, we would like to know which expressions belong to the vocabulary of DS and what formal mechanisms are involved (e.g. axioms and rules of inference). For the cases discussed in this paper, one would need to assume that the comparative morpheme *–er*, negation, Exh, *only*, and the temporal preposition, *before* belong to the relevant vocabulary. This assumption, when coupled with appropriate DS axioms should be able to deliver the required results. More specifically, it seems that the following would have to be theorems of DS:

(89) a. *Universal density*: $\forall d_1,d_2\ \exists d_3\ [(d_1 > d_2) \rightarrow (d_1 > d_3 > d_2)]$
 b. *Lexical monotonicity*: lexical n-place relations are upward monotone.
 c. *Lexical closed intervals*: if R is a lexical n-place relation, whose m-th argument is a degree, then for every w, and for every $x_{1,\ldots,}x_{n\text{-}1}$ $\mathrm{Max}_{\mathrm{inf}}(\lambda d.R(x_1),\ldots,(d),\ldots,(x_{n\text{-}1}))(w)$ is defined.
 d. Commutativity: Two existential quantifiers can be commuted.[53]

[52] See Heim (1984), Lahiri (1998), Guerzoni (2003), and Zucchi (1995), among others, for arguments that contradictory presuppositions yield unacceptability. There, too, it seems that not all contradictions are ruled out, only those that can be plausibly linked to DS:

(1) a. #The man who isn't a man arrived late.
 b. #The man is not a man
 c. *I read even a single book.

[53] (89)*d* is also crucial to deliver the results needed for the proposal in Fox (2000).

Based on this set of assumptions (together with basic logical machinery), DS should be able to prove that a Gajewski-structure for (90)a, provided in (90)b, is contradictory. Similar considerations apply to structures that employ *Exh*.

(90) a. *John is only more than 15_F years old.
b. only ($\{\lambda w$.-er than d ($\lambda d'$.NP d'-AP in w: d a degree$\}$)
λw.-er than d'' ($\lambda d'$.NP d'-AP in w')[54]

This division of labor between DS and a more general pragmatic system allows us to address a variety of problems for what we've said in Sects. 1–4. We start with a problem that we mentioned briefly in an earlier section (footnote 27). Consider our account of (32), repeated below as (91)

(91) *Before when did John arrive?

We explained the unacceptability in the following way: suppose that John arrived at 10, the answer to (91) would have to specify the earliest time after 10, but given the density of time, there could be no such time. However, this explanation seemed to be contradicted by the acceptability of (92).

(92) the earliest time after 10

We can now give a rational for a distinction between the two cases. For DS to derive the inference that (91) doesn't have an appropriate answer, the preposition *before* has to belong to the vocabulary of DS. If it does, and density is assumed, the crucial fact can be derived, i.e. that there is no maximally informative t such that John's arrival time, t', is before t. To derive a similar inference for (92) requires the same assumption for the preposition *after*, the adjective *early*, and the superlative morpheme *est*. It seems reasonable to us that this assumption will not hold for the

[54] The LF that corresponds to (90)b assumes that the comparative morpheme *–er* is a restricted quantifier over degrees that adjoins to a clausal node. The restrictor of *–er* is given by *than d* while the nuclear scope is the matrix clause, which, given abstraction over a degree position triggered by movement of *–er than d,* denotes a predicate of degrees (cf. Heim, 2001). *Only*, as before, is analyzed as a clausal operator whose first argument is a set of alternatives derived by replacing the focused expression in the prejacent with its alternatives. The Gajewski structure can be derived from the LF by replacing any non-logical item with a different variable except in the calculation of the alternatives, which requires that the same variable names be used as in the prejacent.

(1) a. John is only more than 15_F years old.
LF b. only A [[–er than 15]$_i$ [John is d_i years old]].
$A = \{\lambda$w. [[–er than d]$_i$ [John is d_i years old]]: d a degree$\}$
GS c. only A [[–er than d']$_i$ [NP is d_i AP]].
$A = \{\lambda$w. [[–er than d']$_i$ [NP is d_i AP]]: d a degree$\}$

adjective. If so, we would have a distinction that could derive the contrast. More specifically, (93)b might be an appropriate Gajewski-structure for (93)a.[55]

(93) a. Mary arrived at the earliest time after 10 pm.
b. -est ($\{\lambda d.\ \lambda w'.$ NP VP at a d-AP time after t in $w'\}$: t a time) ($\lambda d'$. NP VP at a d'-AP time after t' in w)

To see that (93)b is not DS-contradictory, consider various instantiations of AP, the variable that ranges over degree predicates of times such as *early, late, opportune, etc.* Clearly, a problem akin to the one in (91)a arises if the degree predicate of times that licenses *–est* is based on the inherent linear ordering among times. *Early* seems to be such a predicate since t is earlier than t' iff t is before t'. If *early* were part of the DS vocabulary, the fact that ordering among times is that of temporal precedence would be visible to DS and the system would determine that it is impossible to find a time after t that is earlier than any other time after t. However, if we assume instead that *early* is not part of the DS-vocabulary, *early* would be represented by a variable AP at DS and this would make it impossible to derive a contradiction. To see this, consider an instantiation of AP which orders times on grounds other than temporal precedence, e.g. *opportune* in *the most opportune time after* 10.

In fact, we observe quite generally that sentences that employ *only* associating with a degree expression across a comparative operator will be ruled out as DS-contradictions while seemingly parallel sentence with a superlative operator will not.

(94) a. *John only weighs more than 120 pounds/*John is only heavier than 120 pounds.
b. The smallest weight bigger/above 120 pounds …

(95) a. *John only has more than THREE children.
b. The smallest amount/number of children bigger/above 3 …

This contrast is predicted by our assumptions. Because of the UDM, any degree predicate suffices to produce a contradiction in structures such as (90)b. However, in the superlative cases a contradiction or presupposition failure would result at DS

[55] We assume for concreteness, following Heim (1999), that in the LF that corresponds to (93)b the superlative morpheme *–est* is a restricted quantifier over degrees that adjoins to a clausal node. Unlike *–er* (but very much like *only*), the restrictor of *–est* is a set of focus induced alternatives based on the prejacent of *–est*, which denotes a property of degrees. The truth-conditional import of *–est* involves the existential claim that there is a degree d such that the prejacent is true of d but none of the alternatives (that are not entailed by the prejacent) is true of d. (Note that the definite determiner on *the earliest time after* 10 *pm* is translated as an indefinite.)

(1) a. Mary arrived at the earliest time after 10_F pm.
LF b. [[-est A]$_i$ [Mary arrived at a d_i-early time after 10 pm]].
$A = \{\lambda d.\lambda w.$ [Mary arrived in w at a d-early time after]]: t a time$\}$
GS c. -est A [$\lambda d'$. NP VP at a d'-AP time after t' in w].
A $= \{\lambda d.\lambda w'.$ [NP VP at a d-AP time after t]: t a time$\}$.

only if the degree predicate licensing the superlative, *small*, belonged to the vocabulary of DS.

Consider next the following contrast.[56] If we are correct, (96)a lacks an implicature because DS can derive a contradiction from a structure that contains *Exh*. In (96)b and (96)c we see a contrast that we've observed before with universal and existential modals.

(96) a. John handed in more than 3 assignments.
*Implicature: John handed in exactly 4 assignments.
b. Every student handed in more than 3 assignments.
?Implicature: There is at least one student who handed in exactly 4 assignments.
c. Some student handed in more than 3 assignments.
*Implicature: There is no student who handed in exactly 4 assignments.

More specifically, a universal quantifier over individuals in (96)b, just like a universal modal, can prevent DS from deriving a contradiction, and, thus, allows for the generation of an implicature. An existential quantifier over individuals, like an existential modal, cannot produce this effect.

Since the contrasts are parallel, we would like to have a parallel account. Our account in the modal domain depended crucially on the idea that grammar does not preclude quantification over an infinite set of worlds. That is a necessary assumption if the domain of degrees can ever be infinite (given assumptions about asymmetric entailment). However, for the cases in (96), it is not a necessary assumption. Still, it seems rather natural to us.

If DS does not have axioms that rule out an infinite domain of individuals, it will be unable to prove a contradiction for the structure of (96)b that yields the relevant implicature. To show this, we construct in Appendix 3 a world in which the exhaustive interpretation of (96)b is true. The actual interpretation that we derive for the sentence, once it passes the DS filter is based on a level of granularity. Once the level of granularity is introduced it is as if the domain of degrees contained only natural numbers, and the representation with *Exh* will entail the proposition that not every boy handed in more than 4 assignments.

Independently of assumptions about the size of quantification domains, existential quantifiers over individuals, (96)c, are just like existential modals. In both cases a contradiction can be derived by DS based the fact that two existential quantifiers are commutative.

This type of reasoning will extend to the following contrast pointed out to us by Nathan Klinedinst and Philippe Schlenker. (97)b could have a most informative answer: the proposition that no student handed in d^* assignments could be the most informative proposition of the relevant form (*no student handed in d assignments*). This is true under the UDM if the domain of individuals can be infinite.

(97) a. *How many assignments did John not hand in?
b. (?)How many assignments did no student hand in?

[56] Thanks to Nathan Klinedinst, Philippe Schlenker, and Benjamin Spector for insightful comments on this topic.

However, Klinedinst and Schlenker (p.c) point out yet another contrast:

(98) a. *How many assignments did John not hand in?
b. (?)How many assignments did neither John nor Bill hand in?

In order to account for the (relative) acceptability of (98)b, we would have to claim that DS cannot "prove" that the property *λd. neither John nor Bill handed in d many assignments* is an *N*-open property. However, if the UDM is assumed, it is easy to see that there is no minimal degree, *d*, such that neither John nor Bill handed in *d* many assignments. Assume (with no loss of generality, of course) that John handed in more assignment than Bill, say *d* assignments. The set of degrees above *d* is an open interval with no minimum. This seems to be a real problem, for which we don't yet have a satisfactory solution. However, it is worth pointing out that the scope of the phenomenon is not clear. For example, it is not obvious to us that a parallel puzzle arises in the domain of conjunction. While universal quantification over individuals is able to circumvent a UDM effect in (96)b, the same is not true of ordinary conjunction in (99).

(99) Both John and Bill handed in more than 3 assignments.
*Implicature: Either John or Bill handed in exactly 4 assignments.

At this point, we have to leave the matter as an unresolved puzzle.[57]

5.5 Syntactic contextual restriction

Kroch (1989) notes that negative islands can be circumvented if the context provides an explicit set of alternatives. For example, while (100)a is clearly unacceptable, the specification of a set of alternatives in (100)b gets rid of the problem.

(100) a. *Tell me how many points Iverson didn't score?
b. Among the following, please tell me how many points Iverson didn't score?
A. 20 B. 30 C. 40 D. 50

On initial inspection, this contrast is directly predicted by our approach to negative islands. (100)a is ruled out given our assumption that the set of degrees is dense,

[57] But we can offer some speculations. We might treat *neither John nor Bill* as a negative existential, with a non-logical domain restrictor, hence potentially infinite. We are not quite sure how to implement this idea. One possibility is to de-compose *neither* into a negative morpheme *not* and an existential quantifier *either*, and to assume that the word *or,* itself, is not part of the logical vocabulary. If so, the Gajewski representation of (98)b would be the following:

(1) How$_d$ did not either[x [C [y]]] V d-many NPs?

Since *or* is not a logical word, *C* is replaced with a variable of the appropriate type, in this case a function that takes an individual to functions from individuals to sets. Since the function is not restricted in any way, the range could be any set, among others, an infinite set, that would allow for a most informative answer. Many questions arise at this point, and addressing them would involve various speculative moves, which are very likely pre-mature.

from which it follows that the question cannot be answered: an answer to the question would specify the most informative true proposition in the Hamblin denotation of the question, but there can be no such proposition.

In (100)b, by contrast, the addressee is requested to focus on a discrete sub-set of the propositions in the Hamblin set. This contextually specified set of alternative propositions (*that Iverson didn't score* 20 *points*, *that he didn't score* 30 *points, etc.*) contains a most informative true element which renders the question answerable, hence acceptable.

This observation extends to implicatures and *only* as the data in (101) and (102) show.

(101) Iverson sometimes scores more than 30 points. But today he only scored more than 20_F.

(102) A: How many points did Iverson score last night?
B: I don't know.
A: Was it more than 10, more than 20 or more than 30.
B: He scored more than 20 points
Implicature: he didn't score more than 30.

However, a real question arises at this point. While we expect that the existence of a most informative degree will license the use of Max_{inf}, it is at this point puzzling that contextual information can save the day. Earlier, we argued that levels of granularity, which are contextually determined, are invisible to DS and therefore cannot resurrect DS contradictions. Here we see that a contextually provided set of alternatives saves a structure that would otherwise be ruled out as a DS contradiction.

We can make sense of this apparent paradox if we assume, following e.g. von Fintel (1994), that the contextual restrictions at work in (100)–(102) are represented in the syntax as variables that modify the restrictor of the relevant operator, as illustrated in (103)a for *only*. This yields the Gajewski structure in (103)b.[58] This structure is not ruled out by DS since a contradiction cannot be derived given the presence of *C*.

(103) a. [Only[*A* *C*] [Iverson scored more than 20 points]]
where C = {that Iverson scored more than 20 points, that Iverson scored more than 30 points, …}
b. Only [{that NP_1 V more than d NP_2: d a degree} $\cap$ C] NP_1 V more than d' NP_2.

If the context does not provide a specific set of alternatives, a structure with a free variable *C* will not be licensed. Under such circumstances, the restrictor of *only* will be determined solely based on focus induced alternatives to the prejacent, which, as we have shown above, yields a DS contradiction.

Note the crucial assumption: there are two distinct ways in which the context can affect meaning. Levels of granularity enter into the determination of the truth-conditions quite differently from contextually licensed variables, such as *C* in (103)c.

[58] The structure of the comparatives in (103)b,c needs to be further articulated. See fn. 55 and Appendix 1 for one possibility.

Levels of granularity are among the parameters of the valuation function, and, as such, are not represented in the syntax, and, consequently, are invisible to DS. The desired result is that levels of granularity, in contrast to contextually licensed variables, cannot save a structure from a DS violation.

6. Conclusion

In Sects. 1 through 4, we argued that the semantics of degree constructions does not presuppose the concept of natural number. This argument was based on an intricate set of acceptability judgments which follow from an extremely natural assumption (the CIM) only if degree domains are always dense, i.e., if the set of natural numbers is not treated by the linguistic system as a separate domain of entities.

We think that our arguments fit well with the fact that not all languages have a vocabulary that can be used to refer to the set of natural numbers (Gordon, 2004; Pica et al., 2004), and with the much discussed difficulties in the acquisition of the relevant numerical concepts. These give reason for skepticism about claims that there is a core knowledge system that directly defines the set of natural numbers. Therefore, even if various semantic notions are axiomatized within the linguistic system (for which there is abundant evidence), one would probably not expect the same to be true of the theory of natural numbers. From this perspective, the conclusions which we have argued for are fairly natural.

However, our arguments are in conflict with much work in natural language semantics in which the concept of cardinality plays a prominent role. We suspect that this conflict is not substantial, and that standard results can be recast in terms of contextually determined levels of granularity. If this turns out to be correct, there are far reaching consequences for the architecture of the grammar, which we began to investigate in Sect. 5.

Appendix 1: granularity in the interpretation of comparatives

In the following we implement a few of the crucial assumptions about the way a level of granularity, G, affects the interpretation of degree constructions:

(104) Standard lexical entries (without G):

a. $[[\text{old}]]^{w}(d)(x) = 1$ iff x's age in w is greater or equal to d ($a_{x,w} \geq d$).

b. $[[\text{er/more than}]]\,(D_{\langle d,t\rangle})(\phi_{\langle d,t\rangle}) = 1$ iff $\exists d[d \in [[\phi]]\ \&\ \forall d'[d' \in [[\text{D}]] \rightarrow d > d']$[59]

c. $[[\text{15 years}]] = \{15\}$.

d. $[[\text{only S}]]^{w} = 1$ iff $[[\text{S}]]^{w} = 1\ \&\ \forall\phi[\phi \in [[\text{S}]]_{f}\ \&\ (w) = 1 \rightarrow [[\text{S}]]^{w} \models \phi]$[60] (where $[[\text{S}]]_{f} = \{\lambda w.\ [[\text{S}']]$: S' can be derived from S by appropriate replacement of focus marked constituent$\}$)

[59] We are ignoring the difference between characteristic functions and the sets they characterize.

[60] In order to avoid intricate syntactic representations, we provide a syncategorematic rule. Furthermore, we ignore the distinction between assertion and presupposition, again to keep things simple.

(105) Two dimensional interpretation:
$[[\phi_{\langle d,t\rangle}]]^G = \{d: \exists d'[d'Gd \ \& \ d' \in [[\phi]]\ ]\}$
$[[\phi_{\langle d,\text{et}\rangle}]]^G = \{\langle d,x\rangle: \exists d'[d'Gd \ \& \ \langle d',x\rangle \in [[\phi]]\ ]\}.$
(generalizes in an obvious way to n-place predicates)

Some derivations (building up to an account of (76))

(106) $[[\text{er/more than } D\ \phi]]^G =$ 1 iff $\exists d[d \in [[\phi]]^G \ \& \ \forall d'[d' \in [[\text{D}]]^G \rightarrow d > d']$
iff $\exists d \exists d'[d'Gd \ \& \ d' \in [[\phi]]\ ] \ \&$
$\forall d'\ [\exists d''[d''Gd' \ \& \ d'' \in [[\text{D}]]] \rightarrow [d > d']].$

We will assume that every one of G's equivalence classes (ECs) is an interval (continuous, i.e. with no holes). If a member of $[[\phi]]$, d, belongs to EC(d'), and d' which is above a set of ECs, d must also be above the set of ECs. Hence:

iff $\exists d \in [[\phi]] \ \& \forall d'\ [\exists d''[d''Gd' \ \& \ d'' \in \ [[\text{D}]]] \ \rightarrow d > d']$

(107) $[[\text{er/more than } x \text{ years } \phi]]^G = 1$ iff $\exists d \in [[\phi]] \ \&$
$\forall d'\ [d' \in \ \text{EC}(x) \ \rightarrow d > d']$ (since
$\text{EC}(x) \ = \ \{d: \exists d'[d'Gd \ \& \ d' = \text{x}\}$)

Level of Granularity for Age:
$G \ = \ \{\langle d,d'\rangle: \exists n[n \in N \& d \in [n,n+1) \& d' \in [n,n+1)]\}$

(108) $[[\text{John is more than 15 years old}]]^{G,w} = 1$ (example (72))
iff $[[\text{ more than 15-years } [\lambda d.\text{John is } d\text{-years old}]\ ]]^{G,w} = 1$
(assumption about LF)
iff $\exists d[a_{J,w} = d] \ \& \ \forall d' \in \ \text{EC}(15)\ [d > d']$
iff $\exists d[a_{J,w} = d] \ \& \ \forall d'[d' \in [15,\ 16) \ \rightarrow d > d']$ (by definition of G)
iff $a_{J,w} \ = \ 16$

(109) More generally, we obtain for any $n \in N$ and any $\varepsilon \in [0,1)$
$[[\text{ more than } n+\varepsilon\ [\lambda d.\text{John is } d\text{-years old}]\ ]]^{G,w} = 1$ iff $a_{J,w} \ = \ n+1$
(since $\text{EC}(n+\varepsilon) \ = \ [n,\ n+1]$)

(110) The focus value of *John is more than* 15_{F} *years old* under G:
$[[\text{more than 15 } [\lambda d. \text{ John is } d\text{-years old}]\ ]]_f^G =$
$\{\lambda w.\ [[\text{more than } d\ [\lambda d'.\text{John is } d' \text{ years old}]\ ]]^{G,w}: d \in \ R\} \ =$
$\{\lambda w.\ [[\text{more than n} + \varepsilon\ [\lambda[d'.\text{ John is } d' \text{ years old}]\ ]]^{G,w}: n \in N,\ \varepsilon \in (0,1)\} \ =$
$\{\lambda w.\ a_{J,w} \ = \ n+1: n \in \ N\}$

(111) The focus value of *I can say with certainty that John is more than* 15_F *years old* under G:
$\{\lambda w.\ \forall w'\ [w' \in \ \text{Acc}_w \ \rightarrow \ a_{J,w'} \geq n+1]: n \in \ N\}$ (by the same reasoning)

(112) $[\![$ Only [I can say with certainty that more than 15 λd.] John is d-years old $]\!]^{G,w} = 1$
iff $\forall w'$ $[w' \in \text{Acc}_w \rightarrow a_{J,w} \geq 16$ &
$\forall \varphi$ $[\varphi \in$ focus-of-prejacent–under G & $\varphi(w) = 1 \rightarrow$
$\lambda w.[\![\text{prejacent}]\!]^{G,w} \models \varphi]$.
(where the prejacent is the sister of *only*)

Since: 1. the-focus-of-prejacent-under-G is
$\{\lambda w.\forall w'$ $[w' \in \text{Acc}_w \rightarrow a_{J,w'} \geq n+1] : n \in N\}$ (by (111)), and
2. $\varphi_{16} \models \varphi_n$ iff $n < 17$
where $\varphi_n = \lambda w.\forall w'$ $[w' \in \text{Acc}_w \rightarrow a_{J,w'} \geq n+1]$

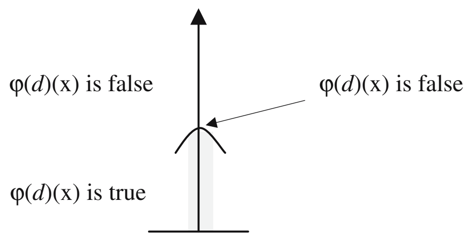
$[\![(112)]\!]^{G,w} = 1$ iff $\forall w'$ $[w' \in \text{Acc}_w \rightarrow a_{J,w'} \geq 16]$ &
$\forall n \geq 17[\varphi_n(w) = 0]$

Exactly the same computations will produce the desired interpretations for examples (2) and (81).

Appendix 2: The UDM and quantification over individuals

In this appendix, we show that the consequences of universal and existential modals for the UDM extend to all universal and existential quantifiers, when no restrictions are imposed on the domain of quantification. To save space, we limit our discussion to upward monotone functions.[61]

Let φ be an upward monotone function of type $\langle d,\langle e,st\rangle\rangle$, where φ is upward monotone if for every d_1,d_2: $d_1>d_2$ iff for every x, $\varphi(d_1)(x)$ is more informative than $\varphi(d_2)(x)$ (asymmetrically entails it). Assume also that φ necessarily describes an open interval, i.e. for every world w, and individual x, $\lambda d.\varphi(d)(x)(w)$ is an open interval. The following drawing represents what holds of φ in every world, given a choice of an individual x:

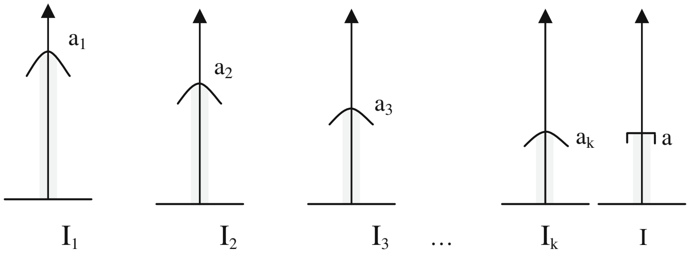


We can see that if we append a universal quantifier over individuals with restrictor C, $\forall_C$ (of type $\langle\langle e,st\rangle, st\rangle$), the modified property, $\lambda d.\ \forall_C\ [\varphi(d)]$, would no longer necessarily describe an open interval. The universal modal could quantify over a dense set of individuals that corresponds to the set of degrees with the result that the

[61] To see that the same holds for downward monotone function, one needs to follow the same steps, with the difference that all open intervals are reversed and the sequence of open intervals converges to a closed interval from the bottom.

modified property describes a closed interval. To see this, consider a sequence of open intervals I_k that converges to a closed interval $I = [0, a]$ from the top ($I_k = [0, a_k)$ where a_k converges to a from the top).

There is a world w in which there is a corresponding sequence of individuals in C, x_k, such that for every n, $\lambda d.\varphi(d)(x_n)(w) = I_n$. In this world, $\lambda d.\forall_C\ \varphi(d)(w)$ denotes the closed interval I. To see this, suppose that d is not in I. This means that d is bigger



than a. Hence, starting with some N, for all n bigger than N, $\varphi(d)(x_n)(w)=0$ (given convergence from the top). Hence, $\forall_C[\varphi(d)](w)=0$. Suppose conversely that d is in I. Given that I is a subset of every I_n, $\forall_C[\varphi(d)](w)=1$. This means that complex predicates over degrees such as λd. *every student handed in more than d assignments* do not necessarily denote an open interval. And this, we suggest, accounts for the fact that such predicates can be the source of an implicature in (96).

As in the modal case, appending an existential quantifier is of no help: the property $\lambda d.\exists_C[\varphi(d)]$ necessarily describes an open interval. To see this, assume that $\lambda d.\exists_C[\varphi(d)]$ denotes a closed interval in a world, w, and that d' is its maximal member. Of course $\exists_C[\varphi(d')](w)=1$. This means that there is an individual, x, in C such that $\varphi(d')(x)(w)=1$. But since φ describes an open interval for every x, there is a degree d'' bigger than d' such that $\varphi(d'')(x)(w)=1$. Hence, d' is not the maximal member in $\lambda d.\exists_C[\varphi(d)](w)$. This, of course, explains the fact that complex predicates over degrees such as λd. *some student handed in more than d assignments* cannot support the relevant implicature.

Appendix 3: Remaining issues in the semantics of questions

The results of Sects. 1–4 were based on the assumption that Maximality (specifically $\mathrm{Max}_{\mathrm{inf}}$) is relevant for the semantics of four different linguistic constructions: sentences with *only*, sentences with scalar implicatures, definite descriptions, and questions. We think that this assumption is straightforward for the first three cases. However the role of Maximality in the semantics of questions is less straightforward, and various conflicting positions have been argued for. (See, among others, Groenendijk & Stokhof, 1984; Dayal, 1996; Higginbotham, 1993; Lahiri, 2002.) While we can't do justice to the extensive literature on this topic, we feel obliged to deal with specific challenges to Maximality that have been raised in the context of degree questions.

Consider the question answer sequence in (113) and (114) discussed by Beck and Rullmann (1999).

(113) Speaker A: How many people can play soccer?
Speaker B: 6 people (indoor soccer), 8 people (small field)
and 11 people (regular)

(114) Speaker A: How many courses are you allowed to take?
Speaker B: Any number between 4 and 6.

The answer given by Speaker B in both cases is inconsistent with a maximality presupposition. There are two possible moves that are available to us at this stage. We can interpret Bs answer as involving a rejection of the question's presupposition, or alternatively we can try to replace the maximality presuppositions of degree questions with a weaker presupposition that would still capture the results described in Sects. 2 and 3. The first move might be appropriate for the dialogue in (113), which can be replicated with a definite description.

(115) Speaker A: What is the number of people that can play soccer?
Speaker B: 6 people (indoor soccer), 8 people (small field) and
11 people (regular)

It is very likely that in this case Speaker B is indicating that three different questions need to be asked and is answering each in turn. However the situation seems to be different in the case of (114), which is totally appropriate and quite different from its counterpart with a definite description:

(116) Speaker A: What is the number of courses you are allowed to take?
Speaker B: #Any number between 4 and 6.
Spealer B': Oh, the rules are not that rigid.
I can take any number of courses between 4 and 6.

We thus need to investigate the second possibility. The need is quite pertinent for our concerns as illustrated by the dialogue in (117), pointed out to us by Irene Heim (p.c.).

(117) A: How much money are you not allowed to bring into this country?
B: $10,000
C: The maximum allowed is $10,000.
= You're not allowed to bring in any amount that exceeds $10,000.

We've claimed that A's question is acceptable—in contrast to the non modalized variant in (118)—because a modal operator can close an N-open interval.

(118) *How much money did you not bring into this country?

Under the assumption that Max_{inf} is employed in the semantics of degree questions, (118) is bad because there can be no minimal degree *d*, such that you did not bring *d*-much money into this country. By contrast, there could be a minimal degree of the relevant sort in (117). For example, Bs answer conveys the information that $10,000 is the minimal (i.e. maximaly informative) amount of money that you are not allowed to bring into this country. However, as pointed out to us by Heim, this line of reasoning would force us to analyze Cs answer as contradicting the presupposition of the question. In his answer, *C* is claiming that there is no minimal amount of money that you are not allowed to bring into this country.

Such an analysis doesn't seem plausible to us, and we would therefore like to tentatively propose an alternative. Specifically, we would like to adopt Hamblin's (1973) proposal that a question denotes a set of propositions, to which we would like to add, as a presupposition, the requirement that it be possible for a member of the set to be a maximally informative answer:[62]

(119) [[how many φ]] , when defined $= \{p\colon \exists d\ (p = \varphi(d))\}$.
(120) A question Q is defined, in a world w, iff it is possible that the conjunction of all true propositions in Q is itself a member of Q.[63]

This presupposition will not be met when the Question property, φ in (119), is N-open. However, it's possible for the question property to end up describing an open interval as long as it was also possible for the property to describe a closed interval (as long as it is not *necessarily* open).

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[62] As in the text, we are following the insight of Dayal (1996), with the necessary weakening that comes from adding the word *possible*.

[63] Note the conjunction of all true propositions to Q is Heim's (1994), Answer-1.

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