On the Composition of Proportional Quantifiers

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The Question

What are the semantic primitives of quantification in natural language?
Quantification in natural language is a form of second order predication where . . .

- [Determiner NP] denote second order predicates (Generalized Quantifiers)
- Determiners denote relations between sets individuals
- NP and VP denote (characteristic functions of) sets of individuals
Primitives of Quantification in Natural Language

Semantic Primitives of Quantification in GQT

1. IP
   DP\(\langle et,t \rangle\)
   D\(\langle et,ett \rangle\)
   Every/some/most/more than half of the
   student/s
   is/are sick

(2) \([\text{every}] \) (A)(B) = 1 iff A \subseteq B
(3) \([\text{some}] \) (A)(B) = 1 iff A \cap B \neq \emptyset
(4) \([\text{most}] \) (A)(B) = 1 iff |A \cap B| > |A-B|
(5) \([\text{more than half of the}] \) (A)(B) = 1 iff |A \cap B| > \frac{1}{2}|A|

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On the Composition of Proportional Quantifiers
Importance of Proportional Quantifiers

Proportional quantifiers such as most, more than half, two thirds, 7 out of 10, every other, . . . set a benchmark because . . .

- Proportional quantifiers are not 1st order definable
- Proportional quantifiers are not sortally reducible

A determiner $D$ is sortally *reducible* iff there is a two-place boolean function $h$ st.
for all $A, B \subseteq E$, $D(A)(B) = D(E)h(A, B)$

(1) \(\left[\text{every}\right](A)(B) = 1\) iff $\forall x[A(x) \to B(x)]$
(2) \(\left[\text{most}\right](A)(B) = 1\) iff $Mx[A(x) \{\neg, \&, v, \to\} B(x)]$
(3) \(\left[\text{most}\right](A)(B) = 1\) iff $|A \cap B| > |A - B|$
(4) \(\left[\text{more than half}\right](A)(B) = 1\) iff $|A \cap B| > \frac{1}{2}|A|$
An possible alternative

An alternative approach would be to take the morpho-syntactic form of proportional determiners like *more than half* seriously and assume that (some) proportional quantifiers are constructed from

- Comparative operator
- Cardinality of function
- Division operator
- . . .
Primitives of Quantification in Natural Language

Reasons to Adopt a New Set of Primitives

Coarseness of Generalized Quantifier Theory

Two Types of Reasons to Choose

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More than half - decomposed

(1) $\exists x. x \leq_i \sigma(*\text{STUDENT}) \land |x| > \frac{1}{2} |\sigma(*\text{STUDENT})|$

more than half $M_1$ of the students

Landman (2004)
Inspired by expressions like *at least three eights of a busload full of*, I’d like to propose the following set of semantic primitives for quantification:

- Degrees (three, 0.5, ...)
- Measure functions (many, much, numerous, ...)
- Measure phrases (dozen, a busload full of, inches, ...)
- Degree modifiers, degree functions (very, -th, ...)
- Degree quantifiers (-er, -est, ...)
- Max, PART
- $\forall x, \exists x$
Commentary: Two different research strategies

- GQT generalizes to the seemingly simplest case \textit{most}
  - \textit{most} is a lexical item, its meaning therefore not derived compositionally
  - complex relations between sets such as those denoted by \textit{most} are semantic primitives

- The alternative generalizes to the worst case: e.g. \textit{at least three eights of a busload full of}
  - all proportional quantifiers are morpho-syntactically complex
  - proportional determiner meanings are not primitives
Why should we adopt the second strategy?

- GQT does not deny that *more than half* is morpho-syntactically complex.
- Instead, the claim is that the internal make-up of a determiners does not affect the determiners "external" behavior.

To argue for an alternative to GQT we need to show that this claim is incorrect!
Coarseness of GQT

- The internal make-up of determiners does not affect their external semantics.

  1. \([\text{no}] (A) (B) = 1 \text{ iff } A \cap B = \emptyset\)
  2. \([\text{zero}] (A) (B) = 1 \text{ iff } A \cap B = \emptyset\)
  3. \([\text{fewer than one}] (A) (B) = 1 \text{ iff } A \cap B = \emptyset\)

- Any two equivalent statements of the TC-import are equally good.

  4. \([\text{no}] (A) (B) = 1 \text{ iff } A \cap B = \emptyset\)
  5. \([\text{no}] (A) (B) = 1 \text{ iff } |A \cap B| = 0\)
  6. \([\text{no}] (A) (B) = 1 \text{ iff } |A \cap B| < 1\)
Coarseness of GQT

Intuitively, the treatment in (7) - (9) is preferable.

(7) \[\text{[no]} (A) (B) = 1 \text{ iff } A \cap B = \emptyset\]
(8) \[\text{[zero]} (A) (B) = 1 \text{ iff } |A \cap B| = 0\]
(9) \[\text{[fewer than one]} (A) (B) = 1 \text{ iff } |A \cap B| < 1\]

Which of the treatments in (10) - (13) are preferable given that \(|A \cap B| > |A - B| \iff |A \cap B| > \frac{1}{2}|A|\)?

(10) \[\text{[most]} (A) (B) = 1 \text{ iff } |A \cap B| > |A - B|\]
(11) \[\text{[most]} (A) (B) = 1 \text{ iff } |A \cap B| > \frac{1}{2}|A|\]
(12) \[\text{[more than half]} (A) (B) = 1 \text{ iff } |A \cap B| > |A - B|\]
(13) \[\text{[more than half]} (A) (B) = 1 \text{ iff } |A \cap B| > \frac{1}{2}|A|\]
Possible reasons to choose

- Language internal reasons:
  - Better correspondence between LF of sentences that contain *most* and $|A \cap B| > |A - B|$ on the one hand and *more than half* and $|A \cap B| > \frac{1}{2}|A|$ on the other.
  - Relies on decomposition of *most* and *more than half*.

- Language external reasons:
  - Establish that $|A \cap B| > |A - B|$ and $|A \cap B| > \frac{1}{2}|A|$ are treated differently by some language external cognitive system.
  - Show that *most* triggers $|A \cap B| > |A - B|$ while *more than half* goes with $|A \cap B| > \frac{1}{2}|A|$.
Cross-linguistic observations about MOST

- Not all that many languages that have a determiner like *most*.
- In languages that have a determiner element comparable to *most* it is morphologically related to the superlative or comparative form of *many* or *numerous*.
- No language can use *FEWEST*, the superlative of the polar opposite of *many* to express a proportional quantifier meaning along the lines of *less than half*
Distribution of MOST and FEWEST

Language specific generalization (German, English)

- *die meisten/most* is ambiguous between a relative superlative and a proportional reading.
- *die meisten/most* does not have a genuine absolute superlative interpretation.
- The constraints that govern the interpretation of superlatives in general govern also the availability of the two readings of *die meisten/most*
- *die wenigsten/fewest* has only a relative superlative reading and is unacceptable in context that don’t allow relative superlatives in general.
It behaves like a superlative - for the most part

Die meisten is ambiguous between a "relative" superlative and a proportional reading but it does not have a genuine "absolute" reading.

(1) Who climbed the highest mountain?
   b. Who climbed a mountain higher than anybody else rel.

(2) Wer hat die meisten Bücher gelesen?
   Who has the most books read?
   b. Who read more books than anybody else? rel.
   c. Who read more than half of the books? prop
It behaves like a superlative - for the most part

*Die wenigsten/the fewest* has no proportional reading and no absolute reading. It is unambiguously a "relative" superlative.

(3) Wer hat die wenigsten Buecher gelesen?

*Who has the fewest books read?*

a. Who read fewer books than anybody else? rel.sup
b. * Who read less than half of the books? prop
c. * Who read no/one the book? abs
It behaves like a superlative - for the most part

When the licensing conditions for a relative superlative reading are not met, *die meisten* is unambiguously proportional while *die wenigsten* is unacceptable or conveys "very few".

(4) Jeder hat die meisten Buecher gelesen
   * Everybody has the most books read.
   a. * Everybody read more books than everybody else. rel.sup
   b. Everybody read more than half of the books. prop
   c. * Everybody read all the books. abs

(5) ?? Jeder hat die wenigsten Buecher gelesen.
   a. *Everybody read fewer books than everybody else. rel.sup
   b. * Everybody read less than half of the books? prop
   c. * Everybody read no/one the book? abs
Even when all the syntactic licensing conditions for a relative superlative reading are met - roughly there has to be a clausemate focused or wh-expression (cf. Szabolcsi 1986) - but resulting relative meaning would we equivalent to a proportional reading die wenigsten is unacceptable or conveys "very few".

(7) Die meisten Studenten sind drINNEN
   *The most students are INside.
   *More students are inside than outside.

(8) ?? Die wenigsten Studenten sind drINNEN
   *The fewest students are INside.
   *Fewer students are inside than outside.
Questions

- Why is there no genuine absolute reading for \textit{die meisten/the most} and \textit{die wenigsten/the fewest}?
- Why is there a proportional reading for \textit{die meisten/most} but not for \textit{die wenigsten/fewest}?
MOST = MANY + EST

(1) \[
\begin{array}{c}
\text{DP} \\
\emptyset \\
\text{-est} \quad C \\
\lambda d \\
\text{NP}_{(et,t)} \\
\text{d-many students}
\end{array}
\]

(2) \[\text{[many]} \ (d) \ (*\text{NP}) = \lambda x. |x| \geq d \ & \ *\text{NP}(x)\]

(3) \[\text{[est]} \ (C) \ (D) = \lambda x. \exists d[D(d) \ (x) \ & \ \forall y[y \in C \ & \ y \neq x \rightarrow \neg D(d)(y)]]\]

(4) \(C = *\text{NP} \text{ if } \text{-est inside DP} \) (Szabolcsi’86, etc.)
MOST = MANY + EST

(5) -est presupposes that $\exists x, y[x \neq y & x \in C & y \in C]$

??You are the best mother I have.

(6) No overlap: $x \neq y$ iff $\neg \exists z[z \text{i-part } x & z \text{i-part } y]$

(7) $\emptyset$

$\lambda x. \text{students}(x) & \forall y[\text{students}(y) & x \neq y \rightarrow |x| > |y|]$

$\lambda x. \text{students}(x) & \forall y[\text{students}(y) & x \neq y \rightarrow |x| > |y|]$

$\lambda d$

NP$_{\langle et, t \rangle}$

d-many students
Motivation
Language Internal Evidence
Verification Studies
Conclusion
Appendix

Cross-linguistic observations
Case study: die meisten and die wenigsten
MOST = MANY+EST

Interim Summary

*FEWEST

(7) \[ \lambda x. \text{students}(x) \& \forall y[\text{students}(y) \& x \neq y \rightarrow |x| < |y|] \]

No plurality can satisfy \( \lambda x. *\text{NP}(x) \& \forall y[*\text{NP}(y) \& x \neq y \rightarrow |x| < |y|] \) !!

Example: Let \( S = \{a,b,c\} \).
(8) \( a+b, a+c, b+c \) will all be more numerous than their complements in \( S \).
(9) However, even the smallest plurality in \( S \), say \( a \), is not less numerous than every plurality in \( S \) different from \( a \). E.g. \( |a| = |b| \).
There is compelling cross-linguistic evidence to suggest that *MOST* is a superlative of *MANY*.

Such an analysis supports the claim that $|A \cap B| > |A - B|$ is closer in form to an LF containing *most* than $|A \cap B| > \frac{1}{2} |A|$.
Does it matter for other cognitive systems?

Assume that LFs inform verification strategies. Verifying $p$ is to collect information that supports $p$ or $\neg p$.

1. $|A \cap B| > \frac{1}{2}|A|$
   - Determine the total number of As.
   - Divide by 2
   - Compare the result to the number of As that are Bs.

2. $|A \cap B| > |A-B|$
   - Compare the number of As that are Bs to the number of As that are not Bs.
Imagine that you get a bag of marbles and your task is to find out whether most/more than half of the marbles in the bag are black.
Method 1

Empty the bag all at once and count the number of black and white marbles.

Problem: Too many degrees of freedom to solve the counting problem.
Reach in with one hand and grab a handful of marbles to see how many black and white marbles there are. Repeat that as often as necessary.

Intuitively, *most* is easier than *more than half*.
Why most is easier

*Most* triggers a form of "vote counting"
- Every handful of marbles is checked whether there are more black than white.
- Keep track of which color leads (and by how much).

*More than half* triggers a form of "counting to a criterion"
- Estimate what half of the number of marbles is.
- Check whether the number of black marbles is bigger than that.
Self-Paced Counting

Self-paced Counting is basically a computerized version of the "bag" modeled after Self-Paced Reading:

- Subjects hear a sentence whose truth/falsity relative to an array of dots they have to determine as fast and as reliable as possible. *Most of the dots are blue.*
- Subjects see an array of initially empty dots.
- The dots are incrementally filled in as subjects press the space bar.
- Previously seen dots are masked.
- Subjects can answer as soon as they have enough information.
“Most/more than half of the dots are blue.”
Self-Paced Counting: most

“Most/more than half of the dots are blue.”
Self-Paced Counting: most

"Most/more than half of the dots are blue."
Self-Paced Counting: most

“Most/more than half of the dots are blue.”

4.pdf
Self-Paced Counting: most

“Most/more than half of the dots are blue.”
“Most/more than half of the dots are blue.”
Self-Paced Counting: most

“Most/more than half of the dots are blue.”

1.pdf

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“Most/more than half of the dots are blue.”
“Most/more than half of the dots are blue.”
Self-Paced Counting: most

“Most/more than half of the dots are blue.”

4.pdf
Self-Paced Counting: most

“Most/more than half of the dots are blue.”
Self-Paced Counting: most

“Most/more than half of the dots are blue.”
Target Items:

- 24 target items: 12 *most* and 12 *more than half*
- There are as many true as false target items.
- Target items differed only with respect to what sound files precede it.
- Dot arrays varied in length between 10 and 12.
- Within the first 3 frames one cannot decide whether the sentence is true or false.
Methods and Materials

Filler Items:
- 36 Filler items: more than 5, only n, n, many, few, some
- 18 true, 18 false.
- Dot arrays ranged from 7 to 12.
- Dot arrays varied in length between 10 and 12.

Practice Items:
- 10 Practice items similar to filler items
Results:

- We analyze only RT from correct answers.
- Subjects were excluded if the percentage of correct answers was below 80.
- We focus on RTs up to frame 3 when it is not yet decidable whether a target sentence is true or false.
Results: Accuracy and total RTs

**Success Rate**

- Most
- Half

**Total RTs**

- "Most"
- "More than half"

(Size Corrected)
RTs over first 4 frames

Most and More Than Half (Size Corrected) 

Screens 1 to 3
Results of Experiment 1

Findings of Experiment 1 (20 subjects)

- *Most* and *More than half* are overall still treated as equivalent.
  - No significant difference in accuracy.
  - No significant difference in overall RT.

- Main effect of Determiner Type st. *most* is consistently faster than *more than half*.
- Main effect of Screen Number st. the later in the array the longer it takes to move to the next screen.
To determine whether SPC tracks reliably complexity of counting we ran a control experiment using *more than n* instead of *more than half* and *at least n+1* instead of *most*

(1) At least seven of the dots are blue.
(2) More than than six of the dots are blue.
Reliability of SPC: At least n and more than n
RTs over first 4 frames

At Least/More Than n (Size Corrected)

Screen for screens p - 3
The findings of Experiment 1 and 2 suggest that *most* should be disproportionately affected by distributional asymmetries.

Figure 5. Schema of experimental items in Exp. 5, 6a,b.
RTs over first all frames - 10 subjects
We gave two converging arguments that the way we describe the TC import of *most* and *more than half* is more constrained than GQT would have it.

GQT is too coarse to make the relevant distinction because it assumes that relations between sets are semantic primitives.

We need a different set of primitives for quantification such as degrees, measure functions and comparative operators.
(1) Most of the dots are blue.
(2) More than half of the dots are blue.

Figure 4. Sequence of events in revised Self-Paced Counting trials.
Ongoing and future experiments

- Yes/No questions
- Distributional Asymmetries
- Size manipulations
- more than $n$/at least $n+1$
- Monotonicity