# On the Composition of Proportional Quantifiers

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# The Question

What are the semantic primitives of quantification in natural language?

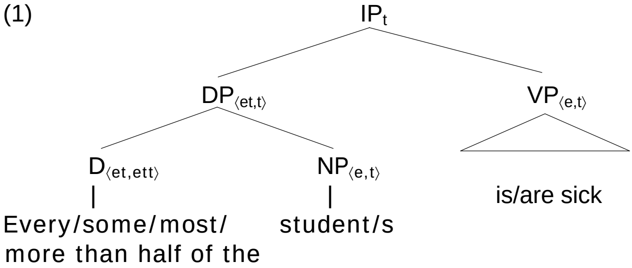
# Semantic Primitives of Quantification in GQT

Quantification in natural language is a form of second order predication where ...

- [Determiner NP] denote second order predicates (Generalized Quantifiers)
- Determiners denote relations between sets individuals
- NP and VP denote (characteristic functions of) sets of individuals

# Semantic Primitives of Quantification in GQT

(1)




(2) $[\![\text{every}]\!]$ (A)(B) = 1 iff $A \subseteq B$
(3) $[\![\text{some}]\!]$ (A)(B) = 1 iff $A \cap B \neq \emptyset$

(4) $[\![\text{most}]\!]$ (A)(B)= 1 iff $|A \cap B| > |A\text{-}B|$
(5) $[\![\text{more than half of the}]\!]$ (A)(B)= 1 iff $|A \cap B| > \frac{1}{2}|A|$

# Importance of Proportional Quantifiers

Proportional quantifiers such as most, more than half, two thirds, 7 out of 10, every other, . . . set a bench mark because . . .

- Proportional quantifiers are not 1st order definable
- Proportional quantifiers are not sortally reducible

A determiner D is *sortally* *reducible* iff there is a two place boolean function h st. for all A,B $\subseteq$ E, D(A)(B)=D(E)h(A,B)

(1) $[\![\text{every}]\!]$ (A) (B)= 1 iff $\forall$x[ A(x) $\rightarrow$ B(x) ]

(2)* $[\![\text{most}]\!]$ (A) (B)= 1 iff Mx[ A(x) { $\neg$, &,v, $\rightarrow$ } B(x) ]

(3) $[\![\text{most}]\!]$(A) (B)= 1 iff $|A \cap B| > |A - B|$

(4) $[\![\text{more than half}]\!]$(A) (B)= 1 iff $|A \cap B| > \frac{1}{2}|A|$

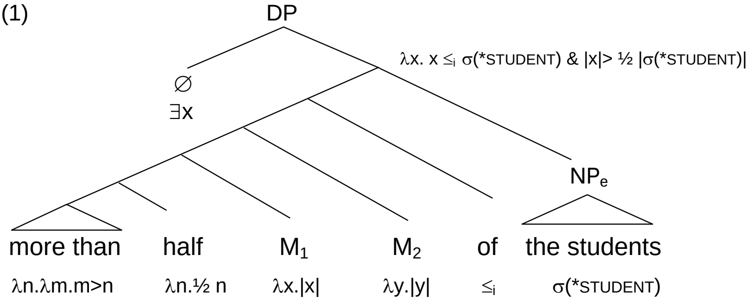
# An possible alternative

An alternative approach would be to take the morpho-syntactic form of proportional determiners like more than half seriously and assume that (some) proportional quantifiers are constructed from

- Comparative operator
- Cardinality of function
- Division operator
- ...

# More than half - decomposed

(1)




Landman (2004)

# A New Set of Primitives

Inspired by expressions like *at least three eights of a busload full of*, I'd like to propose the following set of semantic primitives for quantification:

- Degrees (three, 0.5, . . . )
- Measure functions (many, much, numerous, . . . )
- Measure phrases (dozen, a busload full of, inches, . . . )
- Degree modifiers, degree functions (very, -th, . . . )
- Degree quantifiers (-er, -est, . . . )
- Max, PART
- $\forall x$, $\exists x$

# Commentary: Two different research strategies

- GQT generalizes to the seemingly simplest case *most*
  - *most* is a lexical item, its meaning therefore not derived compositionally
  - complex relations between sets such as those denoted by *most* are semantic primitives
- The alternative generalizes to the worst case: e.g. *at least three eights of a busload full of*
  - all proportional quantifiers are morpho-syntactically complex
  - proportional determiner meanings are not primitives

# Why should we adopt the second strategy?

- GQT does not deny that *more than half* is morpho-syntactically complex.
- Instead, the claim is that the internal make-up of a determiners does not affect the determiners "external" behavior.
- To argue for an alternative to GQT we need to show that this claim is incorrect!

# Coarseness of GQT

- The internal make-up of determiners does not affect their external semantics.

(1) $[\![\text{no}]\!]$ (A) (B) = 1 iff $A \cap B = \emptyset$
(2) $[\![\text{zero}]\!]$ (A) (B) = 1 iff $A \cap B = \emptyset$
(3) $[\![\text{fewer than one}]\!]$ (A) (B)= 1 iff $A \cap B = \emptyset$

- Any two equivalent statements of the TC-import are equally good.

(4) $[\![\text{no}]\!]$ (A) (B) = 1 iff $A \cap B = \emptyset$
(5) $[\![\text{no}]\!]$ (A) (B) = 1 iff $|A \cap B| = 0$
(6) $[\![\text{no}]\!]$ (A) (B) = 1 iff $|A \cap B| < 1$

# Coarseness of GQT

- Intuitively, the treatment in (7) - (9) is preferable.

(7) $[\![\text{no}]\!]$ (A) (B) = 1 iff $A \cap B = \emptyset$
(8) $[\![\text{zero}]\!]$ (A) (B) = 1 iff $|A \cap B| = 0$
(9) $[\![\text{fewer than one}]\!]$ (A) (B)= 1 iff $|A \cap B| < 1$

- Which of the treatments in (10) - (13) are preferable given that $|A \cap B| > |A - B| \Leftrightarrow |A \cap B| > \frac{1}{2}|A|$?

(10) $[\![\text{most}]\!]$ (A) (B) = 1 iff $|A \cap B| > |A - B|$
(11) $[\![\text{most}]\!]$ (A) (B) = 1 iff $|A \cap B| > \frac{1}{2}|A|$
(12) $[\![\text{more than half}]\!]$ (A) (B) = 1 iff $|A \cap B| > |A - B|$
(13) $[\![\text{more than half}]\!]$ (A) (B) = 1 iff $|A \cap B| > \frac{1}{2}|A|$

# Possible reasons to choose

- Language internal reasons:
  - Better correspondence between LF of sentences that contain *most* and $|A \cap B| > |A - B|$ on the one hand and *more than half* and $|A \cap B| > \frac{1}{2}|A|$ on the other.
  - Relies on decomposition of *most* and *more than half*.
- Language external reasons:
  - Establish that $|A \cap B| > |A - B|$ and $|A \cap B| > \frac{1}{2}|A|$ are treated differently by some language external cognitive system.
  - Show that *most* triggers $|A \cap B| > |A - B|$ while *more than half* goes with $|A \cap B| > \frac{1}{2}|A|$.

# MOST = MANY+EST

Cross-linguistic observations about *MOST*

- Not all that many languages that have a determiner like *most*.
- In languages that have a determiner element comparable to *most* it is morphologically related to the superlative or comparative form of *many* or *numerous*.
- No language can use *FEWEST*, the superlative of the polar opposite of *many* to express a proportional quantifier meaning along the lines of *less than half*

# Distribution of MOST and FEWEST

Language specific generalization (German, English)

- *die meisten/most* is ambiguous between a relative superlative and a proportional reading.
- *die meisten/most* does not have a genuine absolute superlative interpretation.
- The constraints that govern the interpretation of superlatives in general govern also the availability of the two readings of *die meisten/most*
- *die wenigsten/fewest* has only a relative superlative reading and is unacceptable in context that don't allow relative superlatives in general.

# It behaves like a superlative - for the most part

*Die meisten* is ambiguous between a "relative" superlative and a proportional reading but it does not have a genuine "absolute" reading.

(1) Who climbed the highest mountain?

| | | |
|---|---|---|
| a. | Who climbed Mt. Everest? | abs. |
| b. | Who climbed a mountain higher than anybody else | rel. |

(2) Wer hat die meisten Buecher gelesen?
*Who has the most books read?*

| | | |
|---|---|---|
| a. | * Who read all the books? | abs. |
| b. | Who read more books than anybody else? | rel. |
| c. | Who read more than half of the books? | prop |

# It behaves like a superlative - for the most part

*Die wenigsten/the fewest* has no proportional reading and no absolute reading. It is unambiguously a "relative" superlative.

(3) Wer hat die wenigsten Buecher gelesen?
*Who has the fewest books read?*

a. Who read fewer books than anybody else? rel.sup
b. $*$ Who read less than half of the books? prop
c. $*$ Who read no/one the book? abs

# It behaves like a superlative - for the most part

When the licensing conditions for a relative superlative reading are not met, *die meisten* is unambiguously proportional while *die wenigsten* is unacceptable or conveys "very few".

(4) Jeder hat die meisten Buecher gelesen
*Everybody has the most books read.*

a. * Everybody read more books than everybody else. rel.sup
b. Everybody read more than half of the books. prop
c. * Everybody read all the books. abs

(5) ?? Jeder hat die wenigsten Buecher gelesen.

a. *Everybody read fewer books than everbody else. rel.sup
b. * Everybody read less than half of the books? prop
c. * Everybody read no/one the book? abs

# *FEWEST

Even when all the syntactic licensing conditions for a relative superlative reading are met - roughly there has to be a clausemate focused or wh-expression (cf. Szabolcsi 1986) - but resulting relative meaning would we equivalent to a proportional reading *die wenigsten* is unacceptable or conveys "very few".

(7) Die meisten Studenten sind drINNEN
*The most students are INside.*
*More students are inside than outside.*

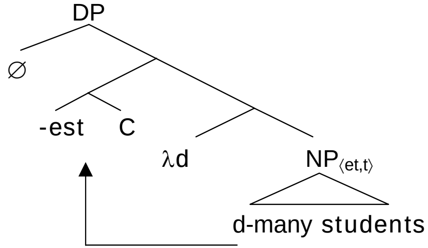
(8) ??Die wenigsten Studenten sind drINNEN
*The fewest students are INside.*
**Fewer students are inside than outside.*

# Questions

- Why is there no genuine absolute reading for *die meisten/the most* and *die wenigsten/the fewest*?
- Why is there a proportional reading for *die meisten/most* but not for *die wenigsten/fewest*?

# MOST=MANY+EST

(1)




(2) $[\![\text{many}]\!]$ (d) (*NP) = $\lambda$x. $|x| \geq d$ & *NP(x)

(3) $[\![\text{est}]\!]$ (C) (D)= $\lambda x.\exists d[D(d)(x)\ \&\ \forall y[\ y \in C\ \&\ y \neq x \rightarrow \neg\ D(d)(y)]]$

(4) C=*NP if -est inside DP( Szabolcsi'86, etc.)

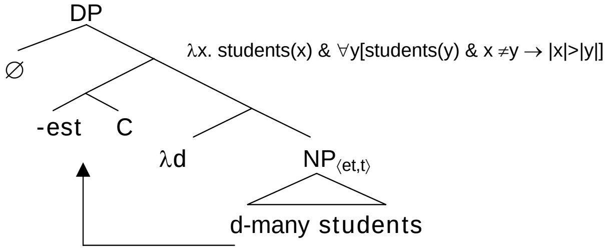
# MOST=MANY+EST

(5) -est presupposes that $\exists$ x,y[x$\neq$y & x$\in$C & y$\in$C]

??You are the best mother I have.

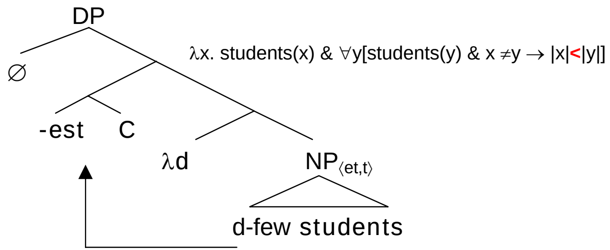
(6) No overlap: x $\neq$y iff $\neg$ $\exists$z[z i-part x & z i-part y]

(7)



# *FEWEST

(7)




No plurality can satisfy $\lambda$x. *NP(x) & $\forall$y[*NP(y) & x$\neq$y $\rightarrow$ |x|<|y|] !!

Example: Let S = {a,b,c}.
(8) a+b,a+c,b+c will all be more numerous than their complements in S.
(9) However, even the smallest plurality in S, say a, is not less numerous than every plurality in S different from a. E.g. |a|=|b|.

# Interim Summary

- There is compelling cross-linguistic evidence to suggest that *MOST* is a superlative of *MANY*.
- Such an analysis supports the claim that $|A \cap B| > |A\text{-}B|$ is closer in form to an LF containing *most* than $|A \cap B| > \frac{1}{2}|A|$.

# Does it matter for other cognitive systems?

Assume that LFs inform verification strategies. Verifying p is to collect information that supports p or ¬p.

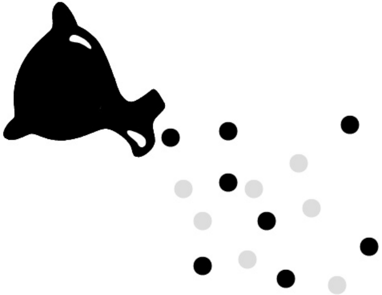
- $|A \cap B| > \frac{1}{2}|A|$
  - Determine the total number of As.
  - Divide by 2
  - Compare the result to the number of As that are Bs.
- $|A \cap B| > |A\text{-}B|$
  - Compare the number of As that are Bs to the number of As that are not Bs.

# The basic idea

Imagine that you get a bag of marbles and your task is to find out whether most/more than half of the marbles in the bag are black.

# Method 1

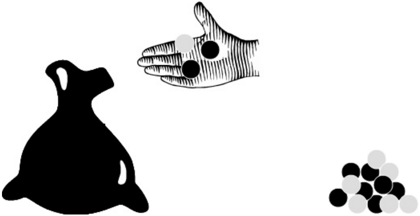
Empty the bag all at once and count the number of black and white marbles.



Problem: Too many degrees of freedom to solve the counting problem.

# Method 2

Reach in with one hand and grab a handful of marbles to see how many black and white marbles there are. Repeat that as often as necessary.



Intuitively, *most* is easier than *more than half*.

# Why most is easier

*Most* triggers a form of "vote counting"

- Every handful of marbles is checked whether there are more black than white.
- Keep track of which color leads (and by how much).

*More than half* triggers a form of "counting to a criterion"

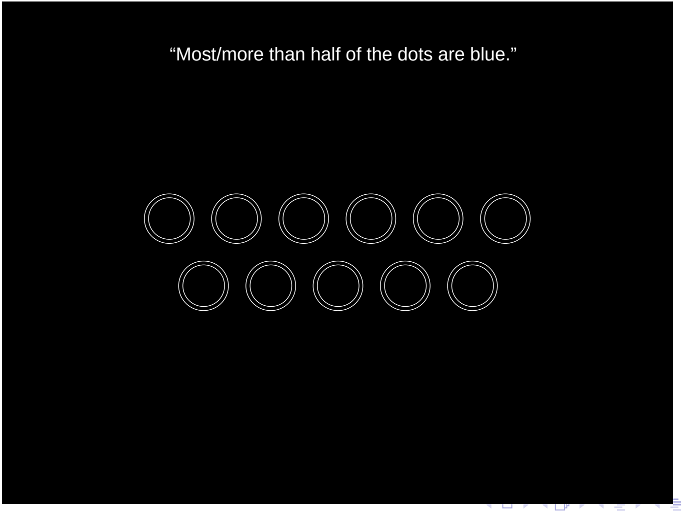
- Estimate what half of the number of marbles is.
- Check whether the number of black marbles is bigger than that.

# Self-Paced Counting

Self-paced Counting is basically a computerized version of the "bag" modeled after Self-Paced Reading:

- Subjects hear a sentence whose truth/falsity relative to an array of dots they have to determine as fast and as reliable as possible. *Most of the dots are blue.*
- Subjects see an array of initially empty dots.
- The dots are incrementally filled in as subjects press the space bar.
- Previously seen dots are masked.
- Subjects can answer as soon as they have enough information.

# Self-Paced Counting: most

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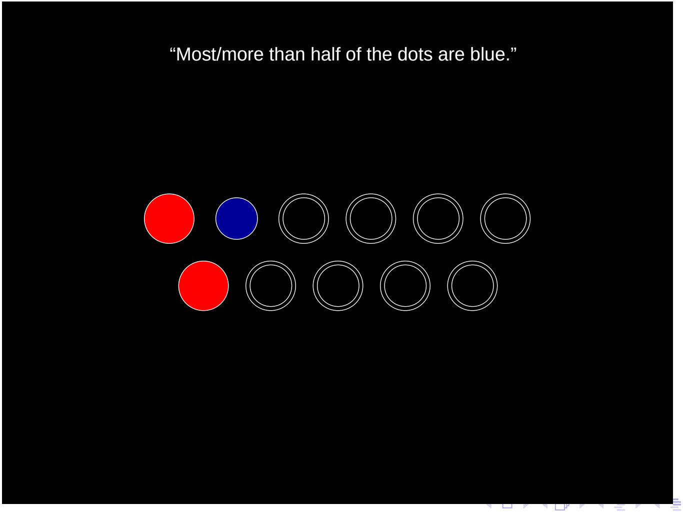
# Self-Paced Counting: most

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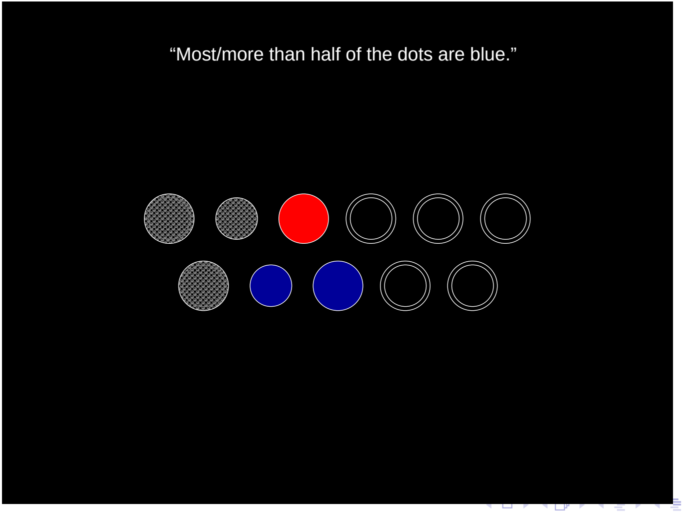
# Self-Paced Counting: most

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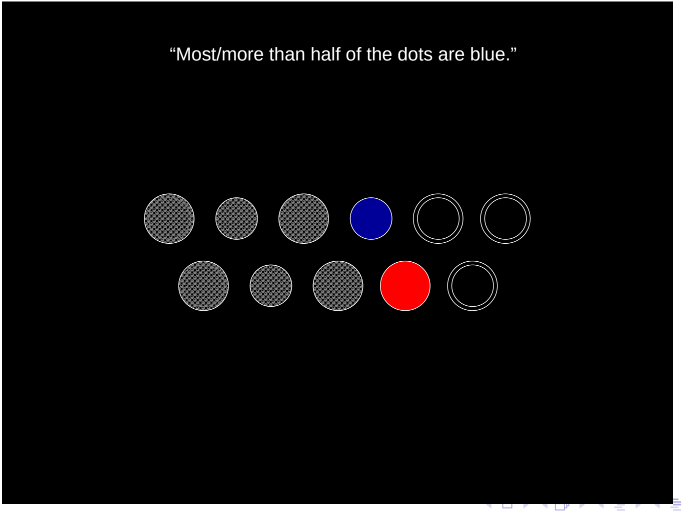
# Self-Paced Counting: most

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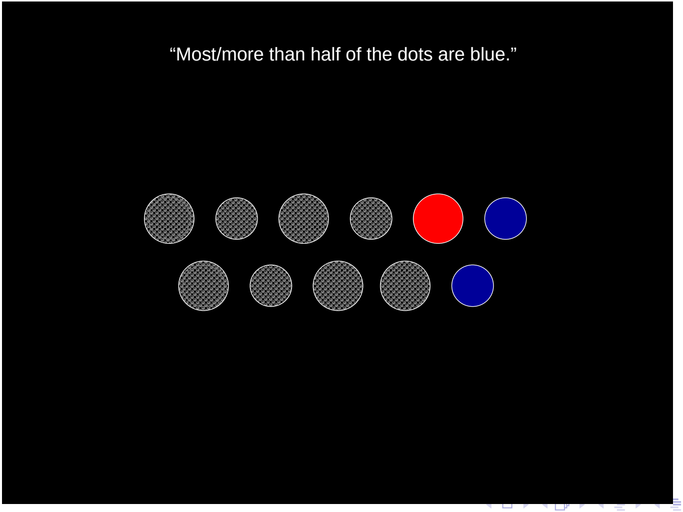
# Self-Paced Counting: most

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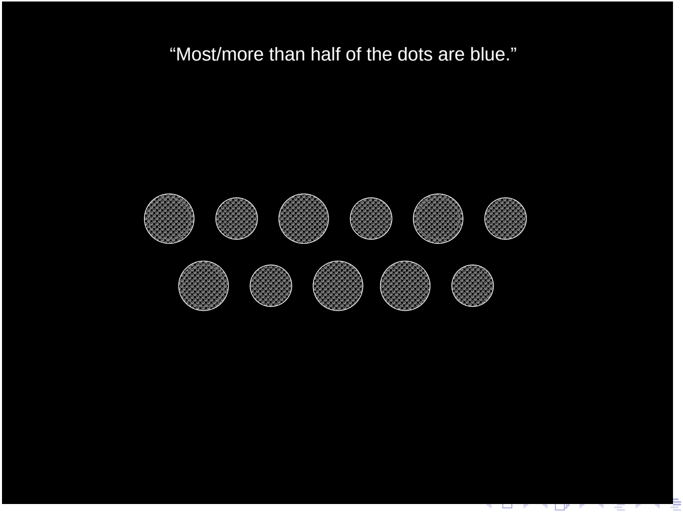
# Self-Paced Counting: most

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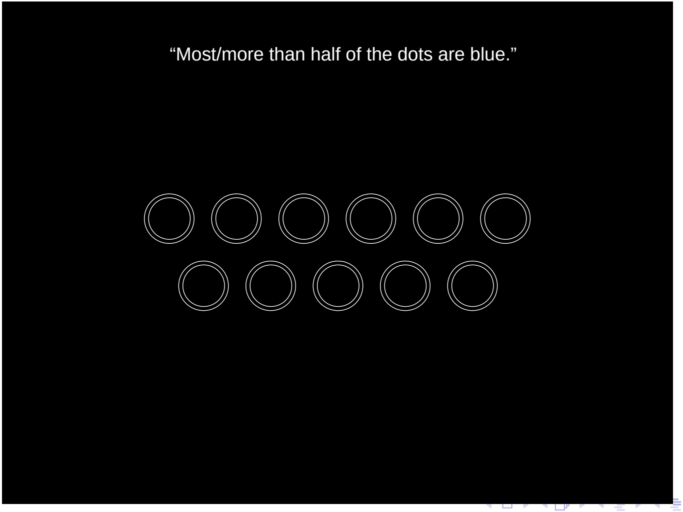
# Self-Paced Counting: most

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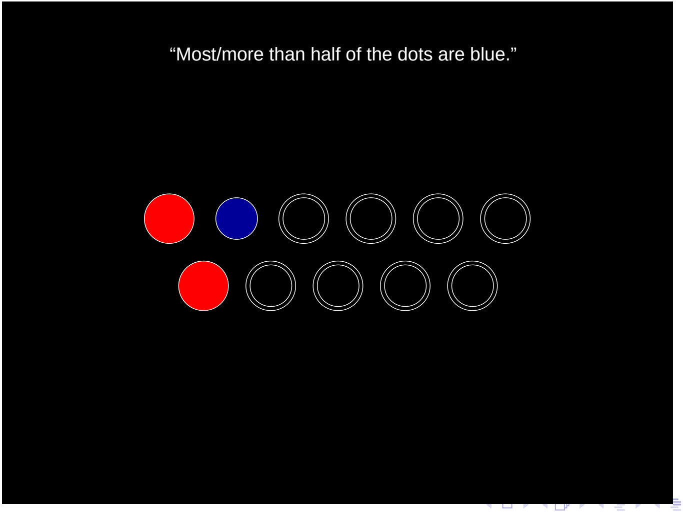
# Self-Paced Counting: most

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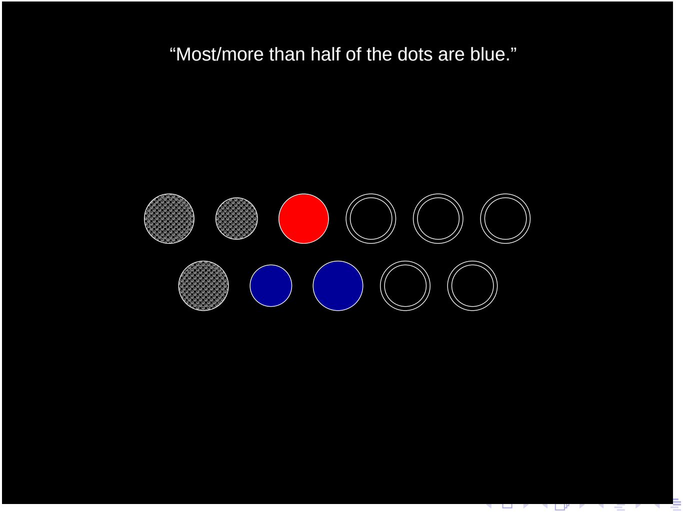
# Self-Paced Counting: most

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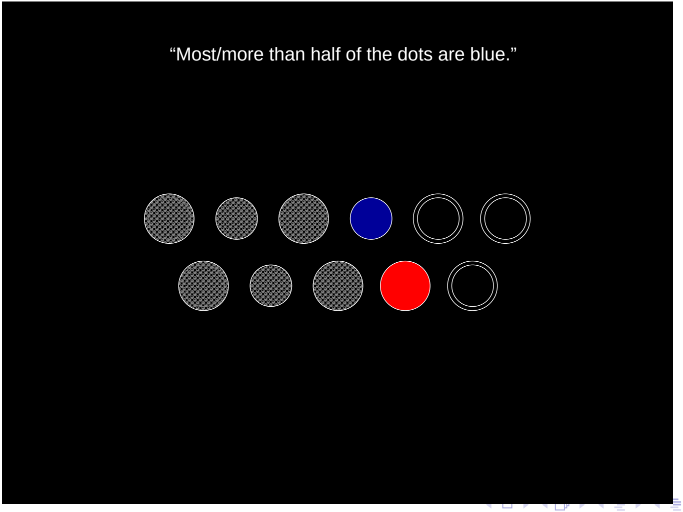


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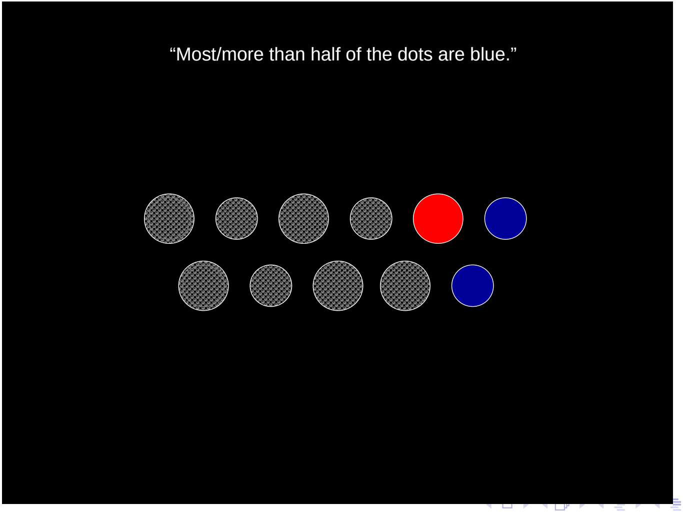
# Self-Paced Counting: most

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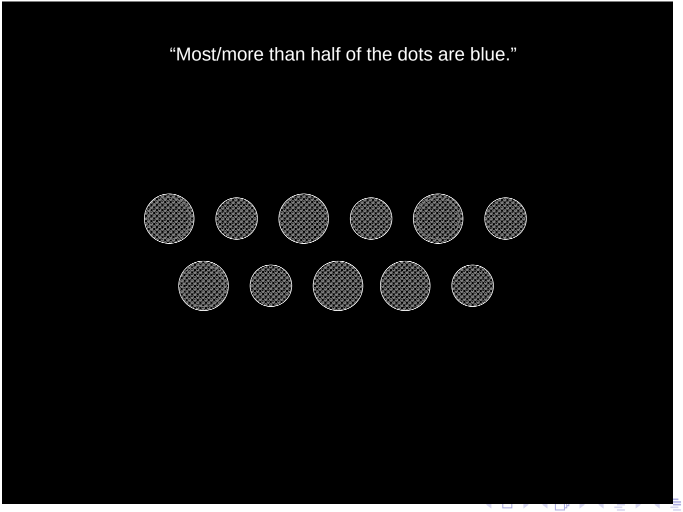
# Self-Paced Counting: most

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# Methods and Materials

Target Items:

- 24 target items: 12 *most* and 12 *more than half*
- There are as many true as false target items.
- Target items differed only wrt. what sound files precedes it.
- Dot arrays varied in length between 10 and 12.
- Within the first 3 frames one cannot decide whether the sentence is true or false.

# Methods and Materials

Filler Items:

- 36 Filler items: more than 5, only n, n, many, few, some
- 18 true, 18 false.
- Dot arrays ranged from 7 to 12.
- Dot arrays varied in length between 10 and 12.

Practice Items:

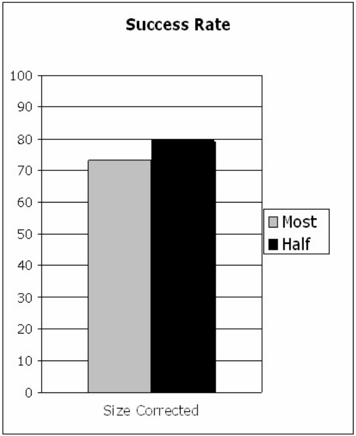
- 10 Practice items similar to filler items

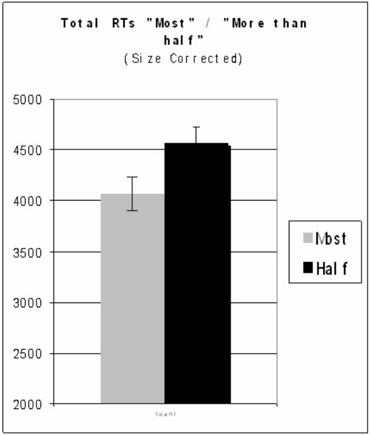
# Analysis

Results:

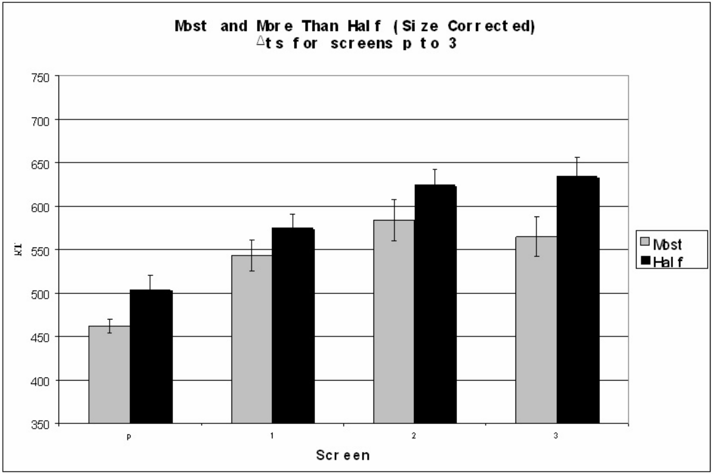
- We analyze only RT from correct answers.
- Subjects were excluded if the percentage if correct answers was below 80
- We focus on RTs up to frame 3 when it is not yet decidable whether a target sentence is true or false.

# Results: Accuracy and total RTs



# RTs over first 4 frames



# Results of Experiment 1

Findings of Experiment 1 (20 subjects)

- *Most* and *More than half* are overall still treated as equivalent.
  - No significant difference in accuracy.
  - No significant difference in overall RT.

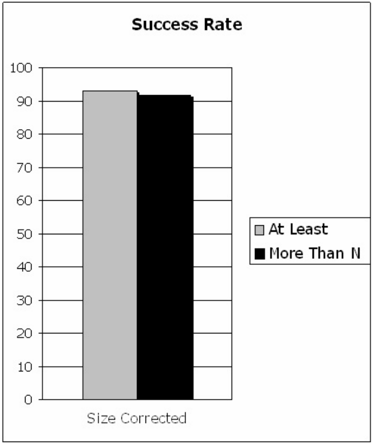
- Main effect of Determiner Type st. *most* is consistently faster than *more than half*.
- Main effect of Screen Number st. the later in the array the longer it takes to move to the next screen.

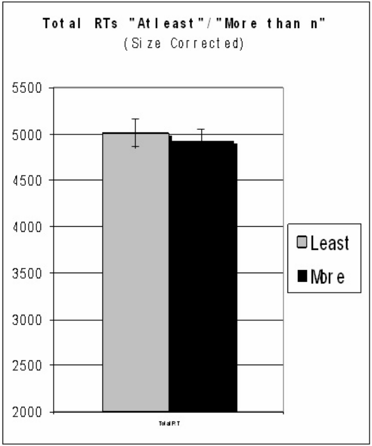
# Reliability of SPC: At least n and more than n

To determine whether SPC tracks reliably complexity of counting we ran a control experiment using *more than n* instead of *more than half* and *at least n+1* instead of *most*

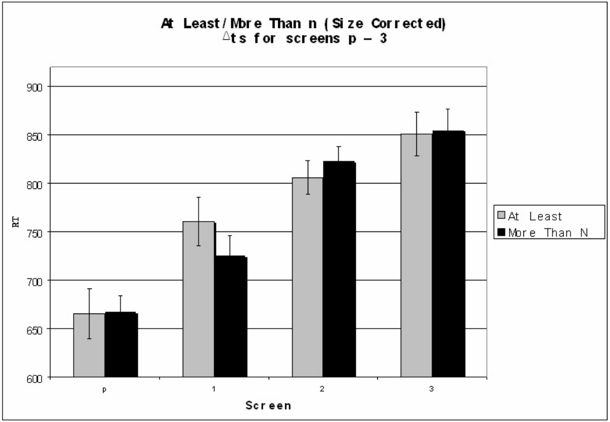
(1) At least seven of the dots are blue.
(2) More than than six of the dots are blue.

# Reliability of SPC: At least n and more than n



# RTs over first 4 frames



# Ongoing Research: Distributional Asymmetries

The findings of Experiment 1 and 2 suggest that *most* should be disproportionately affected by distributional asymmetries

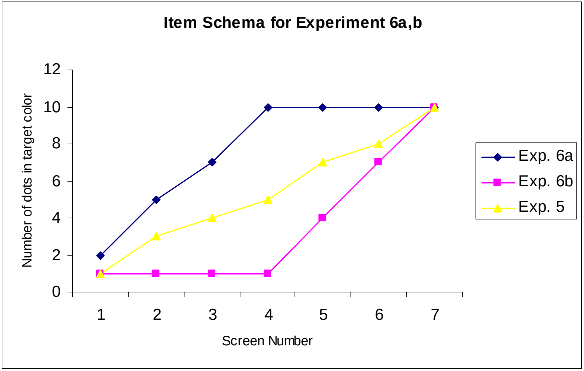
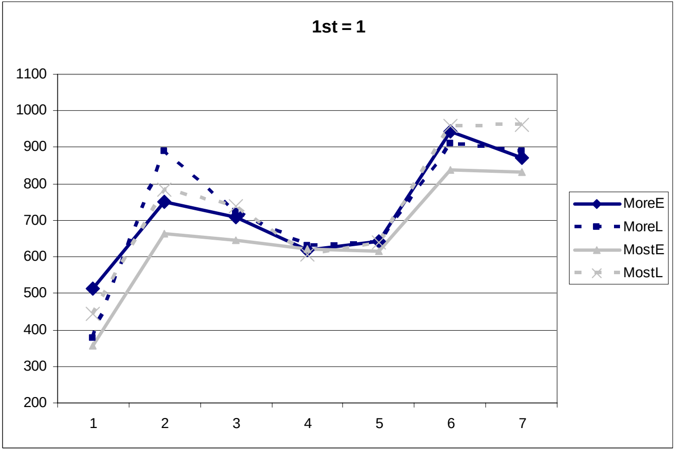



Figure 5. Schema of experimental items in Exp. 5, 6a,b.

# RTs over first all frames - 10 subjects



# Conclusions

- We gave two converging arguments that the way we describe the TC import of *most* and *more than half* is more constrained than GQT would have it.
- GQT is too coarse to make the relevant distinction because it assumes that relations between sets are semantic primitives.
- We need a different set of primitives for quantification such as degrees, measure functions and comparative operators.

# Modifications to the basic design

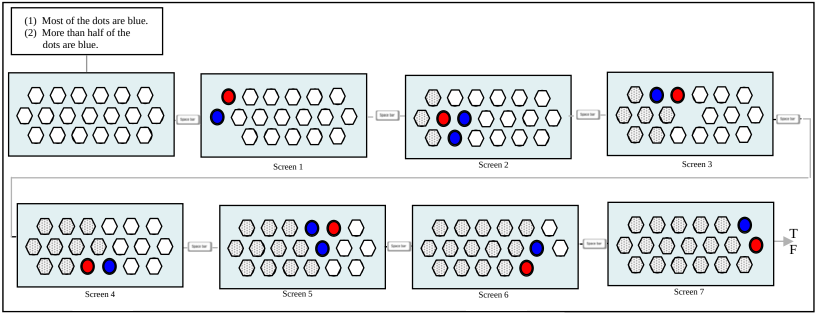



Figure 4. Sequence of events in revised Self-Paced Counting trials.

# Ongoing and future experiments

- Yes/No questions
- Distributional Asymmetries
- Size manipulations
- more than n/at least n+1
- Monotonicity