Note on Managerially Efficient Experimental Designs

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Abstract

There are many examples in marketing where some managerial decisions are more critical than others. In these situations, the researcher may wish to focus experimental precision on estimates of greater managerial interest. For example, in a conjoint analysis to support product design, some decisions may be critical and irreversible while others might be easy to change after the product is launched. However, standard measures of precision (e.g., D-efficiency) do not allow differential focus. We propose alternative "managerial efficiency" criteria (M-errors) and explore their properties. For example, orthogonal experimental designs do not assure orthogonality for estimates of managerial interest (M-orthogonal), nor vice versa. We provide many illustrative examples. One illustrative example shows that managerial efficiency can be improved with a slight reduction in standard efficiency measures. Another illustrative example using the M-criteria suggests that price levels be set closer to feature costs than at extreme levels – the latter a common "best practice."

Assuring managerial focus is complex. Although a simple generalization of D-errors, M_D-errors, focuses precision on managerial decisions, it does not assure M-orthogonality. Thus, we propose alternative definitions of M-error that are minimized by M-orthogonal designs. Existing algorithms to minimize D-errors are readily adopted to M-errors, however, we propose a new algorithm that provides useful starting solutions which are closer to M-orthogonal designs. In a final section we illustrate how the concepts extend to choice data.

Keywords: Conjoint analysis, experimental design, product development, efficiency

1. Motivation

Consider three examples, each stylized and disguised from recent applications.

1) A manufacturer of consumer electronic devices is considering five features that might be added to the device and is planning a conjoint analysis to determine whether consumers' willingness-to-pay exceeds the marginal cost of providing each feature.
2) A medical devices firm is developing a new line of kidney dialysis machines to be launched globally. Two features, coupled in the engineering development, determine the basic technology and lock the company into a strategic direction. The remaining eight features are easy to add or delete without major capital investments. The firm wants to design a conjoint analysis experiment that reflects the differential managerial importance of information about the two basic-technology features.
3) A retailer is considering seven changes in its store layout and wants to design an experiment to evaluate the potential changes. This experiment would be carried out across 50 stores in an experimental region. Three of those changes are fundamental and would change completely the layout of the store while four are less critical and are easily reversible. Furthermore, store-layout constraints are such that any two, but not all three, of the fundamental changes are feasible.

In each of these examples, some *combinations* of partworth estimates are more critical to the managerial decisions than other combinations. All five of the feature decisions faced by the electronic devices manufacturer depend on the partworths of those features compared to the partworth for price. Furthermore, the range on price is likely to cover the potential addition of all five features, even though the willingness-to-pay for any one feature involves a much smaller price difference. The medical devices firm would be willing to sacrifice precision on the eight minor features if it meant greater precision on the two coupled, basic-technology treatments. The retailer needs greater precision on the three fundamental-change combinations (three features taken two at a time) and would be willing to sacrifice precision on the easily-reversible changes. Other examples include magazine cover design (as in Conde Nast's web-based system), services packages (as offered by Comcast, DirecTV, the Dish Network, or Time-Warner Cable), feature packages (as offered by General Motors, Ford, and others), or advertising copy design.

In this note we illustrate that standard measures of efficiency, that are used to evaluate conjoint analysis designs and other experimental designs in marketing, can be modified to take managerial relevance into account. We introduce revised error criteria, which we call managerial efficiency or M-efficiency, and suggest that researchers should seek designs that are M-efficient rather than efficient by standard measures (D-efficiency, A-efficiency, or G-efficiency – defined below). We examine the properties of various M-efficiency criteria, especially as they relate to orthogonality, and suggest algorithms (some existing) that can be used to improve M-efficiency. We begin with a brief review of efficiency criteria.

For the purpose of this note we consider designs that are not adapted for each individual as in adaptive conjoint analysis (ACA) or polyhedral methods, although M-efficiency criteria can be used, post hoc, to evaluate the adapted experimental designs.

2. Efficient Experimental Designs

Researchers (and managers) often seek experimental designs such that the estimates of the magnitude of the experimental treatments, e.g., partworths in conjoint analysis, are (1) uncorrelated with one another, and (2) have the same precision. These goals are achieved with designs that are balanced and orthogonal.

However, balanced and orthogonal designs are not always feasible or cost-effective (Addelman 1962, Kuhfeld, Tobias and Garratt 1994). In response researchers have defined several measures of efficiency by which to evaluate designs. These measures are known as A-efficiency, D-efficiency, and G-efficiency, with the most common being D-efficiency (Bunch, Louviere and Anderson 1994, Kuhfeld, Tobias and Garratt 1994, Huber and Zwerina 1996, Arora and Huber 2001). These measures have three characteristics in common. First, if a balanced and orthogonal design exists, it has maximum efficiency. Second, efficiency is maximized if the corresponding type of error is minimized (A-, D-, or G-error). Third, that error is proportional to a norm defined on the covariance matrix of the estimates of the partworths (or experimental treatments). A-errors are monotonically increasing in the trace of the covariance matrix, D-errors in the determinant of the matrix, and G-errors in the maximum diagonal element. For simplicity of exposition, we focus on modifications to the most popular criterion, D-error. The concepts of this note are extendable to the norms underlying the other criteria. We begin with metric data and generalize our analysis to choice data in Section 8. For metric data and the D-norm, the standard definition is:

$$\text{D-error} = q \det((X'X)^{-1})^{1/n}$$

In this definition, X is the (suitably-scaled) experimental design matrix, q is the number of questions and n is the number of parameters.

3. A Numerical Example

Consider a metric conjoint analysis study, as in the electronic devices example, with five binary attributes plus price. Appendix 1 provides an example of a balanced and orthogonal experimental design (the first column corresponds to the intercept) and the corresponding covariance matrix of the parameters. With this design, the estimates of the six parameters and the intercept are uncorrelated and all have equal variances. This design minimizes D-errors (as well as A-errors and G-errors).

Now consider the managerial decisions that need to be made. Assume that the cost of each feature is \$10 and that the difference between the low and high levels of price is \$50. In this numerical example the manufacturer will include a feature in the device if the willingness-to-pay (WTP) for a feature is at least \$10. Let $u_1, u_2, \ldots, u_5$ represent the partworths (utilities) for the five features, let u_p represent the partworth of a \$50 price reduction, and let C be an intercept in the estimation. The estimates of managerial interest are linear combinations of the partworths, in particular: $m_1 = u_1 - u_p/5$, $m_2 = u_2 - u_p/5$, … , $m_5 = u_5 - u_p/5$. We rewrite these goals in matrix form as follows:

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & -1/5 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1/5 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1/5 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1/5 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1/5 \end{bmatrix} . \begin{bmatrix} C \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_p \end{bmatrix} = M_{WTP} \cdot \begin{bmatrix} C \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_p \end{bmatrix}$$

The covariance matrix of the estimates of managerial interest is proportional to $\Sigma^M = M_{WTP}(X'X)^{-1}M_{WTP}'$ (Judge, et. al. 1985, p. 57). As shown in Appendix 1, even though the estimates of the parameters are uncorrelated, the estimates of managerial interest are not. While minimizing D-errors ensures that the estimates of the partworths are uncorrelated, it does not ensure that the estimates of managerial interest are uncorrelated or that precision is appropriately focused. We might ask whether we can identify a different experimental design in which (1) the estimates of managerial interest are uncorrelated and/or (2) a reasonable definition of managerial error (M-error) is minimized.

4. Defining M-Efficiency

Because the estimates of managerial interest are defined by Σ_M rather than Σ, we focus precision on the partworth combinations that matter most to the manager's decision. As a direct generalization of D-errors, we define M_D-errors as follows for suitably scaled *X* matrices. Let n_M be the number of estimates of managerial interest.

$$M_D\text{-error} = q \det(M(X'X)^{-1}M')^{1/n_M}$$

To focus research on those decisions of high managerial relevance, a researcher might accept higher D-errors in order to reduce M_D-errors. Consider the same orthogonal array, *X*, given in Appendix 1. This design minimizes D-errors but does not necessarily minimize M_D-errors for $M = M_{WTP}$ as defined above. We illustrate that it is possible to decrease M_D-errors with a new "managerial" design, X_M. This algorithm is for illustrative purposes only. We later address more practical algorithms. As an illustration, we perturb $X'_M X_M$ closer to $M'M$ while maintaining full rank. That is, choose X_M such that $X'_M X_M = \alpha[\beta I + (1-\beta)M'M]$ where *I* is an identity matrix, α and β are scalars, $\beta \in [0,1]$, and α is chosen to maintain constant question "magnitudes."[1] By construction, X_M is no longer an orthogonal array, hence D-errors will increase. However, M_D-errors decrease because X_M is more focused on the managerial problem. For example, with β = 1/10, M_D-errors decrease by over 25%.

5. M_D-Efficiency vs. M-Orthogonality

In the electronic devices example, we sacrifice D-errors to decrease M_D-errors. We also sacrifice orthogonality of the design matrix. We now examine further the relationship between M_D-efficiency and M-orthogonality. (We say a design is M-orthogonal if $M(X'X)^{-1}M'$ is diagonal with all its diagonal elements equal.[2]) Recall that for D-errors, "if a balanced and orthogonal design exists, then it has optimum efficiency (Kuhfeld, Tobias, and Garratt 1994, p.

[1] Efficiency criteria tend to push feature levels to the extremes of the feasible space. Thus, to compare different designs we must attempt to keep the "magnitudes" of the features constant. In this example, we temporarily ignore integrality of X_M and keep the trace of $X_M'X_M$ equal to the trace of *X'X*. Our goal is to provide an <u>example</u> where M-efficiency matters. We can also create examples for other constraints – the details would be different, but the basic insights remain.

[2] Having all diagonal elements equal will also be referred to as "M-balance." Hence, unless otherwise specified, by "M-orthogonal" we imply "completely" M-orthogonal (M-orthogonal and M-balanced).

546).” This does not necessarily generalize to M_D-errors. In particular, a design can be maximally M_D-efficient but not M-orthogonal and a design can be M-orthogonal but not maximally M_D-efficient. We provide counterexamples for both directions.

To demonstrate that optimal M_D-efficiency does not imply M-orthogonality, consider the case where M is square and full rank. In this case, a design, X, that minimizes D-errors will also minimize M_D-errors. Specifically, $\det(\Sigma_M) = \det(M(X'X)^{-1}M') = \det(M)\det((X'X)^{-1})\det(M') = \det(M')\det(M)\det((X'X)^{-1}) = \det(M'M)\cdot\det((X'X)^{-1})$. Because M is fixed, this last expression is minimized if $\det((X'X)^{-1})$ is minimized. Appendix 2 demonstrates a square, full-rank M for which X is orthogonal (and hence minimizes M_D-errors), but the covariance matrix $\Sigma^M = M(X'X)^{-1}M'$ is not diagonal (and hence X is not M-orthogonal).

We also demonstrate that M-orthogonality does not imply optimal M_D-efficiency. Appendix 2 provides an example where a design, Y, is M-orthogonal, but does not minimize M_D-errors. (In this example we assume that all factors are continuous). Specifically, although $M(Y'Y)^{-1}M'$ is diagonal, M_D-errors are larger for Y than X (1.00 vs. 0.372). In this example, all factors are not at their extreme levels in Y and the average precision of the managerial quantities is lower for design Y than for design X, even though the managerial estimates of interest are uncorrelated and have the same precision.

Finally, we note that despite counterexamples illustrating that M_D-efficiency and M-orthogonality do not always coincide, we later show cases where M-orthogonal designs decrease M_D-errors.

6. Alternative Definitions of M-Errors that Address M-Orthogonality

Managers or researchers may wish to balance M_D-efficiency and M-orthogonality. Thus, we propose two alternative error measures, M_1-errors and M_2-errors. Let Σ_{ii}^M be the i^{th} diagonal element of Σ^M, that is, the variance of the i^{th} managerially-relevant estimate. Let a and b be constants chosen by the manager ($0 \le a, b \le 1$). The alternative error criteria are:

$$M_1\text{-error} = a\left(\max_i \Sigma_{ii}^M - \min_i \Sigma_{ii}^M\right) + (1-a)\sqrt{\frac{\sum_{i\neq j}\left(\Sigma_{ij}^M\right)^2}{n_M.(n_M-1)}}$$

$$M_2\text{-error} = b\,(M_D\text{-error}) + (1-b)\,M_1\text{-error}$$

M_1-error enables the manager or researcher to choose between an emphasis on balance and an orthogonal (managerial) design matrix. The first term in M_1-error is the difference in the maximum and minimum error variances and rewards balance. The second term in M_1-error is the root-mean-square of the non-diagonal covariances and rewards orthogonality. It is easy to show that M_1-error is always non-negative and that it is a norm on the set of symmetric, positive semi-definite matrices. Because the first term is minimized if the variances of the estimates of managerial interest are equal and the second term is minimized if Σ^M is diagonal, M_1-error will be minimized (at 0) if and only X_M is M-orthogonal.

M_2-error allows balancing M_D-error and M-orthogonality (as measured by M_1-error).

7. Algorithms to Generate M-Efficient Experimental Designs

Most existing algorithms which minimize D-errors are discrete optimization methods that can be adapted readily to the managerial criteria. By substituting M_D-, M_1-, or M_2-errors for D-errors, researchers can develop M-efficient designs with Dykstra's (1971) sequential search method, Mitchell and Miller's (1970) simple exchange algorithm, Mitchell's (1974) DETMAX algorithm, or Cook and Nachtsheim's (1980) modified Federov algorithm (see Tobias and Garratt 1994 for a review).

We propose an additional algorithm for generating M-orthogonal designs using an orthogonal and balanced design as a starting point. This simple algorithm can provide starting solutions for existing algorithms or be used as a means to improve M-efficiency.

Case 1: *M* is a Square, Full-rank Matrix

We first show that any design proportional to XM is M-orthogonal if X is balanced and orthogonal ($X'X = qI$). Specifically, $M(M'X'XM)^{-1}M' = MM^{-1}qI(M')^{-1}M' = qI$. Intuitively, right multiplying X by M makes the design orthogonal and balanced *in the estimates of managerial interest* rather than the initial parameters.

However, XM may not satisfy the integrality constraints imposed by discrete factors. One heuristic adjustment is to scale XM such that all its entries are within the admissible range, then round the elements of the resulting design to the closest "admissible" integers or combination of integers. The resulting design may not be perfectly M-orthogonal, but may have a lower M_1-error than the original design. In Appendix 3 we provide an example in which M_1-error was reduced by 48% with this algorithm. We first generated *M* randomly.[3] The resulting design, X_1, reduces the difference between the maximum and minimum diagonal element from 0.357 to 0.165 and reduces the root mean square of the diagonal elements from 0.060 to 0.051. If $a = ½$, then M_1-error is reduced from 0.417 to 0.216. In this case, M_D-errors are increased from 0.285 to 0.409, hence X_1 has a lower M_2-error than *X* as long as $b < 0.62$.

As a test, we generated 10,000 random *M* matrices and applied the algorithm to obtain a new design. In 24% of the cases, the procedure yielded a design with a lower M_1-error. We conclude that, although the algorithm does not guarantee improvement, it is worth including in one's experimental design tool kit, and may be a valuable starting solution for extant algorithms.

Case 2: *M* is not a Square or Full-rank Matrix

If there are fewer estimates of managerial interest than there are partworths or experimental treatments, then *M* will have more columns (*c*) than rows (*r*). In this case, we augment *M* to make it square and full-rank and apply the Case 1 algorithm. This is possible because an M-orthogonal design is also M-orthogonal in any subset of the estimates of managerial interest. We provide an example in the next section.

If there are more estimates of managerial interest than there are partworths or experimental treatments, then *M* has more rows (*r*) than columns (*c*). In this case, it is not possible to generate an M-orthogonal design. For example, if we have six features but eight estimates of managerial interest, then at least two of the managerial estimates can be written as linear combinations of the other six. Even if the other six estimates are orthogonal, the remaining two estimates will be correlated with the other six. In this case, the researcher might consider focusing orthogonality on a subset of the estimates of managerial interest.

Illustrative Example: Willingness to Pay

We illustrate the algorithm with the electronic-device example where the manager is focused on estimates of willingness to pay. Let *X*, M_{WTP}, and $M_{WTP}(X'X)^{-1}M'_{WTP}$ be as in Appendix 1. We first expand M_{WTP} to M^{+}_{WTP} which is square and full rank and such that all the entries of XM^{+}_{WTP} are between -1 and +1. As demonstrated in Appendix 4, this algorithm provides an

[3] Because the intercept is rarely involved in any estimates of managerial interest, all of the entries corresponding to the intercept are set to 0 in rows 2 to 7. The first row is added to make *M* full rank.

M-orthogonal design, X_{WTP}. In this simple case, the new design is M-orthogonal <u>and</u> reduces M_D-errors (by approximately 4%).

It is interesting that the X_{WTP} generated by this algorithm does not set price at its extreme levels.[4] The prices of the profiles are closer to the cost of providing the features. This *managerial* design contradicts the common wisdom of setting continuous factors at their extreme levels to minimize D-errors. By using non-extreme prices in the conjoint design, we increase the variance of the estimated price partworth, but decrease the variance of the willingness-to-pay estimates, which are *combinations* of partworth estimates.

8. Generalization to Choice-Based Conjoint Designs

In choice-based conjoint designs the covariance matrix, Ω^{-1}, is given by: $\Omega^{-1} = (Z'PZ)^{-1}$ where Z is a probability-centered design matrix and P is a diagonal matrix of choice probabilities. A-, G- and D-errors are then defined with respect to Ω^{-1} (Huber and Zwerina 1996, p. 307 and Arora and Huber 2001, p. 274). For example, for the D-norm, we have:[5]

$$\text{D-error (choice)} = \det((Z'PZ)^{-1})^{1/n}$$

Following the arguments in this note, the asymptotic covariance matrix of the managerial quantities is $M(Z'PZ)^{-1}M'$ and we define M_D-, M_1-, and M_2-errors analogously. The algorithm proposed to derive M-orthogonal designs generalizes readily to choice data.[6] As with metric data, M_D-efficiency does not imply M-orthogonality or vice versa.

We consider an illustrative example, again using WTP as the estimates of managerial interest. We begin with the design X from the previous sections and cyclically generate alternatives as in Arora and Huber (2001) and Huber and Zwerina (1996). With binary features and binary choice sets, this produces a 12-question choice experiment. One profile in the r^{th} choice is represented by the r^{th} row of X and the other provided by minus the r^{th} row. As illustrated in Appendix 5, this orthogonal design does not produce an M-orthogonal design. However, the managerially-generated choice design (based on X_{WTP}) is M-orthogonal.

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[4] Kanninen (2002) reduces D-errors for *choice-based* designs by setting one feature at a non-extreme level.
[5] In this definition we follow the literature and drop the constant, *q*, which does not affect any optimizations.
[6] The algorithm is exact if we use D_0-errors, i.e., the errors obtained if all parameters are assumed to be zero. However, the algorithm is approximate if we use generalized D_p-errors as computed by Huber and Zwerina (1996) and Arora and Huber (2001). For ease of exposition, the example in Appendix 5 uses D_0-errors.

9. Summary

The purpose of this note is to provide easy-to-compute criteria to focus experimental designs on estimates of managerial interest. We explore the properties of the criteria and illustrate how they might be used in existing (and new) algorithms. Specifically,

- Managers often need to focus on combinations of conjoint-analysis partworths and/or experimental treatments.
- Traditional error criteria only focus on the covariance of the estimates of the individual partworths or experimental treatments.
- M-errors focus precision on the estimates of managerial interest.
- Orthogonal designs are not necessarily M-efficient nor vice versa.
- M_D-efficient designs are not necessarily M-orthogonal nor vice versa.
- M_1-, and M_2-errors address this limitation of M_D-error.
- Existing algorithms can be used to optimize the M-error criteria.
- A new, simple algorithm often improves M-efficiency and provides useful starting solutions to existing algorithms.
- The M-criteria extend readily to choice-based conjoint designs.
- The M-criteria can be extended to other norms (A-errors and G-errors).

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Appendices to a Note on Managerially Efficient Experimental Designs

Appendix 1

$$X = \begin{bmatrix} +1 & +1 & -1 & -1 & -1 & -1 & -1 \\ +1 & -1 & +1 & -1 & +1 & +1 & +1 \\ +1 & +1 & +1 & +1 & -1 & +1 & +1 \\ +1 & -1 & -1 & +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 & +1 & +1 & -1 \\ +1 & +1 & -1 & +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & -1 & +1 & -1 & +1 \\ +1 & -1 & -1 & -1 & -1 & +1 & -1 \\ +1 & -1 & +1 & -1 & -1 & -1 & +1 \\ +1 & -1 & +1 & +1 & -1 & -1 & -1 \\ +1 & +1 & +1 & +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 & +1 & +1 & -1 \end{bmatrix}$$

$$\Sigma = (X'.X)^{-1} = \begin{bmatrix} 0.0833 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0833 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0833 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0833 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0833 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0833 \end{bmatrix}$$

$$\Sigma^{M} = M_{WTP}(X'.X)^{-1}.M_{WTP}' = \begin{bmatrix} 0.0867 & 0.0033 & 0.0033 & 0.0033 & 0.0033 \\ 0.0033 & 0.0867 & 0.0033 & 0.0033 & 0.0033 \\ 0.0033 & 0.0033 & 0.0867 & 0.0033 & 0.0033 \\ 0.0033 & 0.0033 & 0.0033 & 0.0867 & 0.0033 \\ 0.0033 & 0.0033 & 0.0033 & 0.0033 & 0.0867 \end{bmatrix}$$

$$M_{WTP} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & -1/5 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1/5 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1/5 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1/5 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1/5 \end{bmatrix}$$

Appendix 2

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & -0.5 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & -0.5 & -0.5 & 0.5 \\ 0 & 0 & 0 & -0.5 & 0 & -0.5 & 0 \\ 0 & 0 & -0.5 & 0 & -0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M.(X'.X)^{-1}.M' = \begin{bmatrix} 0.0833 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0417 & 0 & 0 & 0.0208 & 0 & 0.0208 \\ 0 & 0 & 0.0417 & 0.0208 & 0 & 0.0208 & 0 \\ 0 & 0 & 0.0208 & 0.0625 & 0.0208 & 0.0208 & 0 \\ 0 & 0.0208 & 0 & 0.0208 & 0.0417 & 0 & 0 \\ 0 & 0 & 0.0208 & 0.0208 & 0 & 0.0417 & 0 \\ 0 & 0.0208 & 0 & 0 & 0 & 0 & 0.0208 \end{bmatrix}$$

$$Y = \begin{bmatrix} +1 & 0 & +1 & 0 & +1 & +1 & -1 \\ +1 & 0 & -1 & 0 & 0 & 0 & 0 \\ +1 & +1 & -1 & 0 & -1 & 0 & +1 \\ +1 & 0 & +1 & 0 & 0 & -1 & 0 \\ +1 & 0 & -1 & -1 & 0 & 0 & 0 \\ +1 & +1 & 0 & 0 & -1 & 0 & 0 \\ +1 & +1 & +1 & -1 & +1 & 0 & -1 \\ +1 & -1 & 0 & +1 & 0 & +1 & -1 \\ +1 & 0 & 0 & +1 & +1 & +1 & 0 \\ +1 & -1 & 0 & +1 & 0 & 0 & +1 \\ +1 & 0 & 0 & -1 & 0 & -1 & +1 \\ +1 & -1 & 0 & 0 & -1 & -1 & 0 \end{bmatrix}$$

$$M.(Y'Y)^{-1}.M' = \begin{bmatrix} 0.0833 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0833 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0833 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0833 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0833 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0833 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0833 \end{bmatrix}$$

Appendix 3

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3836 & 0.2037 & -0.4883 & 0.0170 & -0.0136 & 0.2004 \\ 0 & -0.2591 & 0.4604 & -0.2021 & -0.5299 & 0.6187 & 0.1589 \\ 0 & -0.9953 & 0.9203 & 0.8699 & 0.9897 & -0.6112 & -0.9141 \\ 0 & 0.9860 & -0.5529 & 0.2449 & 0.8876 & 0.5883 & -0.9433 \\ 0 & 0.1472 & -0.4780 & 0.7176 & 0.6929 & -0.9939 & -0.2256 \\ 0 & 0.6791 & 0.8712 & -0.0446 & -0.1059 & 0.8777 & -0.1235 \end{bmatrix}$$

$$M.(X_1'.X_1)^{-1}.M' = \begin{bmatrix} 0.1177 & 0.0555 & -0.0133 & -0.0164 & -0.0019 & -0.0003 & 0.0479 \\ 0.0555 & 0.1242 & -0.0236 & -0.0336 & -0.0606 & -0.0015 & 0.0884 \\ -0.0133 & -0.0236 & 0.0892 & -0.0203 & -0.0560 & -0.1128 & 0.0482 \\ -0.0164 & -0.0336 & -0.0203 & 0.2322 & -0.0205 & 0.0593 & -0.0390 \\ -0.0019 & -0.0606 & -0.0560 & -0.0205 & 0.2543 & 0.0680 & -0.0213 \\ -0.0003 & -0.0015 & -0.1128 & 0.0593 & 0.0680 & 0.1653 & -0.0738 \\ 0.0479 & 0.0884 & 0.0482 & -0.0390 & -0.0213 & -0.0738 & 0.1968 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} +1 & -1 & -1 & -1 & -1 & -1 & +1 \\ +1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & -1 & +1 & +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & +1 & +1 & +1 & -1 \\ +1 & +1 & -1 & -1 & +1 & -1 & +1 \\ +1 & -1 & +1 & +1 & +1 & -1 & -1 \\ +1 & +1 & -1 & -1 & -1 & +1 & +1 \\ +1 & -1 & -1 & +1 & -1 & -1 & +1 \\ +1 & -1 & +1 & -1 & -1 & +1 & +1 \\ +1 & -1 & +1 & +1 & -1 & -1 & +1 \\ +1 & -1 & +1 & -1 & +1 & +1 & -1 \\ +1 & -1 & -1 & +1 & +1 & -1 & -1 \end{bmatrix}$$

$$M.(X'.X)^{-1}.M' = \begin{bmatrix} 0.0833 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0390 & 0.0090 & -0.0648 & -0.0030 & -0.0343 & 0.0351 \\ 0 & 0.0090 & 0.0841 & -0.0452 & -0.0680 & -0.1184 & 0.0678 \\ 0 & -0.0648 & -0.0452 & 0.3986 & 0.0086 & 0.1281 & -0.0368 \\ 0 & -0.0030 & -0.0680 & 0.0086 & 0.2801 & 0.0690 & 0.0597 \\ 0 & -0.0343 & -0.1184 & 0.1281 & 0.0690 & 0.1903 & -0.1055 \\ 0 & 0.0351 & 0.0678 & -0.0368 & 0.0597 & -0.1055 & 0.1682 \end{bmatrix}$$

Appendix 4

$$M_{WTP}.(X_{WTP}'.X_{WTP})^{-1}.M_{WTP}' = \begin{bmatrix} 0.0833 & 0 & 0 & 0 & 0 \\ 0 & 0.0833 & 0 & 0 & 0 \\ 0 & 0 & 0.0833 & 0 & 0 \\ 0 & 0 & 0 & 0.0833 & 0 \\ 0 & 0 & 0 & 0 & 0.0833 \end{bmatrix}$$

$$M_{WTP}^{+} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1/5 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1/5 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1/5 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1/5 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1/5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2/5 \end{bmatrix}$$

$$X_{WTP} = \begin{bmatrix} +1 & +1 & -1 & -1 & -1 & -1 & 0.2 \\ +1 & -1 & +1 & -1 & +1 & +1 & 0.2 \\ +1 & +1 & +1 & +1 & -1 & +1 & -0.2 \\ +1 & -1 & -1 & +1 & +1 & -1 & 0.6 \\ +1 & +1 & +1 & -1 & +1 & +1 & -1 \\ +1 & +1 & -1 & +1 & -1 & +1 & 0.2 \\ +1 & +1 & -1 & -1 & +1 & -1 & 0.6 \\ +1 & -1 & -1 & -1 & -1 & +1 & 0.2 \\ +1 & -1 & +1 & -1 & -1 & -1 & 1 \\ +1 & -1 & +1 & +1 & -1 & -1 & -0.2 \\ +1 & +1 & +1 & +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 & +1 & +1 & -0.6 \end{bmatrix}$$

Appendix 5

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1/5 \\ 0 & 1 & 0 & 0 & 0 & -1/5 \\ 0 & 0 & 1 & 0 & 0 & -1/5 \\ 0 & 0 & 0 & 1 & 0 & -1/5 \\ 0 & 0 & 0 & 0 & 1 & -1/5 \end{bmatrix}$$

$$M.(Z'.P.Z)^{-1}.M' = \begin{bmatrix} 0.0867 & 0.0033 & 0.0033 & 0.0033 & 0.0033 \\ 0.0033 & 0.0867 & 0.0033 & 0.0033 & 0.0033 \\ 0.0033 & 0.0033 & 0.0867 & 0.0033 & 0.0033 \\ 0.0033 & 0.0033 & 0.0033 & 0.0867 & 0.0033 \\ 0.0033 & 0.0033 & 0.0033 & 0.0033 & 0.0867 \end{bmatrix}$$

$$M.(Z_{WTP}'.P_{WTP}.Z_{WTP})^{-1}.M' = \begin{bmatrix} 0.0833 & 0 & 0 & 0 & 0 \\ 0 & 0.0833 & 0 & 0 & 0 \\ 0 & 0 & 0.0833 & 0 & 0 \\ 0 & 0 & 0 & 0.0833 & 0 \\ 0 & 0 & 0 & 0 & 0.0833 \end{bmatrix}$$