The application of organic Rankine cycles (ORCs) for small scale power generation is inhibited by a lack of suitable expansion devices. Thermodynamic and mechanistic considerations suggest that scroll machines are advantageous in kilowatt-scale ORC equipment, however, a method of independently selecting a geometric design optimized for high-volume-ratio ORC scroll expanders is needed. The generalized 8-dimensional planar curve framework (Gravesen and Henriksen, 2001, “The Geometry of the Scroll Compressor,” Soc. Ind. Appl. Math., 43, pp. 113–126), previously developed for scroll compressors, is applied to the expansion scroll and its useful domain limits are defined. The set of workable scroll geometries is: (1) established using a generate-and-test algorithm with inclusion based on theoretical viability and engineering criteria, and (2) the corresponding parameter space is related to thermodynamically relevant metrics through an analytic ranking quantity \( f_c \) (“compactness factor”) equal to the volume ratio divided by the normalized scroll diameter. This method for selecting optimal scroll geometry is described and demonstrated using a 3 kW ORC specification as an example. Workable scroll geometry identification is achieved at a rate greater than 3 s \(^{-1}\) with standard desktop computing, whereas the originally undefined 8-D parameter space yields an arbitrarily low success rate for determining valid scroll mating pairs. For the test case, a maximum isentropic expansion efficiency of 85\% is found by examining a subset of candidates selected for compactness factor \( (volume \ expansion \ ratio \ per \ diameter) \), which is shown to correlate with the modeled isentropic efficiency \( \left( R^2 = 0.88 \right) \). The rapid computationally efficient generation and selection of complex validated scroll geometries ranked by physically meaningful properties is demonstrated. This procedure represents an essential preliminary qualification for intensive modeling and prototyping efforts necessary to generate new high performance scroll expander designs for kilowatt scale ORC systems. [DOI: 10.1115/1.4023112]

Keywords: scroll expander design, planar curves, volume ratio, compactness factor, isentropic efficiency kilowatt-scale organic Rankine cycle

Development of a scroll expander optimized for the larger volume ratios encountered at higher temperature ORC applications (3–15 or higher, depending on the temperature and working fluid) would promote the viability of ORC power generation from smaller distributed thermal resources. While several investigations of scroll expander models and validation experiments are described in the literature [2–7], discussion of the choice of the scroll geometry, the single feature upon which all other properties depend, is generally limited to the case of circle involutes in low volume ratio compressor applications [8,9]. In contrast, the present work explores the effect of varying the basic scroll geometry as a method for developing novel scroll machinery at the higher volume ratios needed for many ORC applications. The results demonstrate a computationally efficient process, based on thermodynamically relevant criteria, for converging on a set of near-optimal candidates for the scroll geometry. Details of the algorithm, along with a specific case study, are described in this publication. The complete thermodynamic analysis of a particular scroll expander in an ORC application, such as that performed in Refs. [10,11], is not addressed in this paper but is further developed from the results of this work in Refs. [12] and [13].
Equations Defining Scroll Geometry. The geometry of the scroll, classically based on circle involutes, was more generally described by Gravesen and Henriksen [8] where the parameterization simplifies to the circle involute as a special case, but cases with varying wall thicknesses can also be modeled. As presented in Ref. [8], the planar curves of the scroll wrap are defined by the intrinsic equation linking the arc length $s$, to the tangent direction $\Phi$

$$s = c_1 + c_2 \Phi + c_3 \Phi^2 + c_4 \Phi^3 + c_5 \Phi^4$$

where \(c_1, c_2, c_3, c_4, c_5\) are scalar coefficients. Based on this general fourth-order polynomial for $s$ and the coordinate system used by Gravesen and Henriksen, Cartesian coordinates for the initial scroll wall $x$ can be analytically found, as shown in Eq. (2)

$$x = (\alpha \sin(\Phi) + 2\alpha \cos(\Phi), \beta \sin(\Phi) - 2\beta \cos(\Phi))$$

Parameters $\alpha$ and $\beta$ are related to the tangent direction $\Phi$ and original scalar coefficients as follows

$$\alpha = (c_1 - 6c_3) + (2c_2 - 24c_4)\Phi + (3c_3)\Phi^2 + (4c_4)\Phi^3$$

$$\beta = (2c_2 - 24c_4) + (6c_3)\Phi + (12c_4)\Phi^2$$

Equations for the mating curve, along with the opposite sides of these scroll walls, are found by reflection and symmetry following the method in Ref. [8]. This requires the definition of two additional parameters: $R$, the orbital radius of the moving scroll and $d$, a scalar length related to wall thickness. The range of $\Phi$ over which these walls are considered is constrained by the definition of $N$, the desired number of turns of the scroll spiral, such that $\Phi_{\text{max}} - \Phi_{\text{min}} = 2\pi N$. The $N$ consecutive points of conjugacy between the moving scroll (orbiting at radius $R$) and the fixed scroll, found at $(\Phi_i, \Phi_i + 2\pi, \Phi_i + 4\pi, ...)$ for some initial conjugacy angle $\Phi_0$, represent the terminal points of adjacent internal chambers ("pockets") within which working fluid expansion occurs (Fig. 1). For additional details on the determination of conjugacy points and calculation of pocket volumes, see Refs. [8,9].

Fig. 1 The expansion action of the scroll device works via a series of chambers defined by adjacent conjugate points. High pressure vapor enters at the inlet and expands against the orbiting scroll in an expanding chamber following the spiral. The orbit of radius $R$ translates to rotation with a crank. The mating pairs of scroll curves are formed by reflection across the center point C, accounting for the wall thickness scalar $d$. The scroll geometry parameter space is thus 8-dimensional, defined by \(c_1, c_2, c_3, d, R, N\). While the relationship of these parameters to the scroll wall $x$ is analytically derived in Ref. [8], the valid domains of these parameters and their effects on important scroll characteristics are not. This work addresses this limitation by systematically exploring the parameter domains and relating the eight-dimensional space to relevant design metrics.

Parameter Domain Definition. Because the parameter space for mating scroll pairs based on the intrinsic equation is infinite, the realistic simulation of scrolls to meet physical specifications is greatly aided by knowledge of the parameter domains as they relate to viable examples. To discover envelopes of "viability islands" within the parameter space and assign relationships to relevant criteria, we employ a generate-and-test algorithm for expanding the parameter domains from an initial set that includes known viable scroll geometries, i.e., the archetypal cases of Ref. [8] (circle involute and two examples with increasing wall thicknesses), followed by the identification of parameterizations producing unworkable scroll designs.

Scroll designs are unworkable in cases where, e.g., the intrinsic equation produces a nonmonotonically increasing spiral that crosses itself or its mating curve, violating the conjugacy between orbiting and fixed scrolls upon which the action of the machine depends; these cases are immediately discarded upon identification. To further improve the quality of resulting scrolls, however, we also include three other criteria of relevance to scroll engineering with which to evaluate potential scroll geometries: wall thickness (for mass and mechanical strength), scroll diameter (form factor, with size limited to $D_{\text{max}}$ calculated as a function of $V_{\text{in}}$), and orbital radius $R$ (i.e., the throw of the crank arm for power takeoff). The designer must choose an acceptable range for these values based on material constraints, conformation within the ORC, the mode of power transmission an potential on considerations related to leakage, lubrication, etc.; reasonable default values for a kilowatt-scale scroll expander are given in Table 1.

The creation of a distribution of useful scrolls proceeds as follows: random values for each parameter are generated from within the defined domains, a scroll is generated based on these parameters, the scroll size is normalized to match the specified inlet volume $V_{\text{in}}$ (since the displacement and, hence, power rating, must be similar for a direct comparison among differing but valid scroll geometries), and finally the geometry is tested against the above criteria for viability and engineering utility (Fig. 2). This is repeated until a sufficiently large number of scrolls has been obtained (e.g., 10,000–20,000). At this point, the distribution of each parameter is examined to determine whether the range capable of producing a valid scroll has been circumscribed; if no such "envelope" is detected, the domains are expanded, generating a new scroll distribution, until this condition is met. Examples of the identification envelope relationships are shown in Fig. 3. Constraining the infinite domains of the 8-D parameter space for viable scrolls thus limits an unproductive simulation effort and represents a computationally efficient means for selection of the optimal scroll geometry.

Because the domain envelopes are expected to be interdependent functions of each other, we characterize them in a hierarchy corresponding to the order of operations in the search algorithm (see Table 1) and related to their relative independence in determining the outcome of the scroll geometry. The result of this process is a stable valid-scroll identification success rate of approximately 0.02 (valid scrolls per random parameter set generated), or greater than 3 scrolls $s^{-1}$ using the computational resources described in Table 2 and discussed further in the following text. This success rate is a significant improvement over the case of arbitrary domain limits, where the success rate is very low; implementation of envelope identification provided approximately an order of magnitude improvement relative to arbitrarily searching within identified valid domains.
Table 1 Viability constraints and parameter domains for the planar curve scroll geometry framework. The lower limit on $R_{\text{norm}}$ is proposed to ensure that orbital motion can be practically translated into rotation mechanically, e.g., with a crank. The wall thickness minimum constraint is derived from an empirical correlation using ZR type scroll machines and a power law for force on the scroll wraps as a function of arc length, described in Ref. [12]. The parameter domain functions are defined through the process described in Fig. 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{\text{max}}$</td>
<td>$D_{\text{max}} &lt; 100 \ln (V_{\text{in}})$</td>
</tr>
<tr>
<td>$R_{\text{norm}}$</td>
<td>$R_{\text{norm}} &gt; 2$ mm</td>
</tr>
<tr>
<td>(after normalization)</td>
<td></td>
</tr>
</tbody>
</table>

Viability constraints

\[ t = \frac{(0.0648 \times h + 1.2864)}{0.0648 \times h + 1.2864} \times \theta^2 \]

\[ \ln \left( \frac{0.0648 \times h + 1.2864}{2.75} \right) \]

Wall thickness

$< \text{wall thickness} < 100$ mm

Parameter domains (with envelopes)

$N$

\[ 3 < N < 12 \]

$R$

\[ (2.4241 N - 16.1332) < R < 60 \]

$D$

\[ 0.5R < d < 30 \]

$c_1$

\[ c_1 = 0 \]

$c_2$

\[ -4 < c_2 < 4 \]

\[ 1.0145N - 8.8014 < c_3 < -0.4041N + 18.0718 \]

$c_3$

\[ 0.083725d - 4.8712 < c_3 < 0.54374d + 0.6304 \]

$c_4$

\[ -0.058902 c_3 - 0.016902 < c_4 < -0.083835 c_3 + 1.1011 \]

$c_5$

\[ -0.016401 c_4 - 0.0013445 < c_5 < \min (0.02, 0.38923 e^{-0.64318N} + 0.0014341) \]

Domains for $N$, $R_{\text{max}}$, and $d_{\text{max}}$ are arbitrary and are chosen based on the experience of the researchers to include a reasonable search space. Values of $N$ higher than 12 are possible, but the mechanical tolerances necessary to achieve compliance present an engineering challenge as $N$ increases. Valid scroll geometries can also be found for arbitrarily high values of $R$ and $d$, with the result simply scaled down during normalization. In practice, however, the uniqueness of the scroll geometries is sufficiently captured at some finite domain limit for $R$ and $d$. Finally, $c_1$ is set to zero because it has no effect on the output other than to locate the curve geometry within the coordinate system.

Optimization of Scroll Geometry. Once a viable set of scrolls has been established with the previous method, optimization is possible over a range of criteria. Ideally, candidates should be chosen for further exploration using a mechanistic and thermodynamic model based on $R$, or other geometric data. Gravesen and Henriksen suggested the leakage factor as a possible figure of merit to prioritize further investigation [8], however, provided analytic treatment of only tangential leakage, whereas Chen identifies radial leakage as the dominant mode [11]. Within Gravesen and Henriksen’s generalized framework, radial leakage, which is dependent in large part on arc length, is problematic to compute because the arc length $s_x$, corresponding to the scaled version of $x$ cannot be analytically derived. To avoid this difficulty and to simplify the ranking process, an alternative figure of merit—the compactness factor—is proposed.

Rapid Selection Based on Compactness Factor. In this study we have essentially normalized tangential leakage by normalizing the scroll height $z$, using a relationship we derived empirically from measurements of ZR-type scroll compressors ($N = 5$, $R^2 = 0.97$) manufactured by Copeland

\[ z = 7V_{\text{in}}^{0.58} \]

where $z$ is the scroll wrap height in mm and $V_{\text{in}}$ is in units of cm$^3$. This leaves radial leakage, axial friction, and heat loss from the unit...
as the main relevant physical criteria to inform optimal scroll selection. The analytical calculation of these quantities is both difficult to achieve, as previously noted for the case of radial leakage, and computationally expensive given that sufficiently dense coverage of the viable domain may include tens of thousands of potential scroll geometries. We instead propose that radial leakage, axial friction, and heat losses may be captured to an approximate by aspects of the geometry distilled into a “compactness factor” \( f_c \) as a function of the volume ratio \( R_v \) and the normalized scroll diameter

\[
 f_c = \frac{R_v}{\text{D}_{\text{norm}}}
\]  

where \( f_c \) has units of 1/L. Whereas normalized diameter alone does not capture information regarding efficiency, the compactness factor as a concept is useful in comparing scrolls across ORCs of different size and temperature specifications. For the assumption of a constant leakage gap height, \( f_c \) will be inversely proportional to the radial leakage path as a function of \( \sigma \) for any given \( R_v \). Similarly, by conserving the area and volume for a given \( R_v \), \( f_c \) intrinsically captures an important scale coefficient for both the axial friction losses and heat transfer between the scroll and its surroundings. Thus, searching for maximal \( f_c \) enables a rapid computationally inexpensive selection of a subset of viable scroll geometries that are likely to be close to optimal when evaluating mechanistic and thermodynamic features in an intensive model such as described in Refs. [7,8], including, e.g., refinements for the suction wrap profiles [9,14] and tradeoffs between the tangential and radial leakage as a function of varying \( z \), and the normalization factor.

**Table 2** Computer system hardware and software configuration

<table>
<thead>
<tr>
<th>Workstation</th>
<th>Operating system: Windows Enterprise 64-bit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processor</td>
<td>Intel i7 LGA1366 2.67 GHz</td>
</tr>
<tr>
<td>RAM</td>
<td>6 GB</td>
</tr>
<tr>
<td>Programming environment</td>
<td>Matlab R2011b 32-bit</td>
</tr>
<tr>
<td>Computation time</td>
<td>4.3 scrolls s(^{-1}) (( \sigma = 0.23 ) scrolls s(^{-1}))</td>
</tr>
</tbody>
</table>

Results

In order to illustrate the method proposed in the preceding text, we provide the following case study, wherein we consider the design of an expander for a 3 kWe ORC having the characteristics defined in Table 3. The nominal expander isentropic efficiency combines the assumed values for the expander mechanical efficiency (0.8) and small induction generator efficiency (0.82) derived from the test bench results using the expander-generators based on reversed hermetic scroll compressors and separately tested induction machines [15,16].

The data of Table 3 show the ORC parameters that bear on the expander design, namely, the targeted power output (kW), the working fluid temperature differential, the rotational speed, and the working fluid (suggested by the \( \text{D}_{\text{T}} \)) [17]. The pinch conditions of the heat exchangers may be considered using, e.g., the effectiveness-number of transfer units (\( \varepsilon \)-NTU) or log mean temperature difference (LMTD) method to arrive at actual working fluid conditions, including any superheat, \( f_c \), from knowledge of thermal source temperatures [18].
Power

Table 3: Design specifications for a 3 kW ORC using R245fa

<table>
<thead>
<tr>
<th>Specification</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>3 kWc</td>
</tr>
<tr>
<td>$T_{\text{exp}}$</td>
<td>°C</td>
</tr>
<tr>
<td>$T_{\text{id}}$</td>
<td>°C</td>
</tr>
<tr>
<td>Superheat</td>
<td>°C</td>
</tr>
<tr>
<td>Rotational speed</td>
<td>RPM</td>
</tr>
<tr>
<td>$R_{\text{VP}}$</td>
<td>8.5</td>
</tr>
<tr>
<td>$V_{\text{in}}$</td>
<td>24.5 cc/rev</td>
</tr>
<tr>
<td>$n_t$</td>
<td>0.123 kg/s</td>
</tr>
</tbody>
</table>

From this data the desired volume expansion ratio $R_{\text{VP}}$ of the expander can be inferred from the working fluid properties, assuming 95% volumetric efficiency (reasonable for scrolls) $[2,4,19]$ and using an imputed isentropic efficiency from, e.g., Refs. $[3,4]$, which may be inclusive of downstream losses from a generator. These characteristics define the initial volume displacement ($V_{\text{in}}$) of the expansion pockets formed by the scroll wraps as follows:

$$V_{\text{in}} = \frac{\text{Power} \cdot V_{\text{in}}(T_{\text{id}}, P_{\text{in}}) \cdot 6 \cdot 10^7}{\text{RPM} \cdot \Delta h_{\text{is}}(h_{\text{in}}(T_{\text{id}}, P_{\text{in}}) - h_{\text{in}}(T_{\text{exp}}, P_{\text{in}}))} \text{[cm}^3\text{] (rev}^{-1}\text{)} \quad (7)$$

where Power is in kilowatts. The initial pocket area is $A = V_{\text{in}}/2z$ because two pockets form and discharge per cycle of the orbiting scroll. Using the wrap height $z$ calculated with Eq. (6), the initial pocket area can, in turn, be described as a function of $V_{\text{in}}$:

$$A = 71.4 \cdot 0.42 \quad (8)$$

where $A$ is in units of mm² and $V_{\text{in}}$ is in units of cm³. The search domain is thus set from these criteria: $V_{\text{in}}$ relationships and the specified viability constraints from Table 1.

The optimization algorithm then proceeds according to Fig. 4. Scrolls are simulated, normalized to $V_{\text{in}}$ and checked for viability, with the resulting valid scrolls subselected for appropriate values of $R_{\text{VP}}$, matching the $R_{\text{VP}}$ of the application and ranked based on $f_s$. The optimal results can be displayed for visual comparison; the scroll identified in this case as optimal is shown in Fig. 5. The total computation time to produce this result, along with 137 other potentially useful geometries in the appropriate range of $R_{\text{VP}}$, was approximately 40 min using the previously described workstation for a viable scroll dataset population of n 10,000.

To validate this approach and the use of $f_s$ as a proxy for isentropic expansion efficiency, a subset ($N = 13$) of the scroll dataset for the test specifications is selected for the analytical and numerical thermo-dynamic modeling in order to determine losses due to leakage, friction, and heat transfer, according to the parameterization (gap height, oil film thickness, etc.) described in Ref. [12]. This analysis shows a correlation between the $f_s$ and efficiency ($R^2 = 0.876$), as shown in Fig. 6. Whether or not true circle involutes rank among the results with the highest compactness factor and isentropic efficiency is highly sensitive to the chosen wall thickness constraint. Scroll geometries with varying wall thickness may have the potential for higher isentropic expansion efficiency at reduced mass compared with circle involutes by executing a larger volume expansion per revolution so that fewer wraps ($N$) are needed for a given $R_{\text{VP}}$ application and the corresponding mating arc length (and the potential leakage path) is shorter. They also present an opportunity for distributing the wall thickness in accordance with the dynamic forces on the wrap walls (i.e., wider at the inlet, decreasing thickness towards the outlet).
These observations raise the possibility that unexploited potential for scroll geometry optimization exists that could increase the performance of scroll devices in a variety of applications.

Conclusions

The generalized scroll geometry framework of Gravesen and Henriksen [8] was employed to develop a design tool for ORC scroll expanders by using an algorithm based on the generate-and-test selection process. The computational efficiency is optimized by identifying key domain boundaries within the 8-dimensional parameter space and by implementing physically relevant constraints. The success rate for identifying valid scrolls by this method was approximately 2%, enabling the computation of valid scroll geometries for a given design specification in < 1 h using standard desktop computing available in 2012. A figure of merit from analytic geometry data, the “compactness factor” $f_c$, is proposed and validated as a positive correlate for isentropic efficiency and the objective function for the identification of scrolls with advantageous properties regarding leakage, friction, and heat losses. These procedures are combined into a design tool and its use is demonstrated through selection of the optimal scroll geometry for an example ORC specification. This approach provides a generalizable, rapid, and physically meaningful basis for selecting scroll geometries for subsequent, more computationally intensive, modeling. The results of this method should promote improved outcomes with detailed mechanistic and thermodynamic models and ultimately support the development of high performance scroll machines for various applications, including expanders in kilowatt-scale ORC systems.

Acknowledgment

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