1998 HMMT Algebra Event

1. The cost of 3 hamburgers, 5 milk shakes, and 1 order of fries at a certain fast food restaurant is \$23.50. At the same restaurant, the cost of 5 hamburgers, 9 milk shakes, and 1 order of fries is \$39.50. What is the cost of 2 hamburgers, 2 milk shakes and 2 orders of fries at this restaurant?

2. Bobbo starts swimming at 2 feet/s across a 100 foot wide river with a current of 5 feet/s. Bobbo doesn't know that there is a waterfall 175 feet from where he entered the river. He realizes his predicament midway across the river. What is the minimum speed that Bobbo must increase to make it to the other side of the river safely?

3. Find the sum of every even positive integer less than 233 not divisible by 10.

4. Given that r and s are relatively prime positive integers such that $\frac{r}{s} = \frac{2(\sqrt{2}+\sqrt{10})}{5(\sqrt{(3+\sqrt{5})}}$,

   find r and s.

5. A man named Juan has three rectangular solids, each having volume 128. Two of the faces of one solid have areas 4 and 32. Two faces of another solid have areas 64 and 16. Finally, two faces of the last solid have areas 8 and 32. What is the minimum possible exposed surface area of the tallest tower Juan can construct by stacking his solids one on top of the other, face to face? (Assume that the base of the tower is not exposed.)

6. How many pairs of positive integers (a,b) with $a \le b$ satisfy $\frac{1}{a} + \frac{1}{b} = \frac{1}{6}$?

7. Given that three roots of $f(x) = x^4 + ax^2 + bx + c$ are 2, -3, and 5, what is the value of $a+b+c$?

8. Find the set of solutions for x in the inequality $\frac{x+1}{x+2} > \frac{3x+4}{2x+9}$ when $x \ne -2, x \ne -\frac{9}{2}$.

9. Suppose $f(x)$ is a rational function such that $3f(\frac{1}{x}) + \frac{2f(x)}{x} = x^2$ for $x \ne 0$.

   Find $f(-2)$.

10. G. H. Hardy once went to visit Srinivasa Ramanujan in the hospital, and he started the conversation with: "I came here in taxi-cab number 1729. That number seems dull to me, which I hope isn't a bad omen." "Nonsense," said Ramanujan. "The number isn't dull at all. It's quite interesting. It's the smallest number that can be expressed as the sum of two cubes in two different ways." Ramujan had immediately seen that

$1729 = 12^3 + 1^3 = 10^3 + 9^3$. What is the smallest positive integer representable as the sum of the cubes of *three* positive integers in two different ways?