1998 Harvard/MIT Math Tournament

GEOMETRY Answer Sheet

Name: ______________________________

School: ____________________ **Grade:** ______

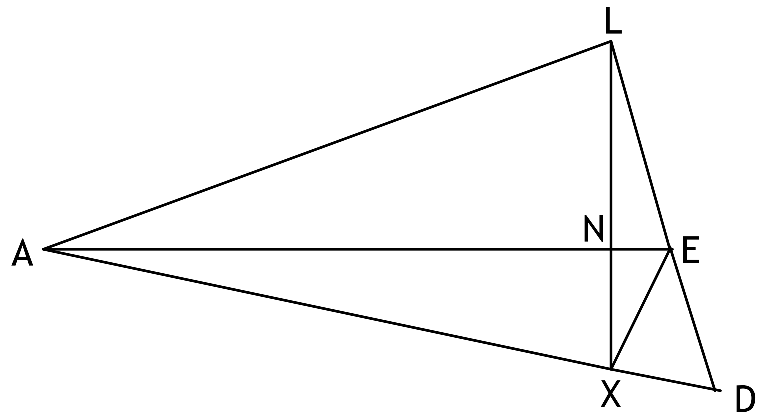
1	________	**7**	________
2	________	**8**	________
3	________	**9**	________
4	________	**10a**	________
5	________	**10b**	________
6	________	**10c**	________

TOTAL: ________

GEOMETRY

Question One. [3 points]

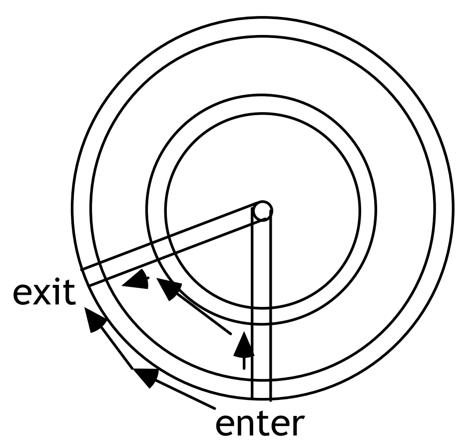
Quadrilateral *ALEX*, pictured below (but not necessarily to scale!) can be inscribed in a circle; with $m\angle LAX = 20°$ and $m\angle AXE = 100°$:



Calculate $m\angle EDX$.

Question Two. [3 points]

Anne and Lisa enter a park that has two concentric circular paths joined by two radial paths, one of which is at the point where they enter. Anne goes in to the inner circle along the first radial path, around by the shorter way to the second radial path and out along it to the exit. Walking at a the same rate, Lisa goes around the outer circle to the exit, taking the shorter of the two directions around the park.



They arrive at the exit at the same time. The radial paths meet at the center of the park; what is the angle between them?

Question Three. [4 points]

MD is a chord of length $\sqrt{2}$ in a circle of radius 1, and *L* is chosen on the circle so that the area of triangle *MLD* is the maximized.

Find $m\angle MLD$.

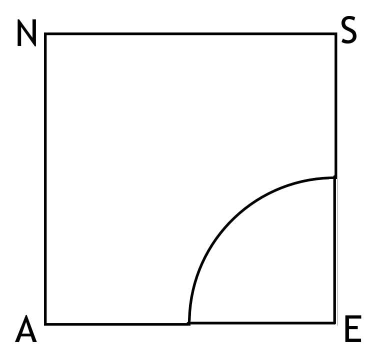
Question Four. [4 points]

A cube with side length 100cm is filled with water and has a hole through which the water drains into a cylinder of radius 100cm. If the water level in the cube is falling at a rate of $1\ {}^{cm}\!/_{s}$, how fast is the water level in the cylinder rising?

Question Five. [5 points]

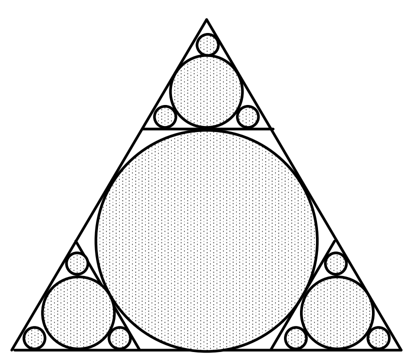
Square *SEAN* has side length 2 and a quarter-circle of radius 1 around *E* is cut out.

Find the radius of the largest circle that can be inscribed in the remaining figure.



Question Six. [5 points]

A circle is inscribed in an equilateral triangle of side length 1. Tangents to the circle are drawn that cut off equilateral triangles at each corner. Circles are inscribed in each of these equilateral triangles. If this process is repeated infinitely many times, what is the sum of the areas of all the circles?



Question Seven. [5 points]

Pyramid *EARLY* has rectangular base *EARL* and apex *Y*, and all of its edges are of integer length. The four edges from the apex have lengths 1, 4, 7, 8 (in no particular order), and *EY* is perpendicular to *YR*.

Find the area of rectangle *EARL*.

Question Eight. [6 points]

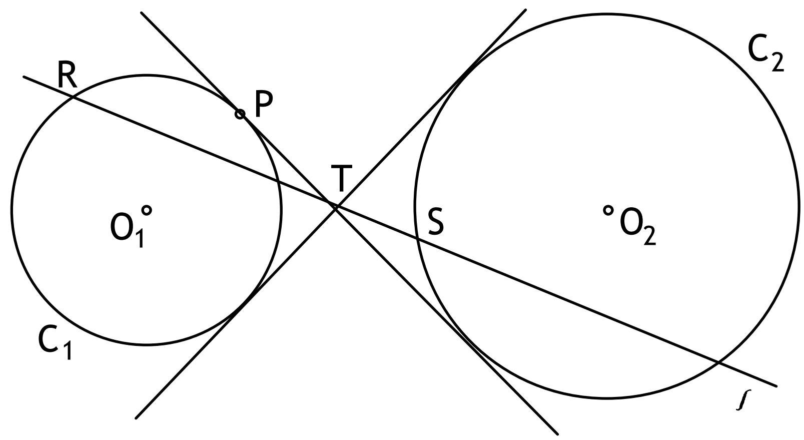
It is not possible to construct a segment of length π using a straightedge, compass, and a given segment of length 1. The following construction, given in 1685 by Adam Kochansky, yields a segment whose length agrees with π to five decimal places:

Construct a circle of radius 1 and call its center *O*. Construct a diameter *AB* of this circle and a line ℓ tangent to the circle at *A*. Next, draw a circle with radius 1 centered at *A*, and call one of the intersections with the original circle *C*. Now from *C* draw an arc of radius 1 intersecting the circle around *A* at *D*, where *D* lies outside of the circle centered at *O*. Draw *OD* and let *E* be its point of intersection with ℓ. Construct *H* on *AE* such that *A* is between *H* and *E*, and *HE*=3.

The distance between *B* and *H* is then close to π; calculate its exact value.

Question Nine. [7 points]

Let T be the intersection of the common internal tangents of circles C_1, C_2 with centers O_1, O_2 respectively. Let P be one of the points of tangency on C_1 and let line ℓ bisect angle O_1TP. Label the intersection of ℓ with C_1 that is farthest from T, R, and label the intersection of ℓ with C_2 that is closest to T, S. If C_1 has radius 4, C_2 has radius 6, and $O_1O_2 = 20$, calculate $(TR)(TS)$.



Question Ten [8 points].

Lukas is playing pool on a table shaped like an equilateral triangle. The pockets are at the corners of the triangle and are labeled C, H, and T. Each side of the table is 16 feet long. Lukas shoots a ball from corner C of the table in such a way that on the second bounce, the ball hits 2 feet away from him along side CH.

a. [5 points]

How many times will the ball bounce before hitting a pocket?

b. [2 points]

Which pocket will the ball hit?

c. [1 points]

How far will the ball travel before hitting the pocket?