

**Algebra Test**  
**Harvard-MIT Math Tournament**  
March 3, 2001

1. Find  $x - y$ , given that  $x^4 = y^4 + 24$ ,  $x^2 + y^2 = 6$ , and  $x + y = 3$ .
2. Find  $(x + 1)(x^2 + 1)(x^4 + 1)(x^8 + 1) \cdots$ , where  $|x| < 1$ .
3. How many times does 24 divide into  $100!$  (factorial)?
4. Given that 7,999,999,999 has at most two prime factors, find its largest prime factor.
5. Find the 6-digit number beginning and ending in the digit 2 that is the product of three consecutive even integers.
6. What is the last digit of  $1^1 + 2^2 + 3^3 + \cdots + 100^{100}$ ?
7. A polynomial  $P$  has four roots,  $\frac{1}{4}$ ,  $\frac{1}{2}$ , 2, 4. The product of the roots is 1, and  $P(1) = 1$ . Find  $P(0)$ .
8. How many integers between 1 and 2000 inclusive share no common factors with 2001?
9. Find the number of positive integer solutions to  $n^x + n^y = n^z$  with  $n^z < 2001$ .
10. Find the real solutions of  $(2x + 1)(3x + 1)(5x + 1)(30x + 1) = 10$ .