

Harvard-MIT Math Tournament

March 17, 2002

Individual General Test: **Part 1**

1. What is the maximum number of lattice points (i.e. points with integer coordinates) in the plane that can be contained strictly inside a circle of radius 1?
2. Eight knights are randomly placed on a chessboard (not necessarily on distinct squares). A knight on a given square attacks all the squares that can be reached by moving either (1) two squares up or down followed by one squares left or right, or (2) two squares left or right followed by one square up or down. Find the probability that every square, occupied or not, is attacked by some knight.
3. How many triples (A, B, C) of positive integers (positive integers are the numbers $1, 2, 3, 4, \dots$) are there such that $A + B + C = 10$, where order does not matter (for instance the triples $(2, 3, 5)$ and $(3, 2, 5)$ are considered to be the same triple) and where two of the integers in a triple could be the same (for instance $(3, 3, 4)$ is a valid triple).
4. We call a set of professors and committees on which they serve a *university* if
 - (1) given two distinct professors there is one and only one committee on which they both serve,
 - (2) given any committee, C , and any professor, P , not on that committee, there is exactly one committee on which P serves and no professors on committee C serve, and
 - (3) there are at least two professors on each committee; there are at least two committees.What is the smallest number of committees a university can have?
5. A square and a regular hexagon are drawn with the same side length. If the area of the square is $\sqrt{3}$, what is the area of the hexagon?
6. A man, standing on a lawn, is wearing a circular sombrero of radius 3 feet. Unfortunately, the hat blocks the sunlight so effectively that the grass directly under it dies instantly. If the man walks in a circle of radius 5 feet, what area of dead grass will result?
7. A circle is inscribed in a square dartboard. If a dart is thrown at the dartboard and hits the dartboard in a random location, with all locations having the same probability of being hit, what is the probability that it lands within the circle?
8. Count the number of triangles with positive area whose vertices are points whose (x, y) -coordinates lie in the set $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2)\}$.
9. Real numbers a, b, c satisfy the equations $a + b + c = 26, 1/a + 1/b + 1/c = 28$. Find the value of
$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} + \frac{a}{c} + \frac{c}{b} + \frac{b}{a}.$$
10. A certain cafeteria serves ham and cheese sandwiches, ham and tomato sandwiches, and tomato and cheese sandwiches. It is common for one meal to include multiple types of sandwiches. On a certain day, it was found that 80 customers had meals which contained both ham and cheese; 90 had meals containing both ham and tomatoes; 100 had meals containing both tomatoes and cheese. 20 customers' meals included all three ingredients. How many customers were there?