Continuous-Time Adaptive Sampling and Forecast Assimilation for Autonomous Vehicles

Franz Hover
Department of Mechanical Engineering
Massachusetts Institute of Technology
22 October 2008
The stability of a vehicle operating in a 2-D plane (e.g., surveying a ship hull) performing localization using relative range-and-bearing measurements of point features with closed-loop control is investigated.

Stability can be assessed by designing a nominal trajectory, modeling the system using a linear(ized) time-varying state space framework which considers deviations from the trajectory.

Analysis of the transition matrix spectral norm yields a necessary and sufficient stability criterion, based on the boundedness and convergence of the norm. Can be used as a design tool to assess robustness.

Brendan Englot, ICRA09 sub.
Particle Filtering with Interpolation

- Particle filters are computationally expensive because every 'particle' entails a simulation.
- Instead, simulate particles at collocation points.
  - Construction of an interpolant, an approximation of the true system equations.
  - Interpolant can be evaluated in place of the true system to propagate many particles cheaply.
- ‘IPF’ achieves significant computational savings over conventional particle filters
- Compatible with other particle filter enhancements, e.g. Rao-Blackwellisation and MCMC.

Above: filter errors for double pendulum angles and angular velocities. The IPFR-49/500 only requires 49 simulations per time step, and achieves nearly a tenfold reduction in computation while matching the error of a 500-particle particle filter.

Josh Taylor, IEEE TSP, sub.
Adaptive Sampling

Coordinated Behavior

Sonars

Uncertain Communication

Self-navigating Network

Advanced Sensors

GPS and Remote Sensing Satellites

Autonomous Surface And Underwater Vehicles

Surface Traffic

Adaptive Sampling

Coordinated Behavior

Sonars

J. Leonard and H. Schmidt
Increased capabilities in deployed autonomous systems

Single vehicle $\rightarrow$  
*Multiple, heterogeneous vehicles*

Few permanent at-sea capabilities $\rightarrow$  
*Major investment in at-sea, long-term equipment*

Closely-gaurded single channel comms $\rightarrow$  
*Robust, distributed network of agents and links*

Data storage on board $\rightarrow$  
*Frequent or near real-time data dissemination*

Pre-programmed trajectories $\rightarrow$  
*Adaptive trajectories, reconfigurable on the fly*
The essential problem

Large-scale computer models of ocean and atmospheric processes, with error estimates

Fixed and moving autonomous agents, with infrastructure for operations, sensing, comms, control

How are they best combined to gain understanding and predictive capability?
Flavors of Adaptive Sampling

Defining the trajectory at launch time so as to take into account the current field prediction (e.g., F. Chavez, as in Lermusiaux 2007)

Tracking a gradient with a formation using real-time data analysis (Fiorelli et al., 2006)

Developing trajectories specifically for error reduction

   Heaney et al. (2007) – genetic algorithm, forecast field and error extrema

   Yilmaz et al. (2008) – mixed integer linear programming (MILP), forecast error extrema

   Lermusiaux (2007) – trial and error, best post-assimilation errors
Path Planning Integrated with Data Assimilation (PPIDA)

• We go for a **key connection** between operation planning and the data assimilation process:
  – Assume
    • Near real-time dissemination of data
    • Frequent model runs with latest assimilation (e.g., NWP)
  – Objectives
    • *Complete integration, leading to best possible assimilation result*
    • Algorithm scalable to many agents
    • Incorporate realistic vehicle dynamic behavior
    • Account for sensor noise properly
    • Eventually accommodate operational concerns such as currents, power budgets, collisions and traffic, vehicle navigation uncertainty, etc…
PPIDA Procedure in Words

• State vector:
  – states of all vehicles, and
  – elements of field variable error covariance matrix

• Minimize a cost function:
  – Vehicle speed deviations, turning rates, energy expense, etc., plus
  – Error covariance trace, plus
  – Terminal condition (if needed)

• Subject to:
  – Vehicle dynamic behavior
  – Propagation of the field variable forecast error
Close Relation to Simultaneous Localization and Mapping (SLAM)

• How do we design a robot path for maximal information (minimum error) in creating a feature map?
• Huang et al. (2005) proposed a solution with multi-step look-ahead planning with a single vehicle, using an MPC-like strategy and a low number of control actions (brute force search)
• Observed VERY MILD improvements over a greedy (one-step search) and even over a basic circle: max 50% improvement  Is it worth it? (One vehicle, seven targets)
Major Issue: Scaling

- Problem demands we keep track of vehicle and field error states over the time and space domain:
  \[ P = MZ + \frac{N^2}{2}, \quad \text{where } Z \text{ vehicles, } M \text{ states/vehicle, } N \text{ field states} \]

- Direct optimization methods (MILP, DP, ADP) can have extremely poor scaling in the number of states involved, and generally require quantizing \( \rightarrow \) problem size becomes \( q^P \)

- Two components in a practical solution:
  - Spectral representation of field variable dynamics
  - Continuous-variable optimization
Spectral Methods in Ocean and Atmospheric Modeling

• Numerical Weather Prediction:
  – Global models and some regional models, horizontal plane (see Kalnay, 2003)
  – Hurricane prediction using empirical orthogonal functions (EOF’s) (Zhang & Krishnamurthi, 1997)

• Ocean Circulation:
  – Five EOF’s may cover 90%+ of the variations in a high-fidelity model run (Fukumori and Melanotte-Rizzoli, 1995)
  – In general case, O(100) EOF modes may be needed (Buehner et al., 2003)
  – Error Subspace Statistical Estimation (ESSE, Lermusiaux et al., 2007)
• For the continuous approach, go back to Pontryagin’s Maximum Principle and a first-order gradient method for smooth trajectories

Results are often “good enough” with reasonable cost, BUT
• Requires a realistic first guess
• Difficult to actually reach a minimum; asymptotic approach
• No guarantee that global minimum is found
• Inequality constraints are clunky

• One Bell and One Whistle
  – **Adaptive gain**, enforcing small steps where needed and allowing large steps where suitable
  – Augment gradient method with **simulated annealing**, to better find a global minimum. *Simulated annealing enforces random, large steps whether they improve the cost or not → provable convergence to a global minimum in the limit*
Agents’ state variables satisfy: \( \dot{x}_v = f_v(x_v, u, t) \)
Agents’ controls: \( u(t) \)
Field state variables: \( \hat{x}_f \)

Riccati equation for field error covariance \( P \):
\[
\dot{P} = F(\hat{x}_f, t) P + PF(\hat{x}_f, t)^T + Q(\hat{x}_f, t) - \\
P H^T(x_v, t) R^{-1}(\hat{x}_f, x_v, t) H(x_v, t) P
\]
Setting up the optimization:

- aggregate state is $x = [x_f, x_P]^T$.
- $\dot{x} = f(x, u, t)$ with $f$ comprised of two parts:
  - agent dynamics $f_v()$
  - the state-space equivalent of the $\dot{P}$ eqn.
- total cost is defined as
  \[ J = \Psi(x(T)) + \int_0^T l(x, u, t)dt \]
  with $l(x, u, t) = tr(P) + \cdots$
- $\Psi(x)$ is the cost of the terminal state at fixed time $T$. 
Outline of gradient optimization:

1. given control trajectory $u$ and $x(t = 0)$.

2. propagate $\dot{x} = f [x, u, t]$.

3. evaluate $\lambda(T) = \partial \Psi [x(T)] / \partial x$.

4. sweep backward in time: $\dot{\lambda} = -f_x(x, t) \lambda$.

5. modify the control: $\delta u = -K \left[ f_u^T [x, t] \lambda + l_u^T \right]$.

6. go to step 2; repeat until suitable convergence.

- $\lambda$ is the adjoint;
- $f_x$ and $f_u$ are the derivatives of $f$ with the aggregate state and the control, respectively;
- $l_x$ and $l_u$ are the derivatives of the cost integrand $l(x, u, t)$. 
## Contrasts: Applications in the Ocean

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Elegant theory and significant accomplishments in the field with gliders</td>
<td>Computational tools well-defined: MILP and genetic algorithm</td>
<td>Gradient optimization, EKF, and simulated annealing: prototype</td>
</tr>
<tr>
<td>Unstructured model for error propagation (length and time scales only); regional placement of sensors</td>
<td>Optimization is based on daily error forecasts</td>
<td>Detailed error model and continuous optimization with assimilation</td>
</tr>
<tr>
<td>Inequality constraints</td>
<td>Smooth trajectories and few constraints</td>
<td></td>
</tr>
<tr>
<td>Simple paths with low computing overhead</td>
<td>Simple paths; reasonable cost</td>
<td>Complex paths; high cost</td>
</tr>
</tbody>
</table>
Example Case Field Conditions

- Ten complex modes – each amplitude is driven by colored noise with a peak at 0.1 rad/s
- Each mode is a traveling wave with same scales as ocean surface waves (i.e., $O(100\text{m length and } 10\text{s period})$)
- Omnidirectional – each component has random direction in $[0, 2\pi]$. 
Example Case Path Planning Setup

- Simulation time of 50s covers about five cycles in the field variables
- Vehicle model is based on maneuverable ASC (SAIC)
- First guess vehicle trajectories: 5m/s, straight line at a given heading
- Adaptive gain for optimization
- 4th-order Runge-Kutta in forward and reverse time
- Simulated Annealing perturbations:
  - Hit frequency begins at 10%, 1/e decay in 1000 trials
  - Each perturbation is a random rudder impulse on a randomly chosen vehicle
- “Assimilation” cost function is dominated by time integral of the field error trace - compare with integral of the local error squared (“Extrema,” e.g., Heaney, Yilmaz)
Relative Performance in Minimizing Integral Covariance Trace: Ten Vehicles

<table>
<thead>
<tr>
<th>Path Type</th>
<th>All East Initial Guesses *</th>
<th>Spread Initial Guess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight</td>
<td>4635</td>
<td>1.17</td>
</tr>
<tr>
<td>Extrema</td>
<td>1120</td>
<td>1.62</td>
</tr>
<tr>
<td>Assimilation</td>
<td>1.14</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Minimum dimensional value: 13.0
Ten Vehicles: Spread Guesses

White dot: Origin of all vehicles

Blue lines: Extrema paths

Red lines: Assimilation paths
Relative Performance in Minimizing Integral Covariance Trace: Two Vehicles

<table>
<thead>
<tr>
<th>Path Type</th>
<th>East-West Initial Guess</th>
<th>North-East Initial Guess</th>
<th>East-East Initial Guess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight</td>
<td>1.48</td>
<td>5.43</td>
<td>163</td>
</tr>
<tr>
<td>Extrema</td>
<td>10.8</td>
<td>3.91</td>
<td>60.6</td>
</tr>
<tr>
<td>Assimilation</td>
<td>1.13</td>
<td>1.00</td>
<td>1.06</td>
</tr>
</tbody>
</table>

Minimum dimensional value: 393
Two Vehicles, North-East Guesses

White dot: Origin of both vehicles

Blue lines: Extrema paths

Red lines: Assimilation paths
Relative Performance in Minimizing Integral Covariance Trace: One Vehicle

<table>
<thead>
<tr>
<th>Path Type</th>
<th>East Initial Guess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight</td>
<td>21.2</td>
</tr>
<tr>
<td>Extrema</td>
<td>11.2</td>
</tr>
<tr>
<td>Assimilation</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Minimum dimensional value: 2920

Scaling laws for the example:  
\[ J \sim 20^4 / \text{veh}^3 \text{ for extrema} \]  
\[ J \sim 20^3 / \text{veh}^2 \text{ for assimilation} \]
One Vehicle: East Guesses

White dot: Origin of vehicle

Blue lines: Extrema paths

Red lines: Assimilation paths

north, m
0 200

east, m
0 200
Typical Convergence of the Optimization
The Two- and Ten-Vehicle Cases

Convergence of the extrema-seeking and assimilation optimizations is dependent on the specific problem, and share many of the same attributes.
Computational Costs

• Cost of each forward/backward integration scales with $N_f^3$: $N_f$ is the number of error covariance states
• Cost of each forward/backward integration scales \textit{linearly} with time steps $N_t$
• Number of iterations required is NOT strongly dependent on $N_f$ or $N_t$
• Current examples have $\max(N_f) = 210$ and $N_t = 500$
  60 vehicle states, 20 control channels
PPIDA

- Fully integrated planning and assimilation in environmental monitoring
- Example cases show intuitive tradeoff between array size and performance, and suggests when the PPIDA approach will pay off or not
- Observed error reductions may warrant significant computational effort for path planning
- Take the prototype version for a drive on 1 November: web.mit.edu/hovergroup
Assimilation of Data into Modeling

Vehicle Operations

Model Run

#1

#2

#3

#4

#5

#6

#7

Time