

# Basic MR image encoding

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# Physical Foundations of MRI

What is NMR?

The basic signal we excite and detect.

Tricks of NMR

The gradient and spin echo

How do we encode an image?

slice select, frequency and phase encoding.

What are some problems (artifacts) relevant to our application.

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## Physical Foundations of MRI

**NMR:** 60 year old phenomena that generates the signal from water that we detect.

**MRI:** using NMR to generate an image

Three magnetic fields (generated by 3 coils)

- 1) static magnetic field  $B_0$
- 2) RF field that excites the spins  $B_1$
- 3) gradient fields that encode spatial info  
 $G_x, G_y, G_z$

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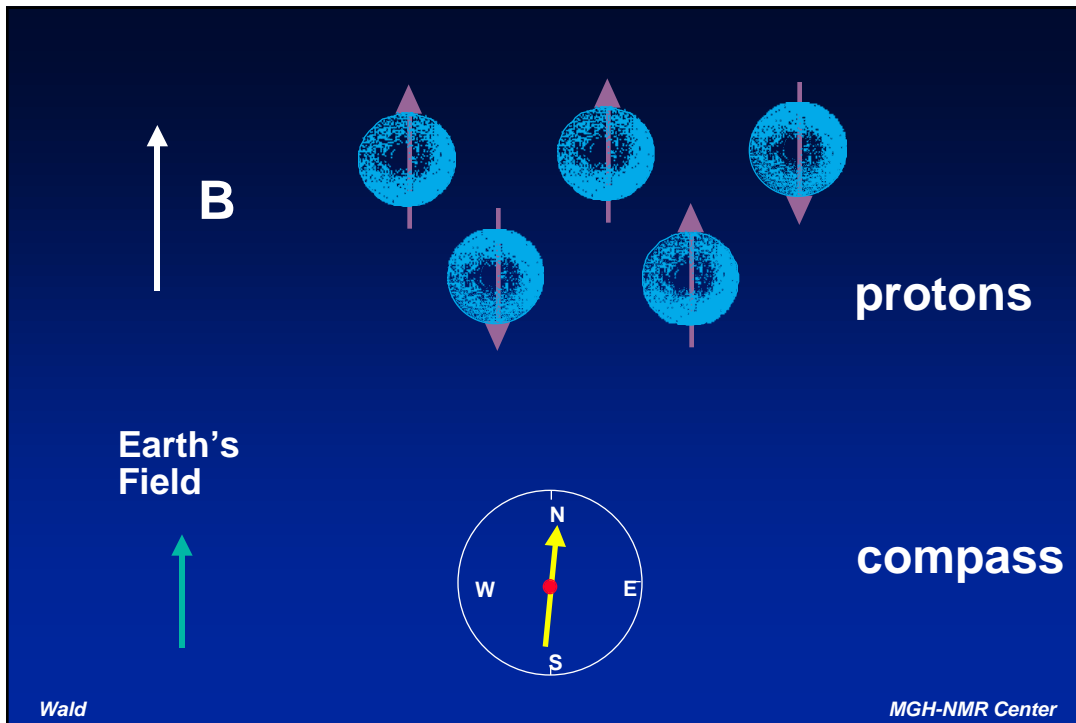
## What is NMR?

**NUCLEAR**  
**MAGNETIC**  
**RESONANCE**

A magnet, a glass of water,  
and a radio wave source and detector....

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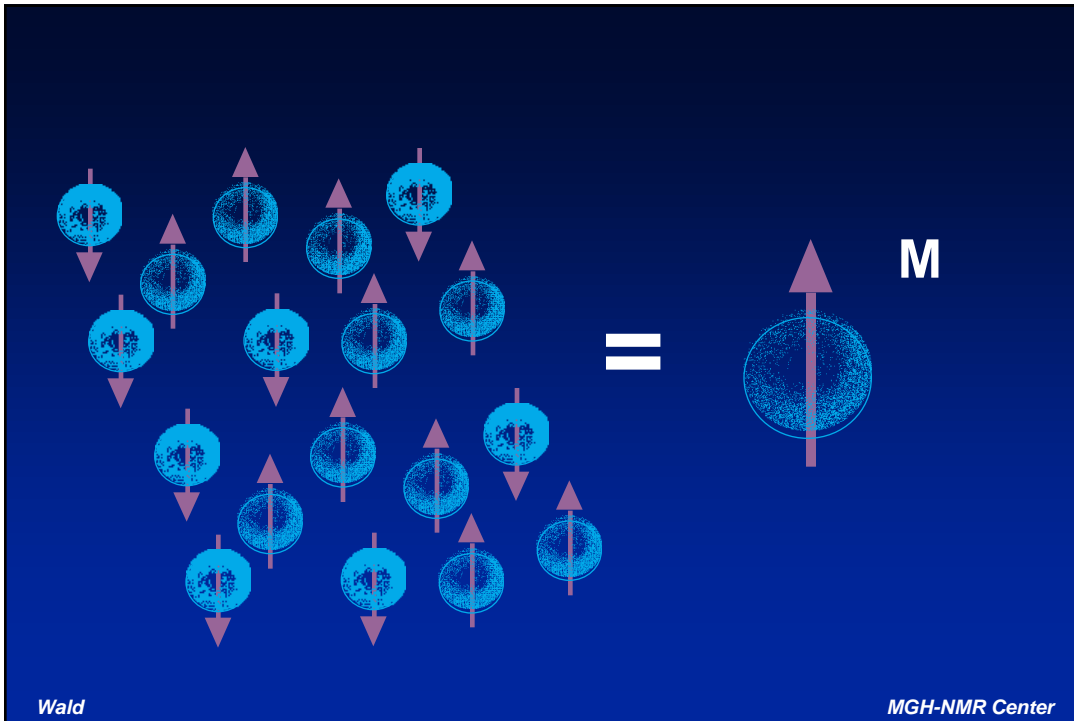
## **Nuclei and Magnetic Fields**

**Not every nucleus lines up with applied magnetic field.**

**Why?**

**Direction of spins becomes randomized by thermal motion.**

**protons at 1.5 Tesla, at room temperature  
net # aligned with field is 1 part in 100,000**



## Compass needles

The vector sum of all the nuclei can be viewed as a compass needle.

Points North. (aligns along the magnetic field lines of the external field (earth or MR magnet))

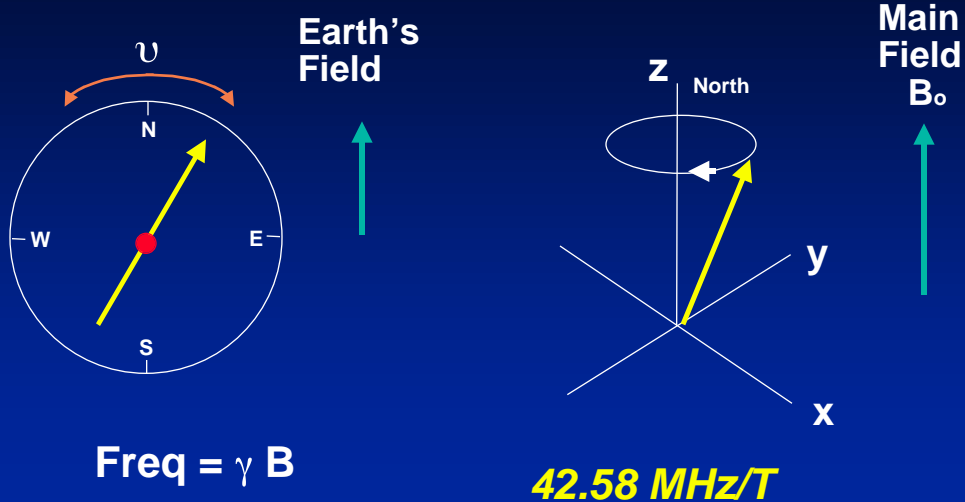
If displaced from North, it will wobble about north with a characteristic frequency (called Larmor freq.)



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## Compass needles



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## **EXCITATION** : Displacing the spins from Equilibrium (North)

**Problem:** It must be moving for us to detect it.

**Solution:** knock out of equilibrium so it oscillates

How? 1) Tilt the magnet or compass suddenly

2) Drive the magnetization (compass needle) with a periodic magnetic field

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## Excitation: Resonance

Why does only one frequency efficiently tip protons?

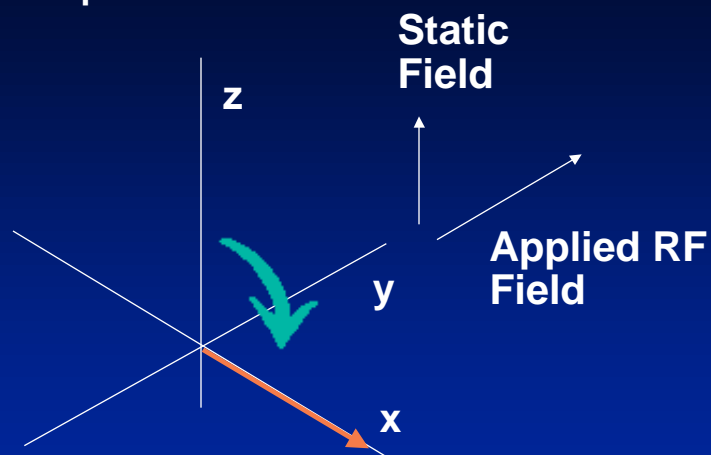
**Resonant driving force.**

It's like pushing a child on a swing in time with the natural oscillating frequency.

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z is "longitudinal" direction  
x-y is "transverse" plane



**The RF pulse rotates  $M_0$  about applied field**

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## "Exciting" Magnetization

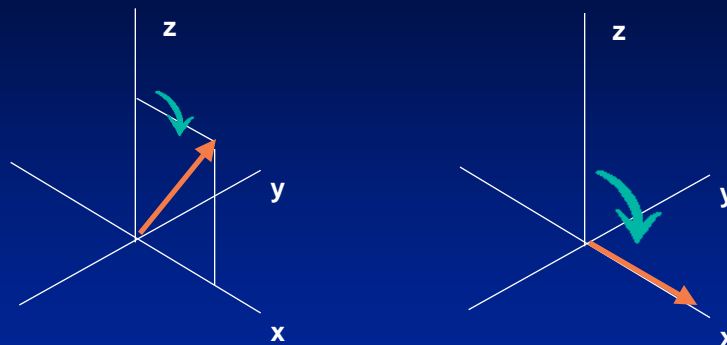
Magnetization processes about new axis (of oscillating RF B field) as long as resonant field is applied.

Total amount vector processes is called the "tip angle" of the excitation.

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## "Exciting" Magnetization tip angle



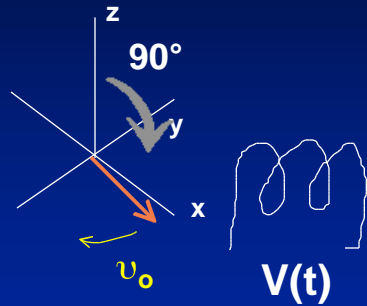
45°

90°

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## Detecting the NMR Signal



A moving bar magnet induces a Voltage in a coil of wire. (a generator...)

The RF coil design is the #1 determinant of the system SNR

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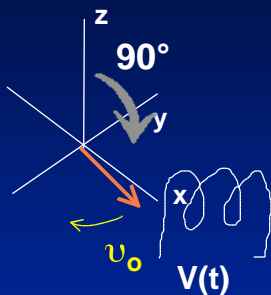
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## Detecting the NMR: the Signal

The coil should be close to the head.

Reciprocity Theorem:

The detection efficiency of a coil detecting a spin at location  $(x,y,z)$  is proportional to the  $B_1$  field it would produce at that location if the coil was driven with 1 amp of current.



Detection efficiency map  $\propto B_1(x,y,z)$

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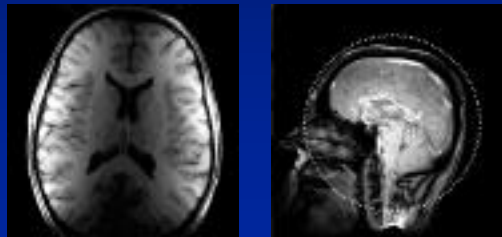


## Detecting the NMR: the Signal

Detection efficiency map  $\propto B_1(x,y,z)$

Big is bad.

Small might mean that  $B_1(x,y,z)$  is spatially varying.



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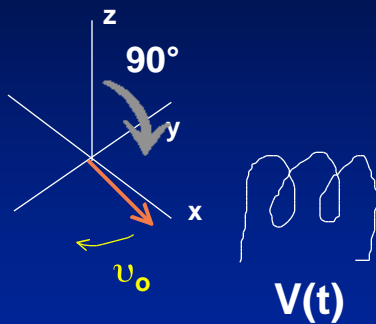
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## Detecting the NMR: the noise

Noise comes from electrical losses in the resistance of the coil or electrical losses in the tissue.

For a resistor:  
 $P_{\text{noise}} = 4kTRB$

- Noise is white.  
>> Power  $\propto$  bandwidth
- Noise is spatially uniform.
- R is dominated by the tissue. >> big coil is bad.



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## Signal to Noise Ratio in MRI

Most important piece of hardware is the RF coil.

$\text{SNR} \propto \text{voxel volume}$  (# of spins)

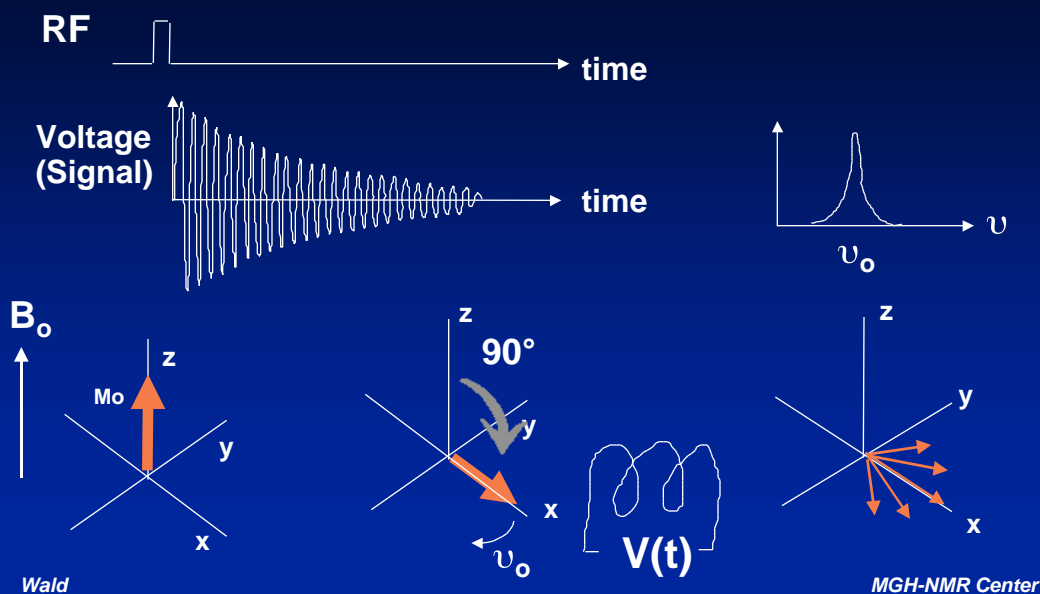
$\text{SNR} \propto \text{SQRT}(\text{total time of data collection})$

SNR is also dependent on the amount of signal you throw away to get contrast.

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## Review: the NMR Signal



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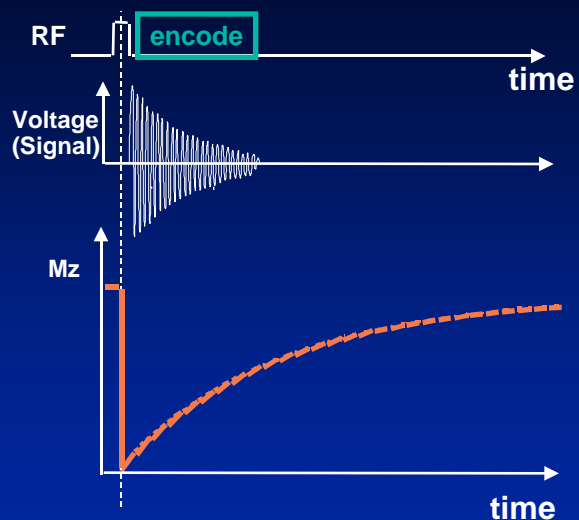
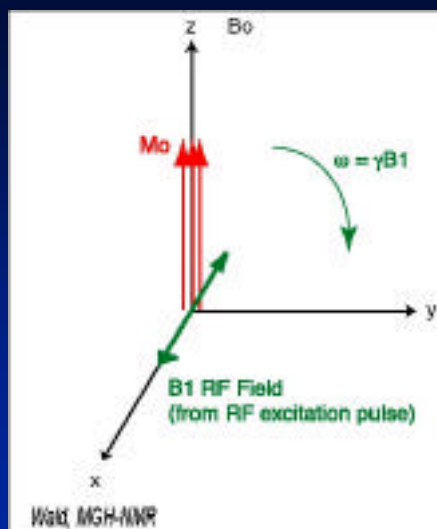
## Three Steps in MR:

- 0) Equilibrium (magnetization points along  $B_0$ )
- 1) RF Excitation (tip magn. away from equil.)
- 2) Precession induces signal, dephasing (timescale =  $T_2$ ,  $T_2^*$ ).
- 3) Return to equilibrium (timescale =  $T_1$ ).

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## Magnetization vector during MR



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## Three places in process to make a measurement (image)

0) Equilibrium (magnetization points along  $B_0$ )

1) RF Excitation (tip magn. away from equil.)

proton  
density  
weighting

2) Precession induces signal, allow to dephase for time  $TE$ .

T2 or T2\*  
weighting

3) Return to equilibrium (timescale  $=T_1$ ).

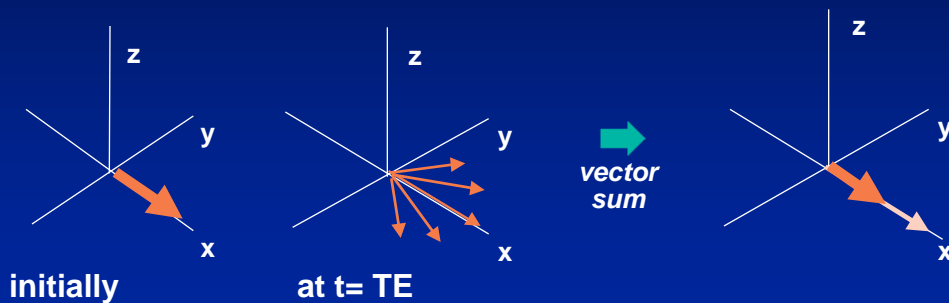
T1 Weighting

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## T2\*-Weighting

Wait time  $TE$  after excitation before measuring M.  
Shorter  $T_2^*$  spins have dephased

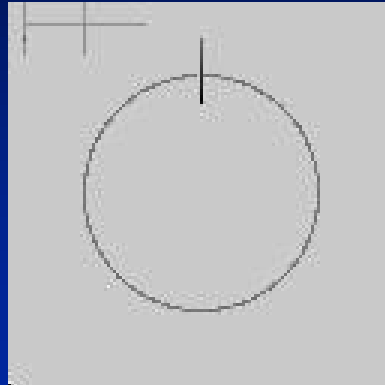


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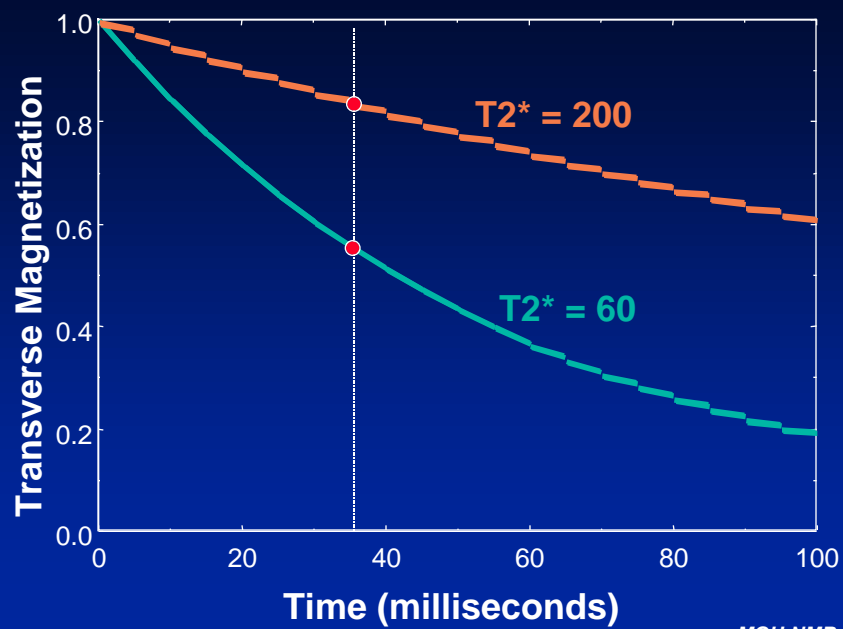
# T2\* Dephasing

Just the tips of the vectors...



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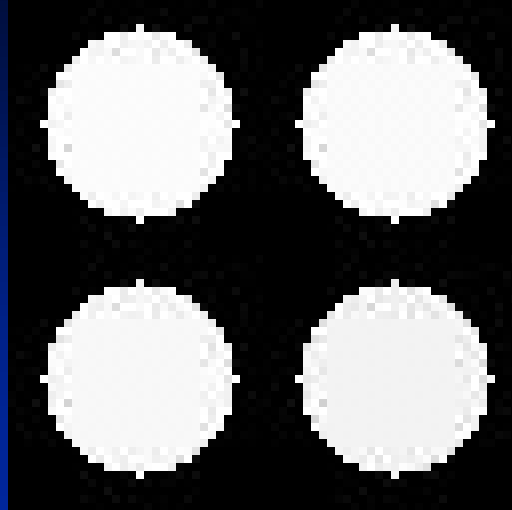
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## T2 Weighting

Phantoms with  
four different T2 decay rates...

There is no contrast  
difference immediately  
after excitation, must  
wait (but not too long!).

Choose TE for max.  
inten. difference.

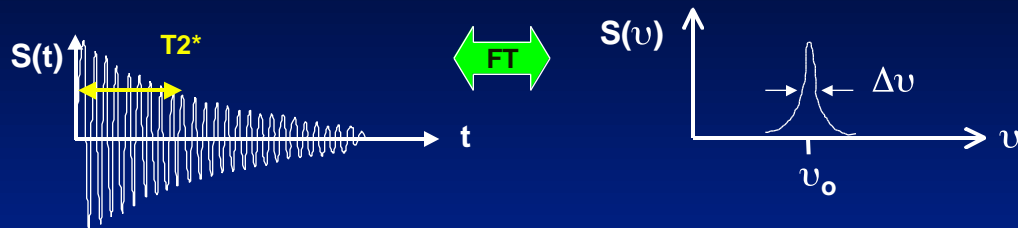


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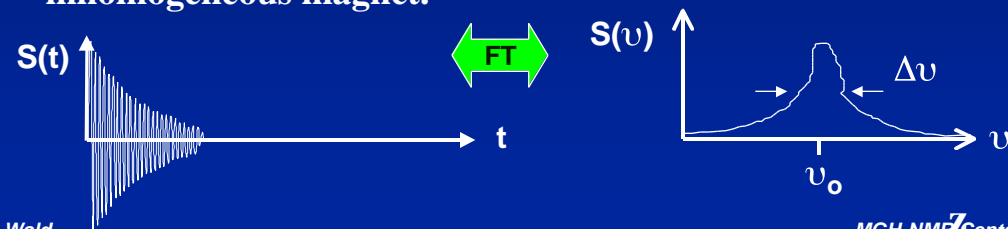
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## Dephasing: local field variations

homogeneous magnet.



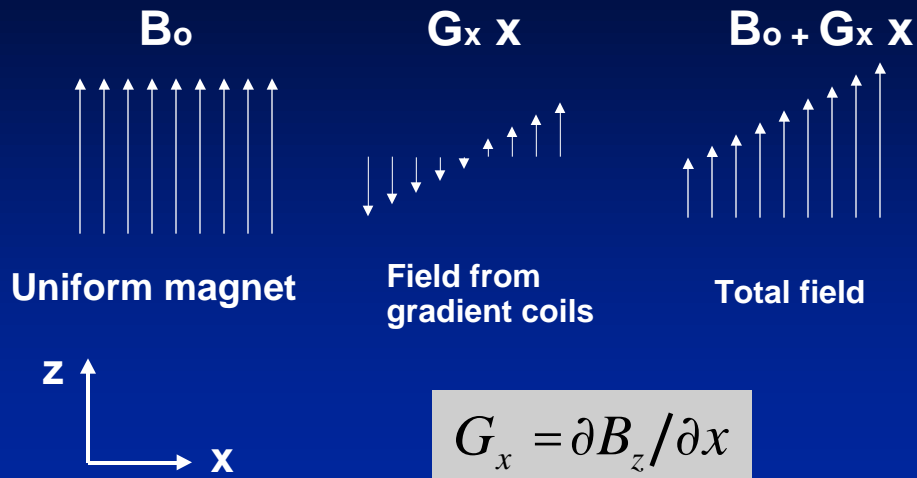
inhomogeneous magnet.



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## Aside: Magnetic field gradient



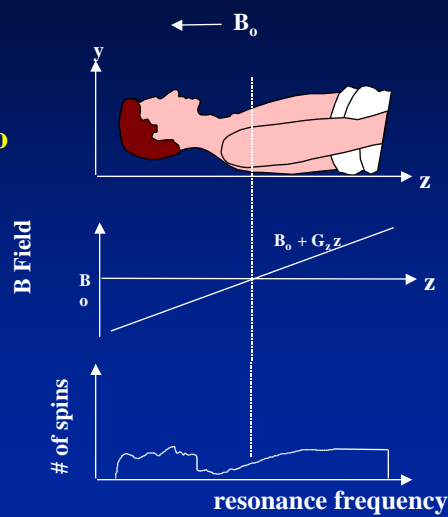
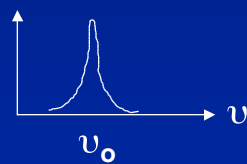
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## A gradient causes a spread of frequencies

MR frequency of the protons in a given location is proportional to the local applied field.

$$\nu = \gamma \mathbf{B}_{TOT} = \gamma (\mathbf{B}_0 + \mathbf{G}_z z)$$



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## If I control it, I can reverse it... Gradient echo

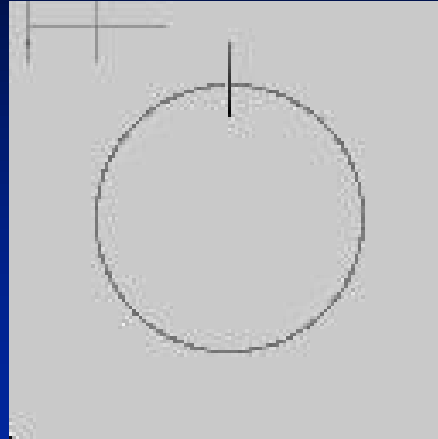
$G_z$  vs time

$$\nu = \gamma \mathbf{B}_{\text{TOT}} = \gamma B_0 + \mathbf{G}_z \mathbf{z}$$

$$\Delta\nu = \gamma \Delta \mathbf{B}_{\text{TOT}} = \gamma \mathbf{G}_z \mathbf{z}$$

Gratuitous manipulation...  
(?)

What happens if the spin moves?



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## Less trivial manipulation... the Spin Echo

Refocus the dephased signal without resorting to  
direct control of the  $B_0$  field.

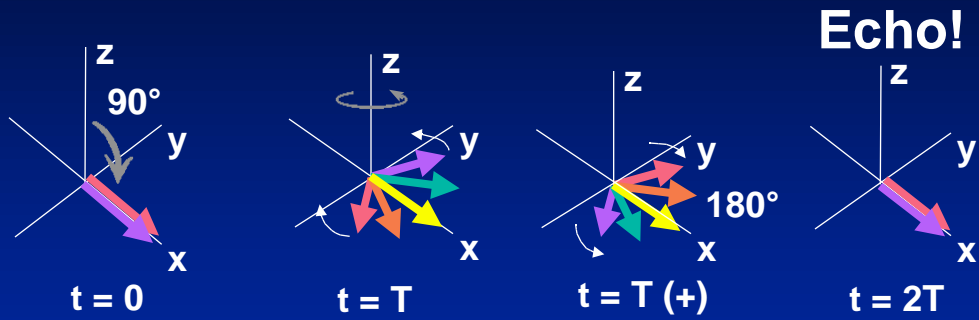
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# Spin Echo

Some dephasing can be refocused because its due to static fields.

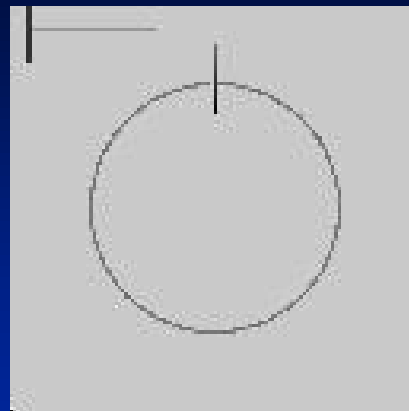
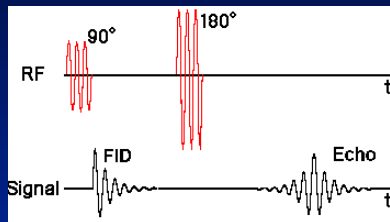


Blue/Green arrows precesses faster due to local field inhomogeneity than red/orange arrow

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# Spin Echo



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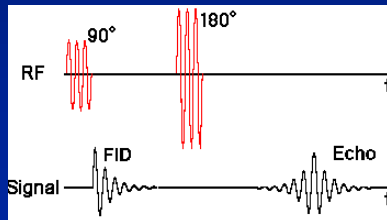
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# Spin Echo

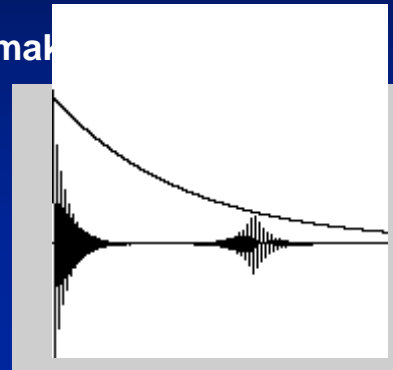
180° pulse only helps cancel static inhomogeneity

The “runners” can have static speed distribution.

If a runner trips, he will not make it with the others.



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## Part II

# Image encoding

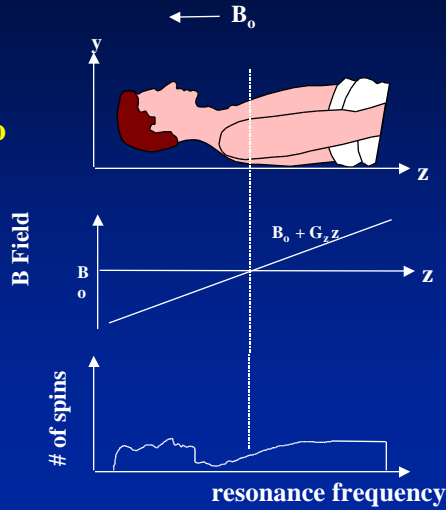
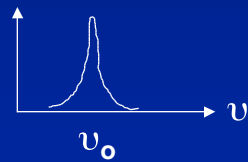
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# 1D projection image

MR frequency of the protons in a given location is proportional to the local applied field.

$$\nu = \gamma \mathbf{B}_{TOT} = \gamma(\mathbf{B}_0 + \mathbf{G}_z z)$$

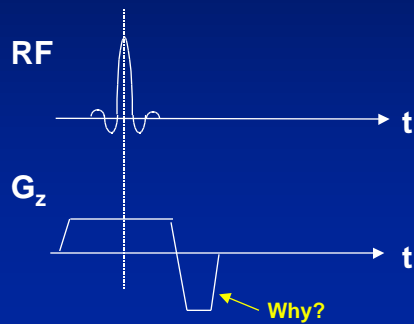
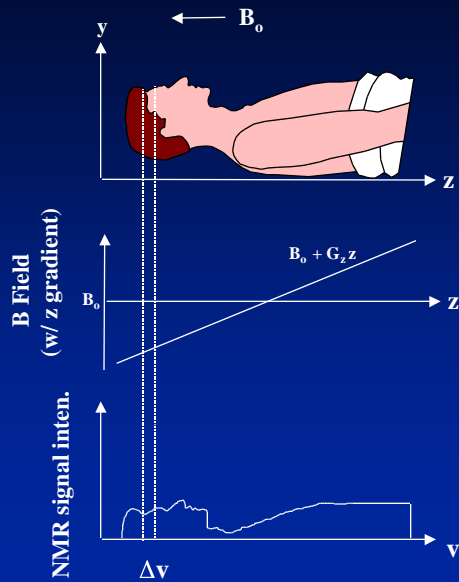


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# Step one: excite a slice

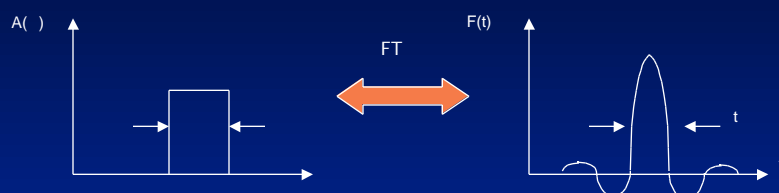
While the grad. is on, excite only band of frequencies.



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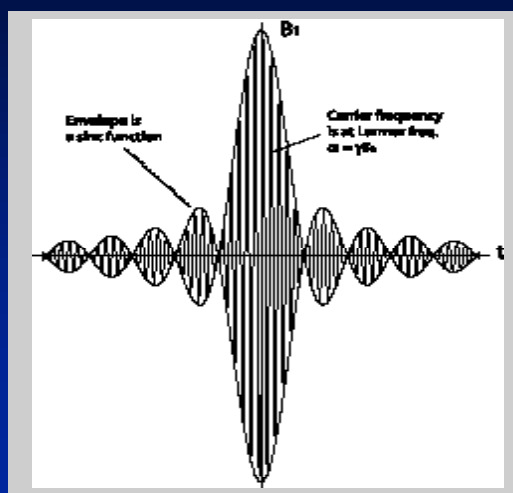
## Slice profile considerations



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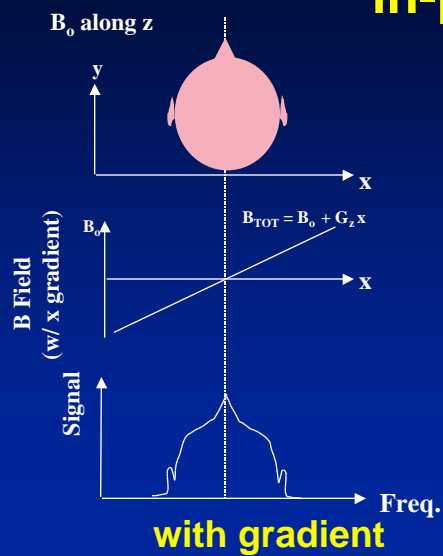
## Slice selective RF pulse



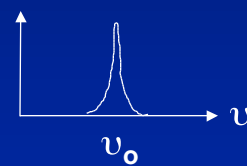
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## Step two: encode spatial info. in-plane



“Frequency encoding”

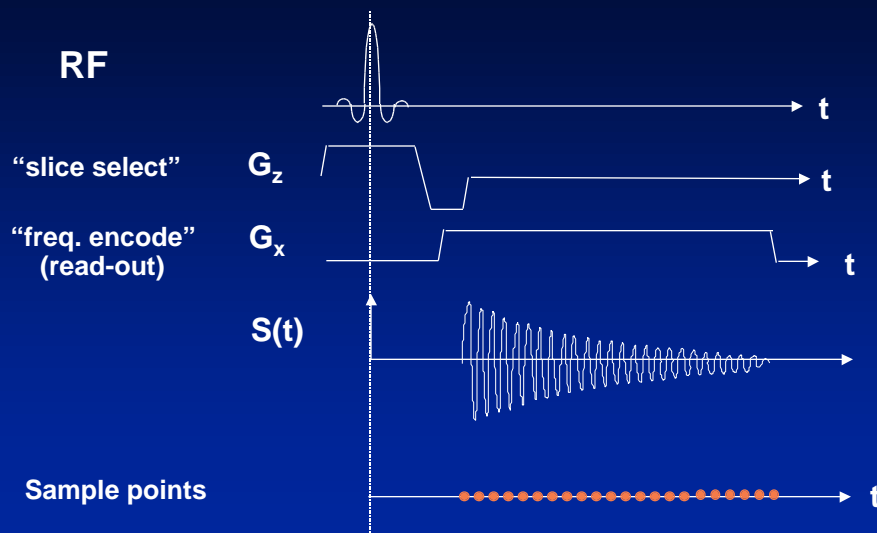


without gradient

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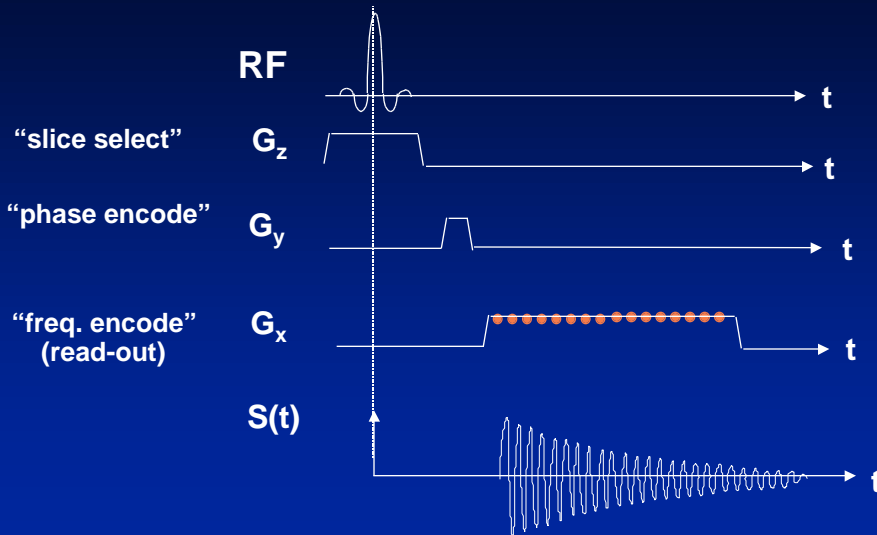
## ‘Pulse sequence’ so far



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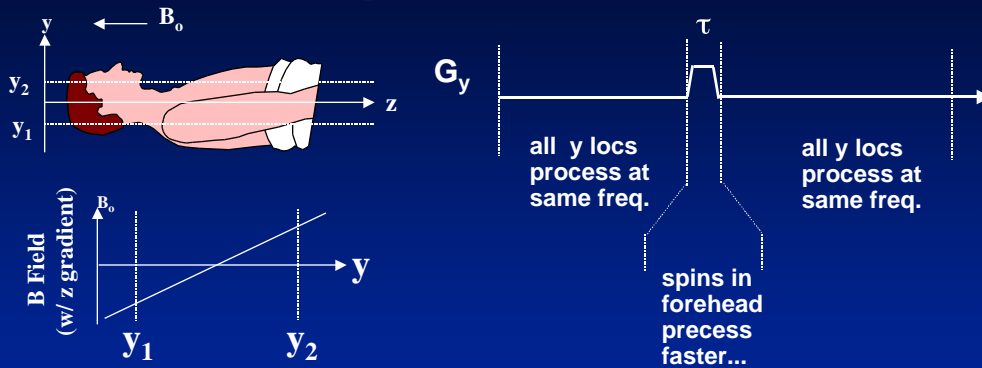
# “Phase encoding”



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# How does blipping on a grad. encode spatial info?



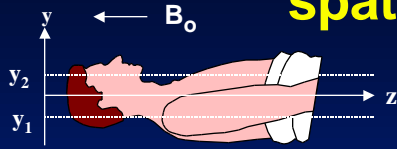
$$v(y) = \gamma \mathbf{B}_{TOT} = \gamma B_0 \Delta y G_y$$

$$\theta(y) = v(y) \tau = \gamma B_0 \Delta y (G_y \tau)$$

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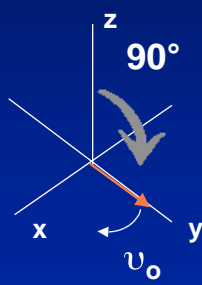
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## How does blipping on a grad. encode spatial info?

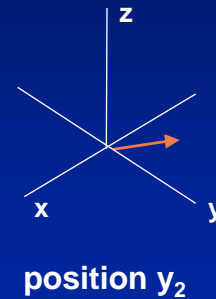
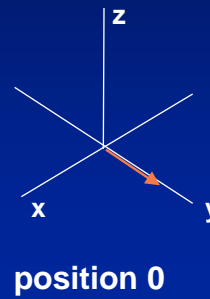
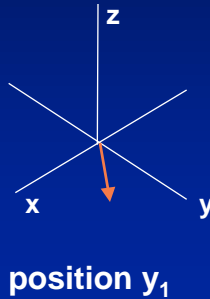


$$\theta(\mathbf{y}) = v(\mathbf{y}) \tau = \gamma B_0 \Delta y (G_y \tau)$$

after RF



After the blipped y gradient...



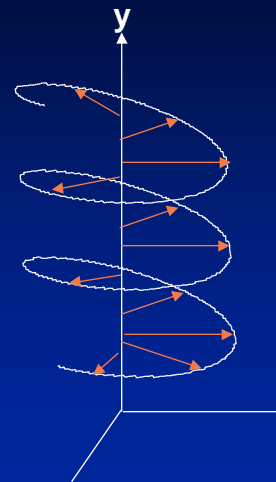
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## How does blipping on a grad. encode spatial info?

The magnetization vector in the xy plane is wound into a helix directed along y axis.

Phases are 'locked in' once the blip is over.

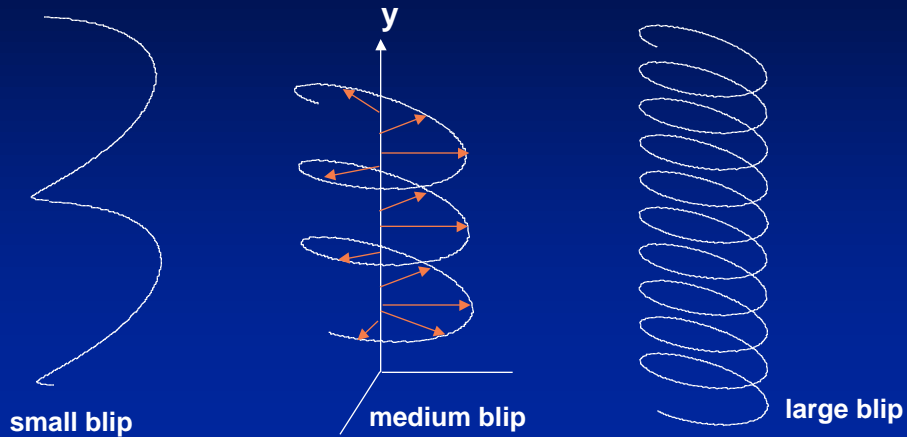


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## The bigger the gradient blip area, the tighter the helix

$$\theta(\mathbf{y}) = v(\mathbf{y}) \tau = \gamma \mathbf{B}_0 \Delta \mathbf{y} (\mathbf{G}_y \tau)$$

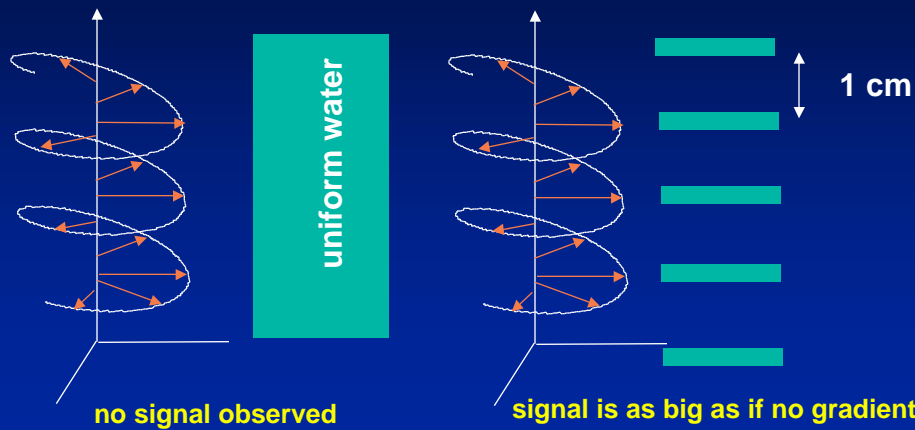


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## What have you measured?

Consider 2 samples:

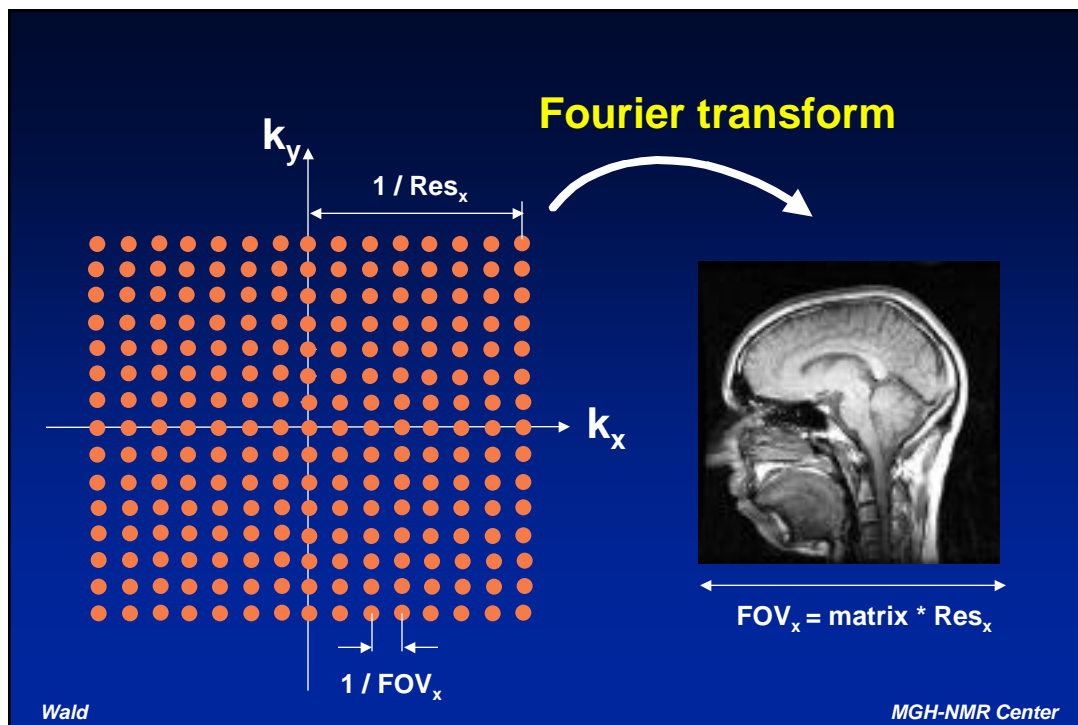
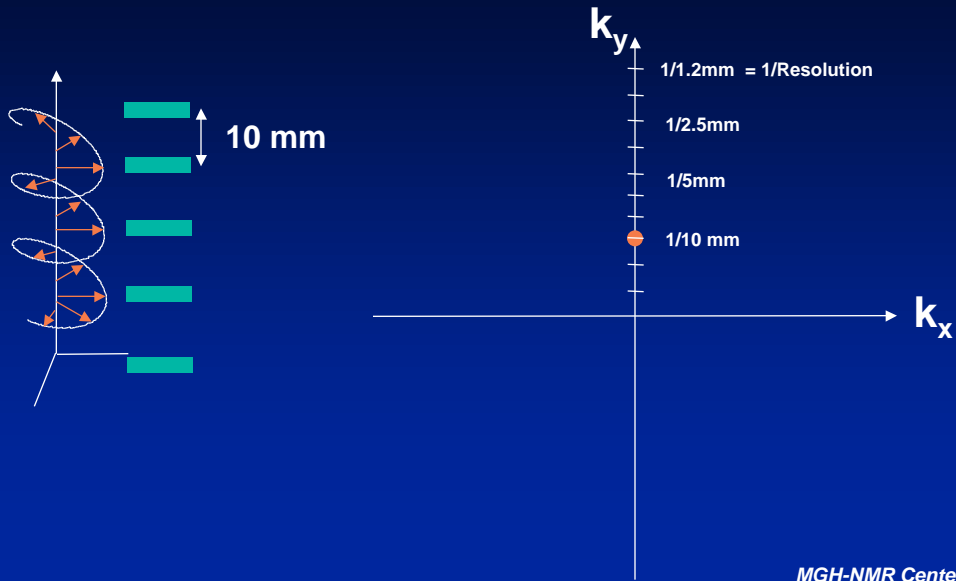


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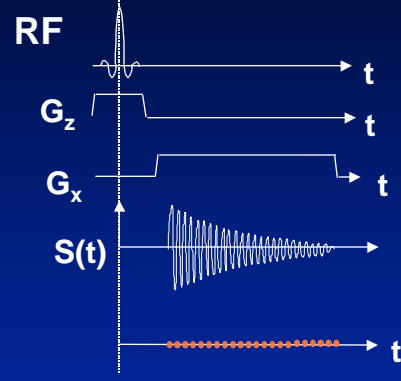
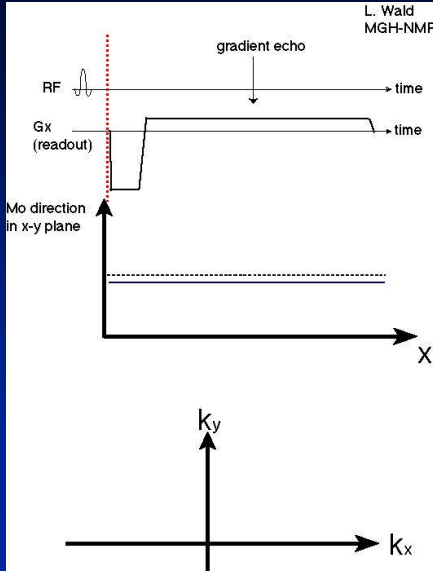
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## Measurement intensity at a spatial frequency...



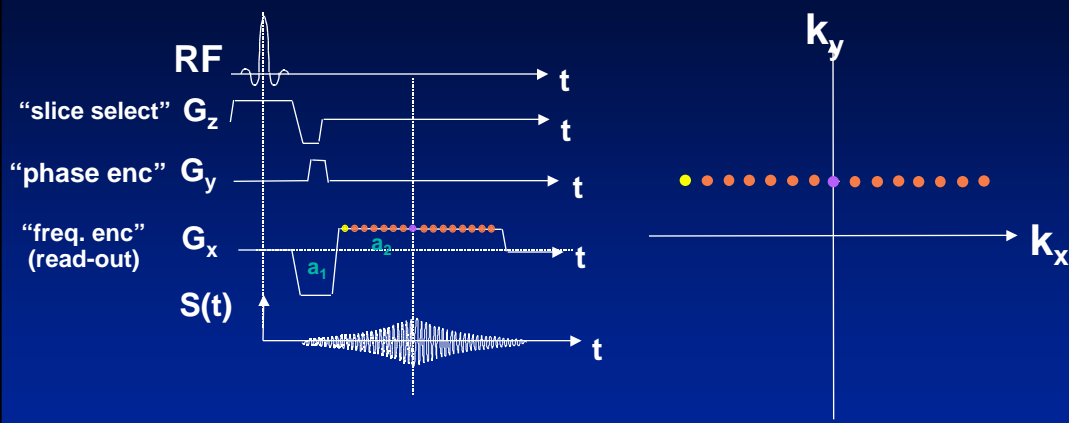
# Frequency encoding revisited



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# "Spin-warp" encoding



one excitation, one line of kspace...

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## “Spin-warp” encoding mathematics

The “image” is the spin density function:  $\rho(x)$

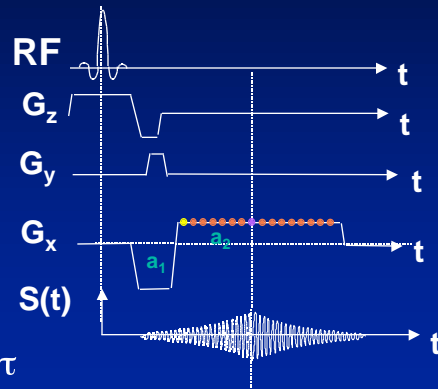
Phase due to readout:

$$\theta(t) = \omega_0 t + \gamma G_x x t$$

Phase due to P.E.

$$\theta(t) = \omega_0 t + \gamma G_y y \tau$$

$$\Delta\theta(t) = \omega_0 t + \gamma G_x x t + \gamma G_y y \tau$$



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## “Spin-warp” encoding mathematics

Signal at time  $t$  from location  $(x,y)$

$$S(t) = \rho(x,y) e^{i\gamma G_x x t + i\gamma G_y y \tau}$$

The coil integrates over object:

$$S(t) = \int_{\text{object}} \rho(x,y) e^{i\gamma G_x x t + i\gamma G_y y \tau} dx dy$$

Substituting  $k_x = - G_x t$  and  $k_y = - G_y \tau$ :

$$S(k_x, k_y) = \int_{\text{object}} \rho(x,y) e^{-ik_x x - ik_y y} dx dy$$

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## “Spin-warp” encoding mathematics

View signal as a matrix in  $k_x, k_y \dots$

$$S(k_x, k_y) = \int_{\text{object}} \rho(x, y) e^{-ik_x x - ik_y y} dx dy$$

∴  
Solve for  $(x, y)$

$$\rho(x, y) = FT^{-1} [S(k_x, k_y)]$$

$$\rho(x, y) = \int_{\text{k-space}} S(k_x, k_y) e^{ik_x x + ik_y y} dk_x dk_y$$

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