

# Basic Principles of Magnetic Resonance

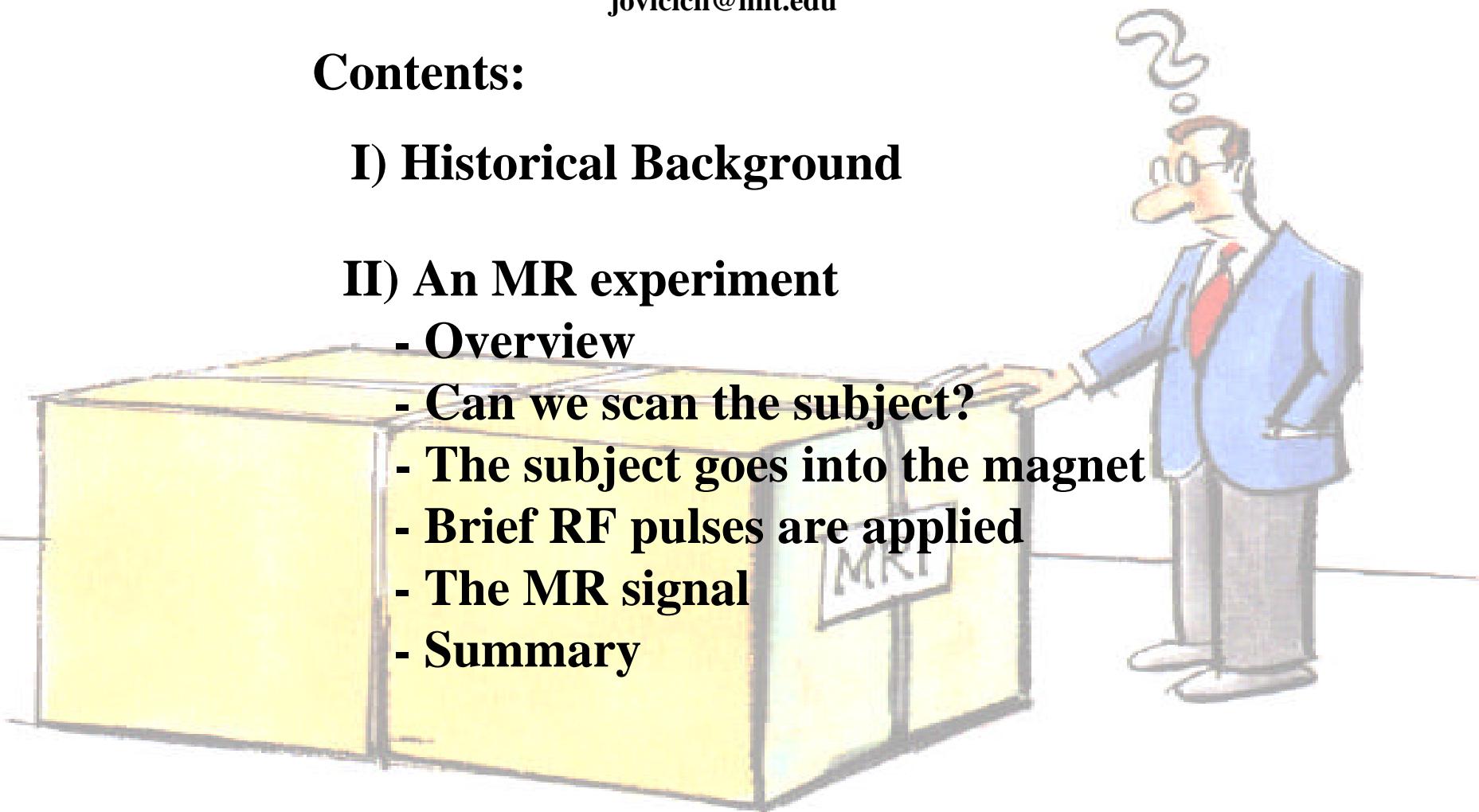
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## Contents:

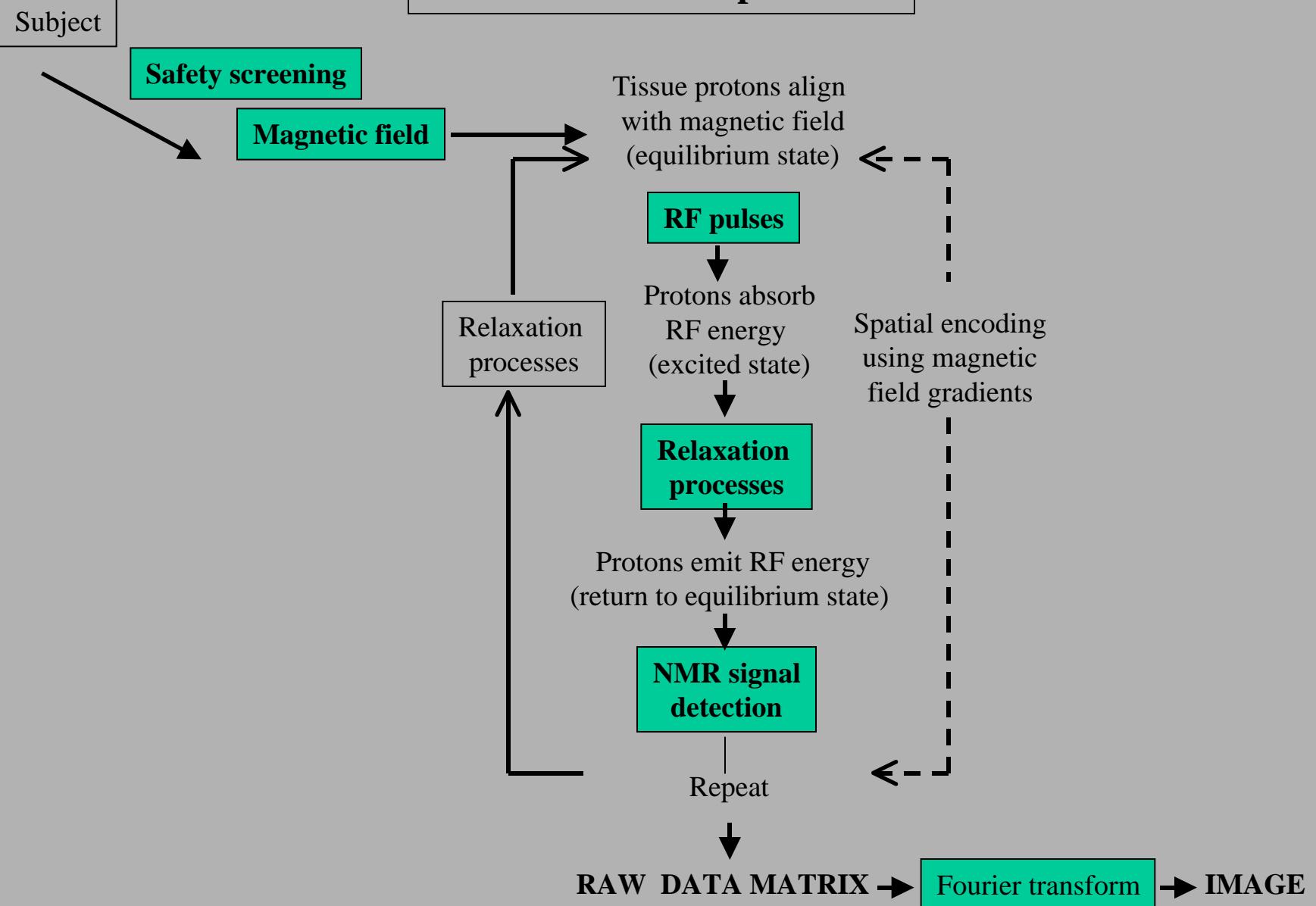
### I) Historical Background

### II) An MR experiment

- Overview
- Can we scan the subject?
- The subject goes into the magnet
- Brief RF pulses are applied
- The MR signal
- Summary



## Overview of a MRI procedure

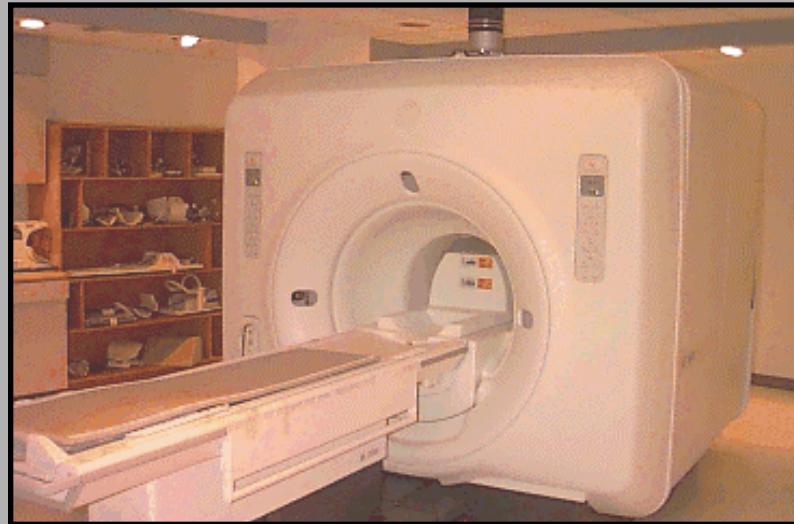


## **Can we scan the subject?**

### **Safety Issues**

- Not everybody can undergo a MR examination
- Screening is mandatory to ensure safety / quality during the MR examination

## A typical MRI scanner



### The main magnetic field:

- is **VERY strong** (  $3T \sim 30,000$  times the earth's magnetic field)
- is **ALWAYS ON !!** (superconducting currents)
- its strength decays  $\sim 1 / r^3$  ( $r$  is the distance from the center)

## Can we scan the subject? **Safety Issues**

- Risks:
  - Ferromagnetic materials (unsafe)
    - will be subject to force / torque in the main magnetic field
    - risks associated with motion / displacements
    - active biomedical implants may not function properly

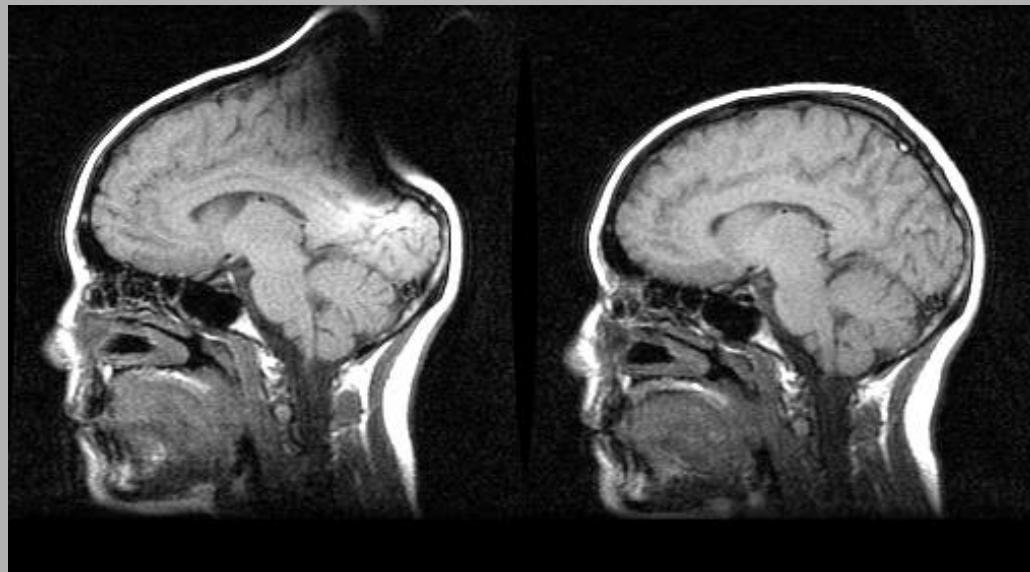


# Can we scan the subject?

## Safety Issues

- Risks:
  - Ferromagnetic materials (unsafe)
  - Non ferromagnetic materials (safe but give image distortions)

Sagittal MRI  
of a normal  
female subject



With hair rubber band\*      Without hair rubber band

# Can we scan the subject?

## Safety Issues

- Risks:

- Ferromagnetic materials (unsafe)
- Non ferromagnetic materials (safe but give image distortions)
- Claustrophobia (subject unhappy      image distortions)



## **Can we scan the subject?**

### **Safety Issues**

- Risks:

- Ferromagnetic materials (unsafe)
- Non ferromagnetic materials (safe but give image distortions)
- Claustrophobia (subject unhappy      image distortions)
- Movement during examination (potential problem for dynamic studies)
- Acoustic protection (mandatory)

## Overview of a MRI procedure

Subject

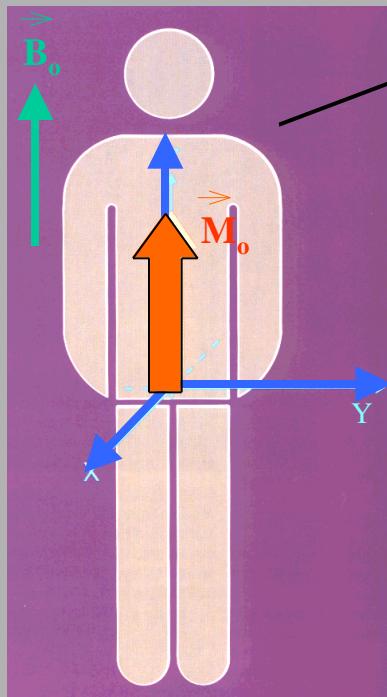
Safety screening

Magnetic field

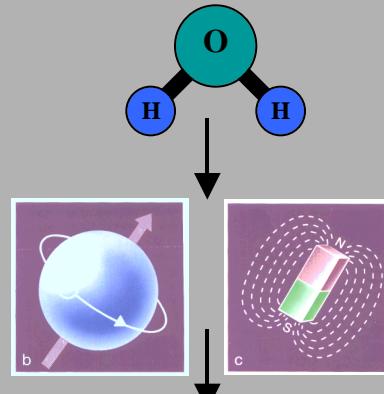
We will introduce the following concepts:

- equilibrium magnetization
- dynamics of the magnetization
- rotating coordinate system

## The subject goes into the magnet...

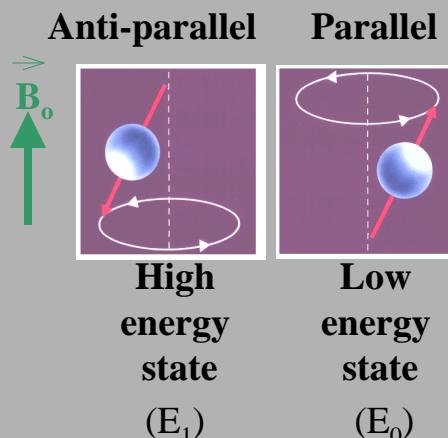


Single nucleus



Water molecules

Hydrogen nucleus:  
magnetic moment  $\vec{\mu}$



Two energy states:

$$E = E_1 - E_0$$

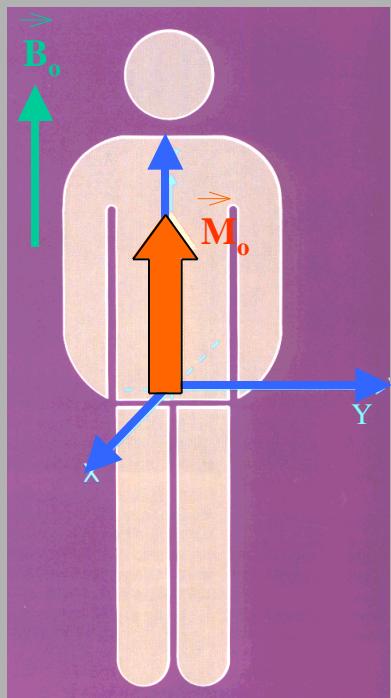
$$E = \hbar\omega_0$$

Precession frequency:  
 $\omega_0 = \gamma B_0$

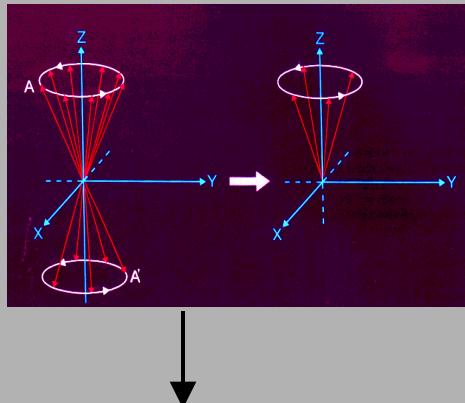
$\vec{B}_0$ : uniform static magnetic field

$\vec{M}_0$ : static macroscopic magnetization

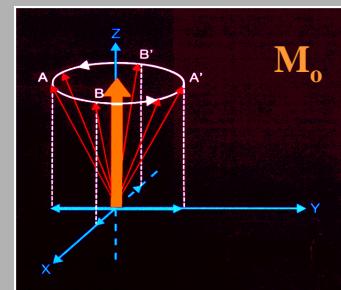
## The subject goes into the magnet... (continued)



A group of protons

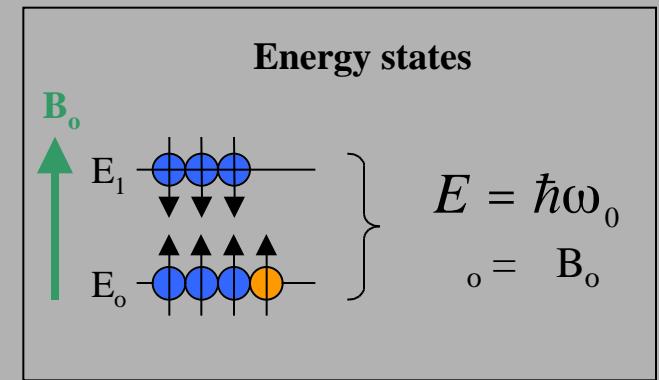


Net magnetization  $M_0$



Equilibrium magnetization  $M_0$ :

- $M_z$  aligned with  $B_0$
- $M_{xy} = 0$



Spin states distribution

$$\frac{N_1}{N_0} = \exp -\frac{\hbar\gamma B_0}{kT}$$

$N_1$  = # of protons in state  $E_1$

$N_0$  = # of protons in state  $E_0$

$k$  : Boltzmann's constant

$T$  : temperature

## Reminder of three main steps in MRI

0) Equilibrium ( $M_o$  along  $B_o$ )



1) RF excitation  
(tip  $M_o$  away from equil.)



2) Precession of  $M_{xy}$  induces signal  
(dephasing for a time  $TE$ )



3) Return to equilibrium  
(recovery time  $TR$ )



Dynamics of  
magnetization

# Dynamics of the magnetization

- Equation of motion of  $\vec{M}$  in external  $\vec{B}_{ext}$
- Classical mechanics formalism
- First, model a single nucleus
- Then, add up for sample
- Finally, consider relaxation (Bloch equations)

## Equation of motion for a **single nucleus**:

- Mechanical moment      angular momentum (spin)

$$\vec{T} = \frac{d\vec{L}}{dt}$$

- Magnetic moment      spin

$$\vec{\mu} = \gamma \vec{L}$$

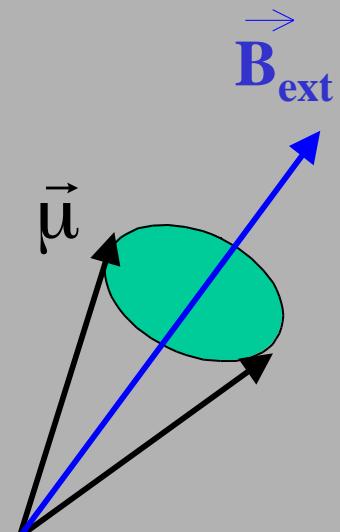
- Magnetic moment and magnetic field interaction

$$\vec{T} = \vec{\mu} \times \vec{B}_{ext}$$



$$\boxed{\frac{d\vec{\mu}_{(t)}}{dt} = \vec{\mu}_{(t)} \times \gamma \vec{B}_{ext(t)}}$$

Precession of  
 $\vec{\mu}$  about  $\vec{B}_{ext}$   
with frequency  
 $\omega = \gamma B_{ext}$



## Equation of motion for the **magnetization vector**:

- Assuming no interaction between nuclei

$$\vec{M} = \vec{\mu}_0 + \vec{\mu}_1 + \vec{\mu}_2 + \dots = \vec{\mu}_i \quad (\text{spin excess})$$

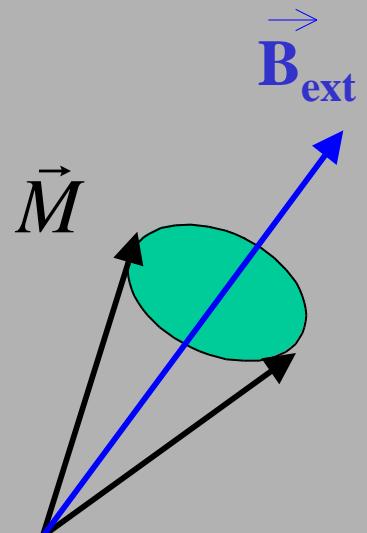
- And since for each nuclei

$$\frac{d\vec{\mu}_{i(t)}}{dt} = \vec{\mu}_{i(t)} \times \gamma \vec{B}_{ext(t)}$$



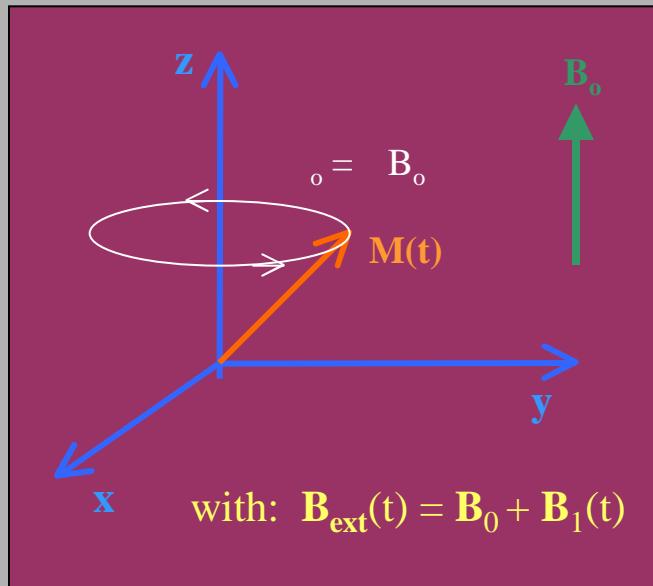
$$\boxed{\frac{d\vec{M}_{(t)}}{dt} = \vec{M}_{(t)} \times \gamma \vec{B}_{ext(t)}}$$

Precession of  
 $\vec{M}$  about  $\vec{B}_{ext}$   
with frequency  
 $\omega = \gamma B_{ext}$

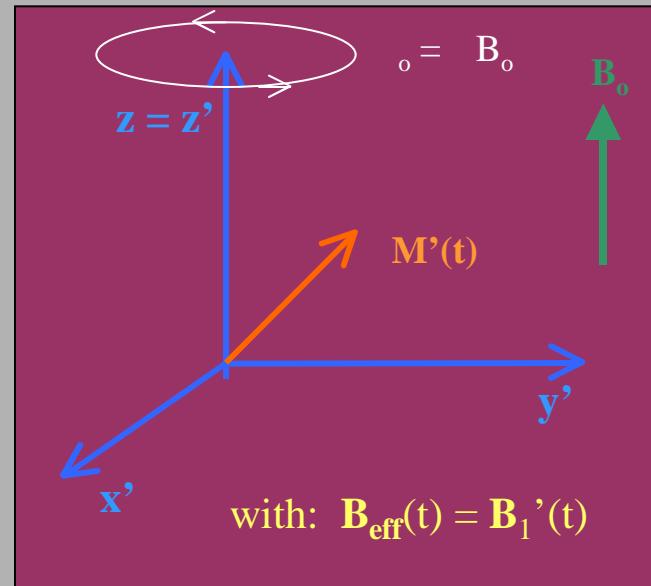


To describe the magnetization we need  
to choose a coordinate systems

Laboratory coordinate system



Rotating coordinate system



$$\frac{d\vec{M}_{(t)}}{dt} = \vec{M}_{(t)} \times \gamma \vec{B}_{\text{ext}(t)}$$

$$\frac{d\vec{M}'_{(t)}}{dt} = \vec{M}'_{(t)} \times \gamma \vec{B}_{\text{eff}(t)}$$

Example:

On-resonance spins:  $_{\text{o}}$

Off-resonance spins:  $_{\text{o}+}$

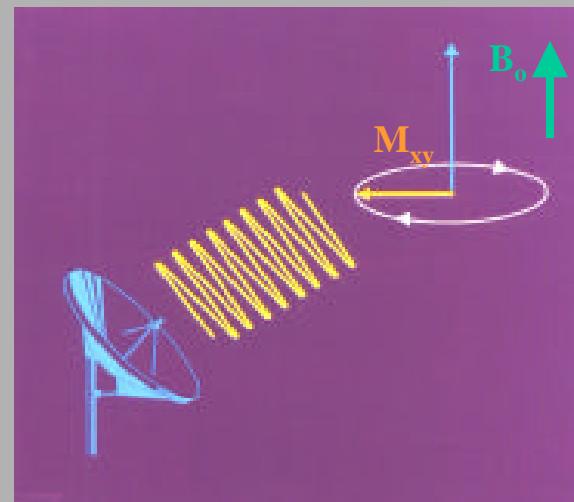
static

## The NMR signal

- At equilibrium no signal:
  - static longitudinal magnetization  $M_o$

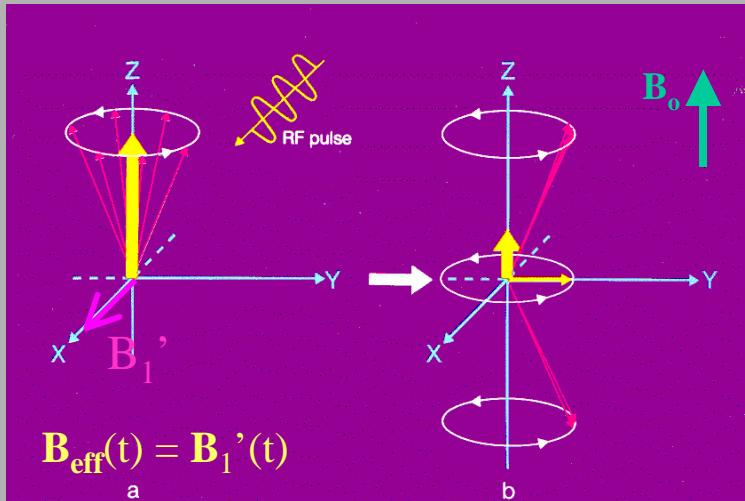


- We need net transverse magnetization:
  - precession of  $M_{xy}$  about  $B_0$
  - rotating magnetic field
  - induces current in a coil: MR signal



## The NMR signal (continuation)

- With an RF pulse on resonance we can rotate  $\underline{M}_0$



### Rotating frame:

- Dynamics:  $\frac{d\vec{M}^{(t)}}{dt} = \vec{M}^{(t)} \times \gamma \vec{B}_{\text{eff}(t)}$
- $B_1'(t)$ :  $B_1'$  constant and  $\perp$  to  $B_0$
- $M'$  precesses about  $B_1$  with  $\omega_1 = \gamma B_1'$
- Flip angle  $\theta$  :  $\theta = \frac{\pi}{2} - \frac{\phi}{2}$

- Signal relaxation, system goes back to equilibrium

$M_z \rightarrow M_o$  ( $T_1$  relaxation)

$M_{xy} \rightarrow 0$  (signal loss,  $T_2$  &  $T_2^*$  relaxation)

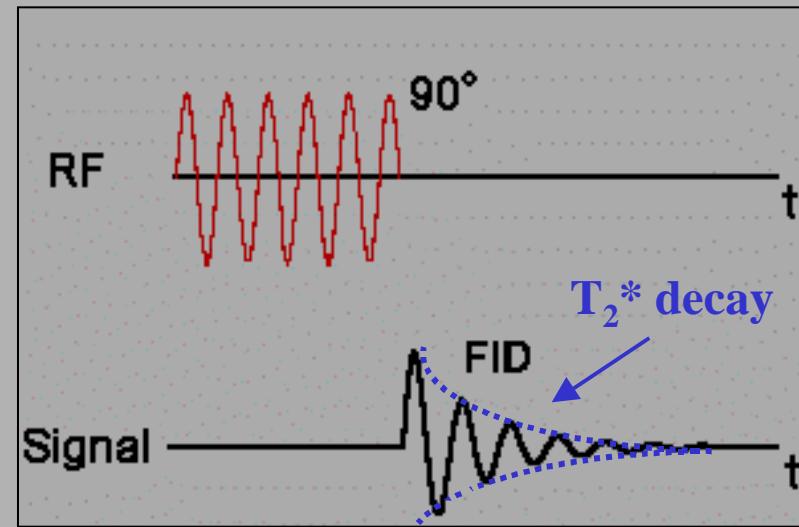
- Relaxation mechanisms give tissue contrast

## The NMR signal (continuation)

**Radio-frequency pulse**  
(oscillating  $B_1(t)$  rotating at  $\omega_0$ )

**NMR signal**  
(transverse magnetization decay)  
*'Free Induction Decay'*

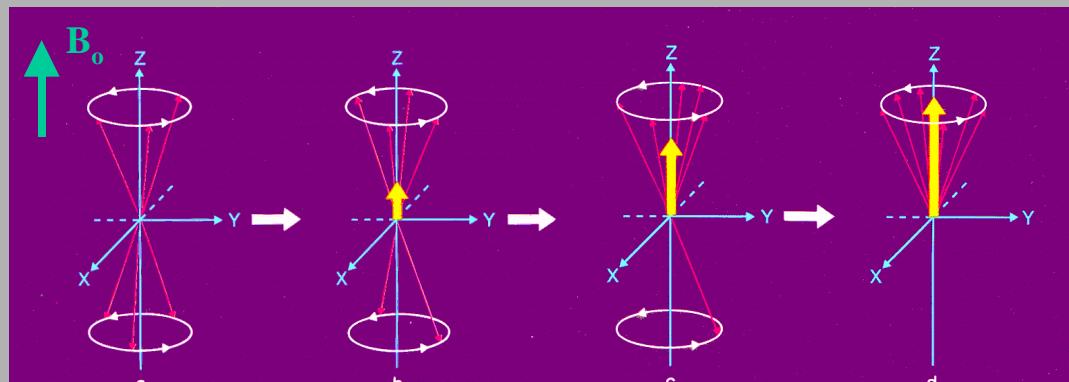
**Schematic representation:**



## Relaxation mechanisms

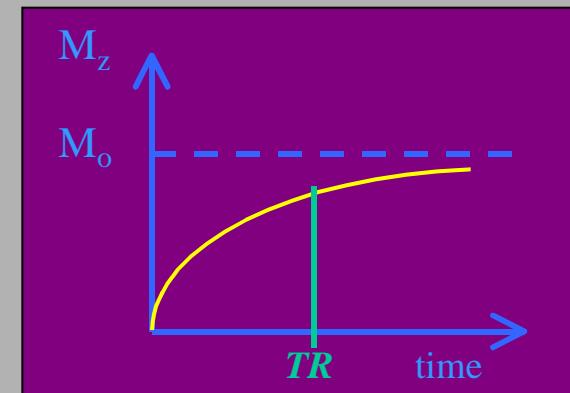
### $T_1$ or spin-lattice relaxation (longitudinal magnetization)

- $M_z$  defined by spin excess population between two energy states
- $M_z$  recovery    transitions between spin states
- transitions    fluctuating transverse field on resonance    molecular motion
- exponential recovery ( $T_1 = 100 - 3000$  ms, longer for higher  $B_0$ )



$M_z=0$   
after 90°  
RF pulse

$M_z=M_o$   
at equilibrium



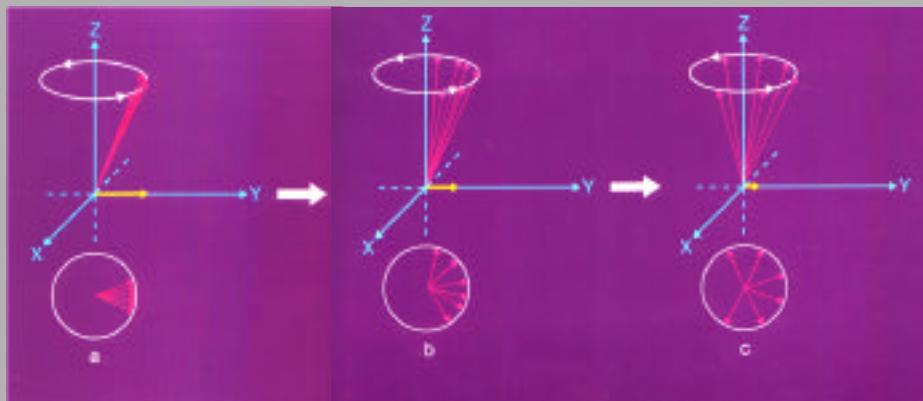
$$\frac{dM_z}{dt} = \frac{M_o - M_z}{T_1}$$

**Repetition time (TR):** time allowed for recovery, defines  $T_1$  contrast

## Relaxation mechanisms (continued)

### $T_2$ or spin-spin relaxation (transverse magnetization)

- incoherent exchange of energy between spins
- molecular motion fluctuations in local  $B_z$  resonance frequency variations
- dephasing of transverse magnetization signal decay
- exponential decay ( $T_2 \sim 70 - 1000$  ms)

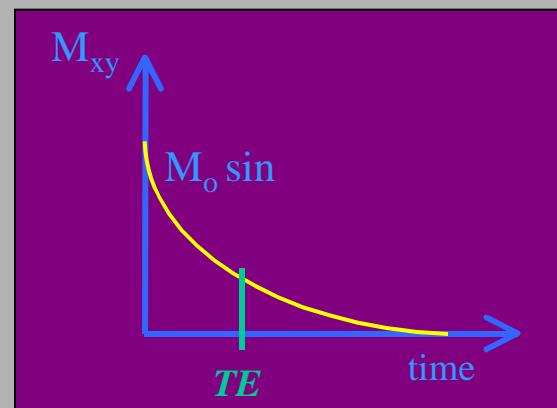


$M_{xy}$  max  
after 90°  
RF pulse

→ spins dephase →

$M_{xy} = 0$   
at equilibrium

$M_{xy} \leftrightarrow$  NMR signal



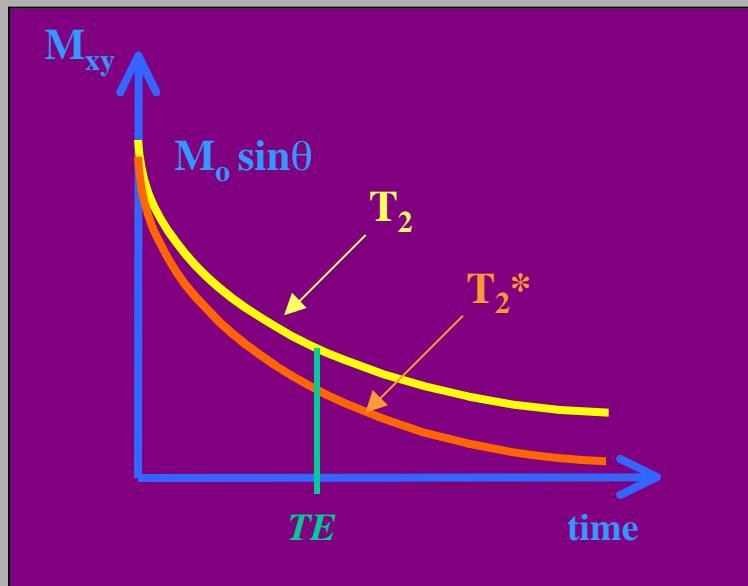
$$\frac{dM_{xy}}{dt} = -\frac{M_{xy}}{T_2}$$

**Echo time (TE): time allowed for dephasing, defines  $T_2$  contrast**

## Relaxation mechanisms (continued)

### $T_2^*$ relaxation

- dephasing of transverse magnetization due to both:
  - microscopic molecular interactions ( $T_2$ )
  - spatial variations of the external main field  $B$   
(tissue/air, tissue/bone interfaces)
- exponential decay ( $T_2^* \approx 30 - 100$  ms, shorter for higher  $B_o$ )

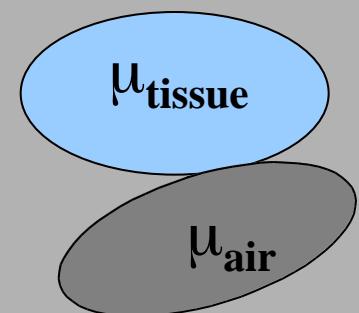


$$B = \mu \mu_0 H$$

Boundary conditions

} Field distortions  
( $B$ )

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \gamma \frac{\mu_{\text{tissue}} - \mu_{\text{air}}}{2}$$



## Bloch Equations

- Dynamics of the magnetization + Relaxation effects:

$$\frac{d\vec{M}_{(t)}}{dt} = \vec{M}_{(t)} \times \gamma \vec{B}_{ext(t)} - \frac{(M_x \hat{i} + M_y \hat{j})}{T_2^*} - \frac{(M_z - M_0)}{T_1} \hat{k}$$

- Geometrical description: damped precession (SUMMARY)

