Physics of MR Image Acquisition

HST-583, Fall 2004

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- **Review: NMR signal**
- **Fourier Transform Concepts**
- **Spatial Encoding of NMR signal**

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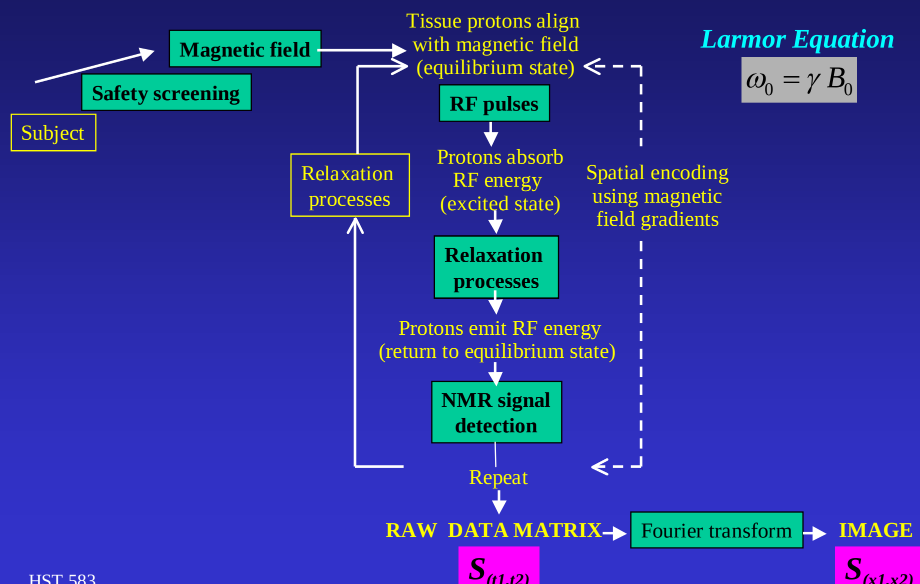
Overview of an MRI procedure



Larmor Equation

$$\omega_0 = \gamma B_0$$

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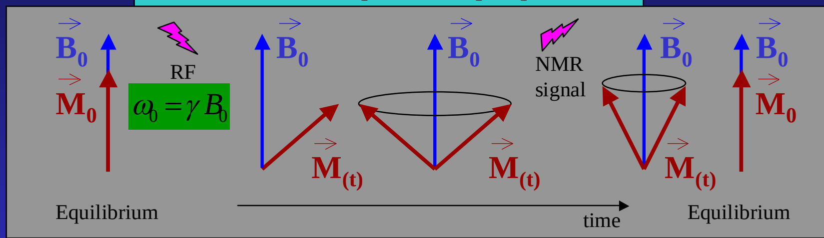
Dynamics of the Magnetization

• Geometrical description: damped precession

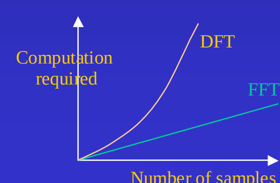


• Mathematical Description: precession + relaxation (Bloch equations)

$$\frac{d\vec{M}_{(t)}}{dt} = \vec{M}_{(t)} \times \gamma \vec{B}_{ext(t)} - \frac{(M_x\hat{i} + M_y\hat{j})}{T_2^*} - \frac{(M_z - M_0)}{T_1}\hat{k}$$

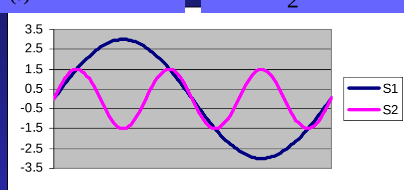
Fourier Transform concepts

- Any signal can be fully described in two different perspectives:
  - The time or spatial domain (our lab measurements)
  - The frequency domain (or frequency spectrum)
- We can go between these two domains with the Fourier Transform (FT)
- Continuous FT: $S_{(\omega)} = \int_{-\infty}^{+\infty} s_{(t)} e^{-i\omega t} dt$ $\quad s_{(t)} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{(w)} e^{i\omega t} d\omega$
- Real measurements are discrete → discrete FT (DFT) (many complex multiplications and additions)
- Fast FT (FFT): simplified discrete FT

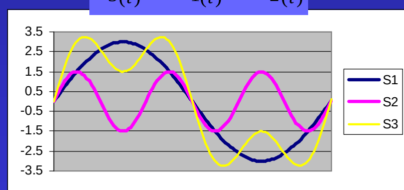


Fourier Transform concepts

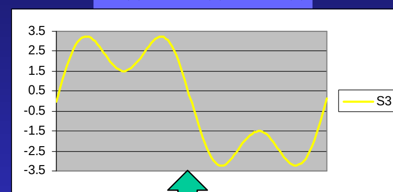
$s_{1(t)} = A\sin(w_0 t)$ $\quad s_{2(t)} = \frac{A}{2}\sin(3w_0 t)$

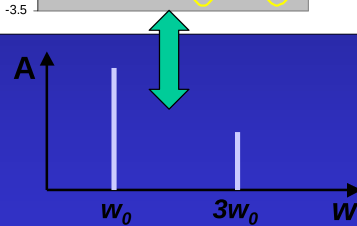


$s_{3(t)} = s_{1(t)} + s_{2(t)}$



$s_{3(t)} = s_{1(t)} + s_{2(t)}$



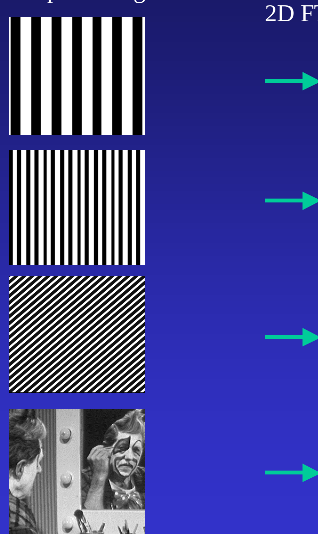


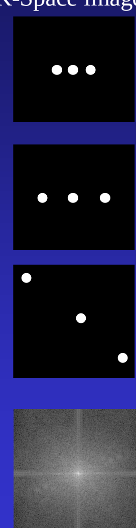
Fourier Transform concepts: Spatial frequency

Real space images

2D FT

K-Space images





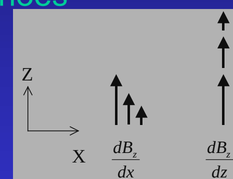
Modified from: http://homepages.inf.ed.ac.uk/rbf/HIPR2/fourier.htm

Spatial Encoding in MRI

Three magnetic fields (generated by 3 coils)

1) Static magnetic field Bo
2) RF field that excites the spins B1
3) Linear Gradient fields for spatial encoding and generating 'echoes'

$$G_x = \frac{dB_z}{dx}, G_y = \frac{dB_z}{dy}, G_z = \frac{dB_z}{dz}$$



While a linear gradient $\vec{G}$ is on, the resonance frequency of spins becomes a linear function of spatial location:

$$\omega_{(\vec{r})} = \gamma \left(B_0 + \vec{G}.\vec{r} \right)$$

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Spatial Encoding in MRI

Key concept: $$\omega_{(\vec{r})} = \gamma \left(B_0 + \vec{G}.\vec{r} \right)$$

- **Slice Selection**
  - Location
  - Thickness
  - Rephasing/Refocussing
- **Frequency Encoding**
  - Fourier Transform
  - FOV
  - Gradient Echo Formation
- **Phase Encoding**
  - Phase / Frequency Equivalency
  - FOV

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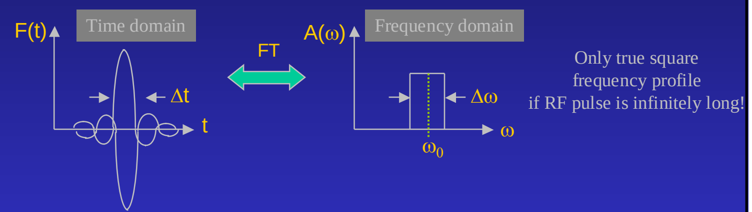
Slice Selection

- How can I selectively excite magnetization from a 2D slice?
- Apply a short RF pulse (oscillator frequency ω_0 modulated)

  This defines a profile of excitation frequencies



- Simultaneously, apply a gradient orthogonal to the desired slice, so that the slice center and slice thickness match the excitation profile from the RF pulse.

Slice center $\omega_0 = \gamma\,(B_0 + G_z\, z_0)$

Slice thickness $\Delta\omega = \gamma\,(B_0 + G_z\, \Delta z)$

Only spins within this slice will be excited because they have the right resonance frequencies defined by the RF pulse. Other spins don't absorb energy.

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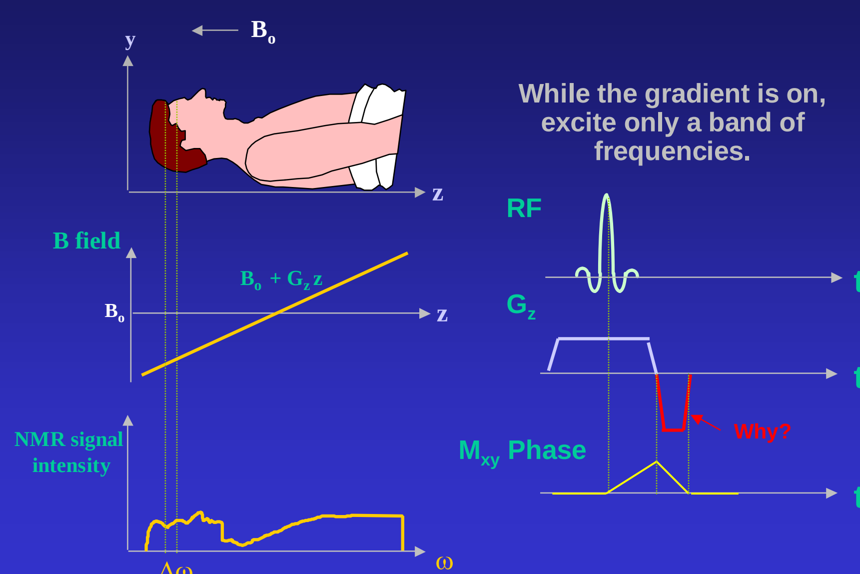
Slice Selection: Schematic Representation



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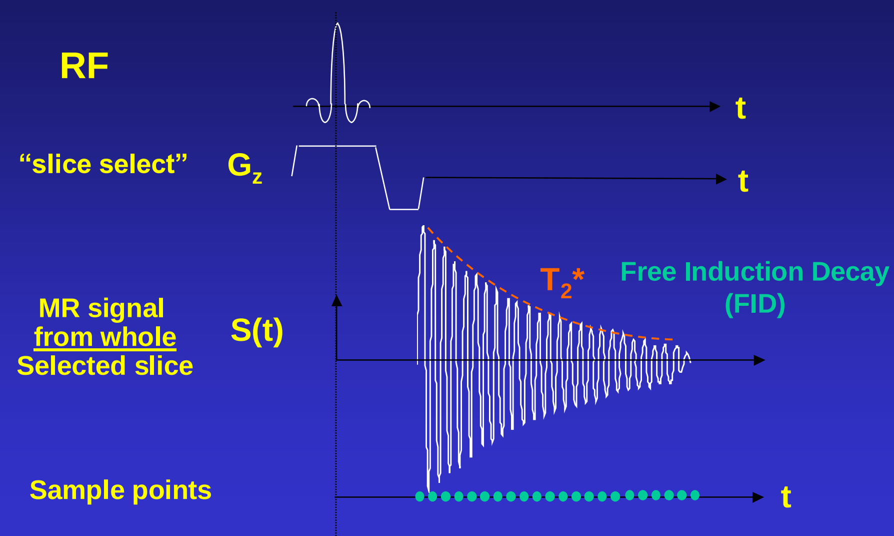
Courtesy of L. Wald

‘Pulse sequence’ so far



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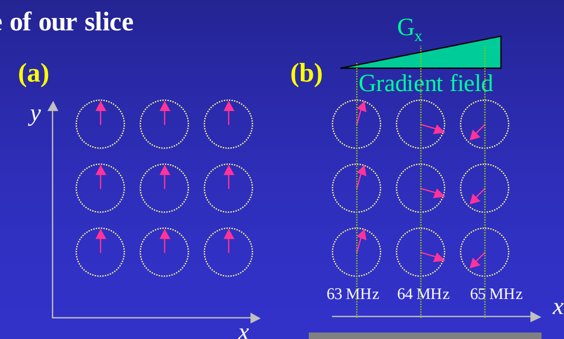
Courtesy of L. Wald

Frequency Encoding

- After slice selection all spins in the selected plane have the same frequency and phase (a)
- If a magnetic field gradient G_x is turned on during the signal acquisition (b), then the measured frequencies related to locations
- The Fourier transform allows us to map from intensity of various frequencies to intensities at various locations. We have a the intensity x-profile of our slice



$$w_{(x)} = \gamma\,(B_0 + G_x\,x)$$

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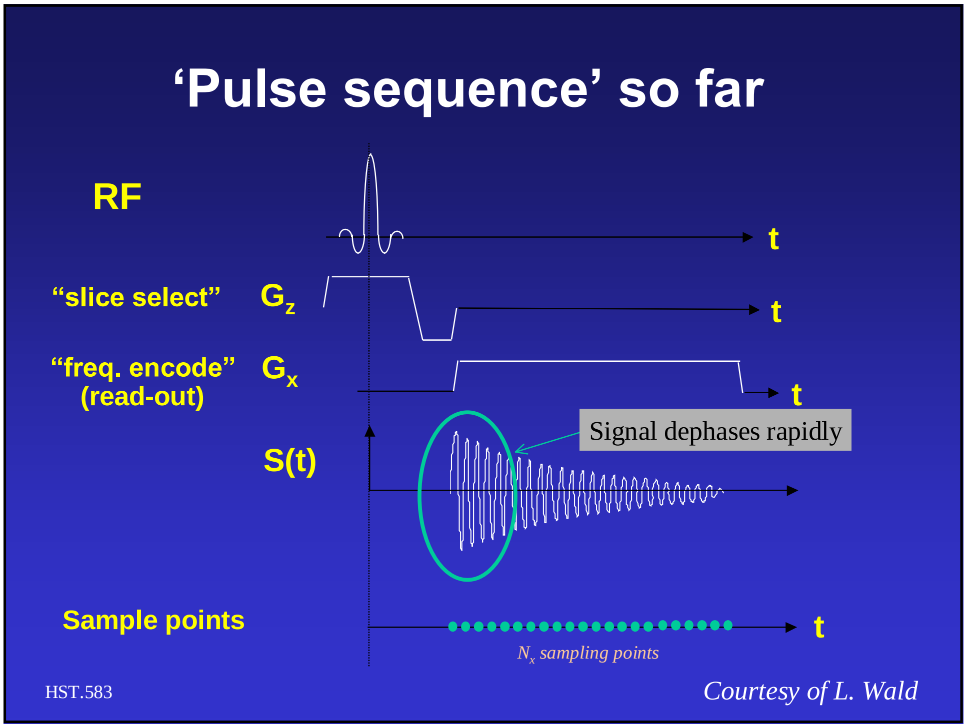
'Pulse sequence' so far
RF
t
"slice select"
Gz
t
"freq. encode" (read-out)
Gx
t
S(t)
Signal dephases rapidly
Sample points
t
Nx sampling points
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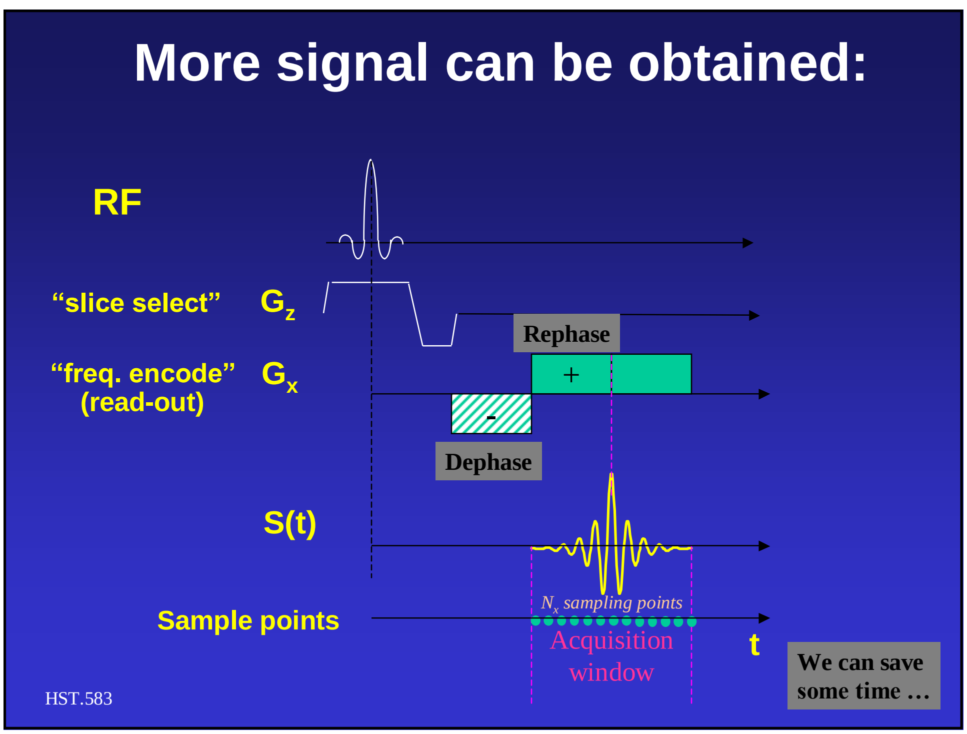
Courtesy of L. Wald


More signal can be obtained:
RF
"slice select"
Gz
Rephase
+
"freq. encode" (read-out)
Gx
-
Dephase
S(t)
Nx sampling points
Sample points
Acquisition window
t
We can save some time …
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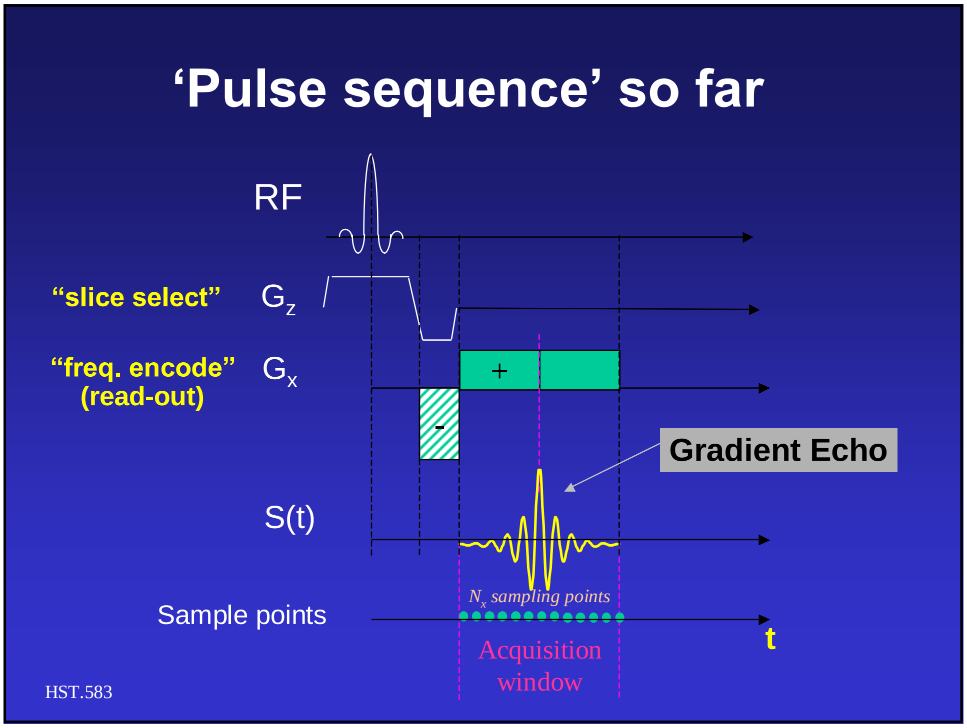
'Pulse sequence' so far
RF
"slice select"
G_z
"freq. encode"
(read-out)
G_x
+
-
Gradient Echo
S(t)
N_x sampling points
Sample points
t
Acquisition
window
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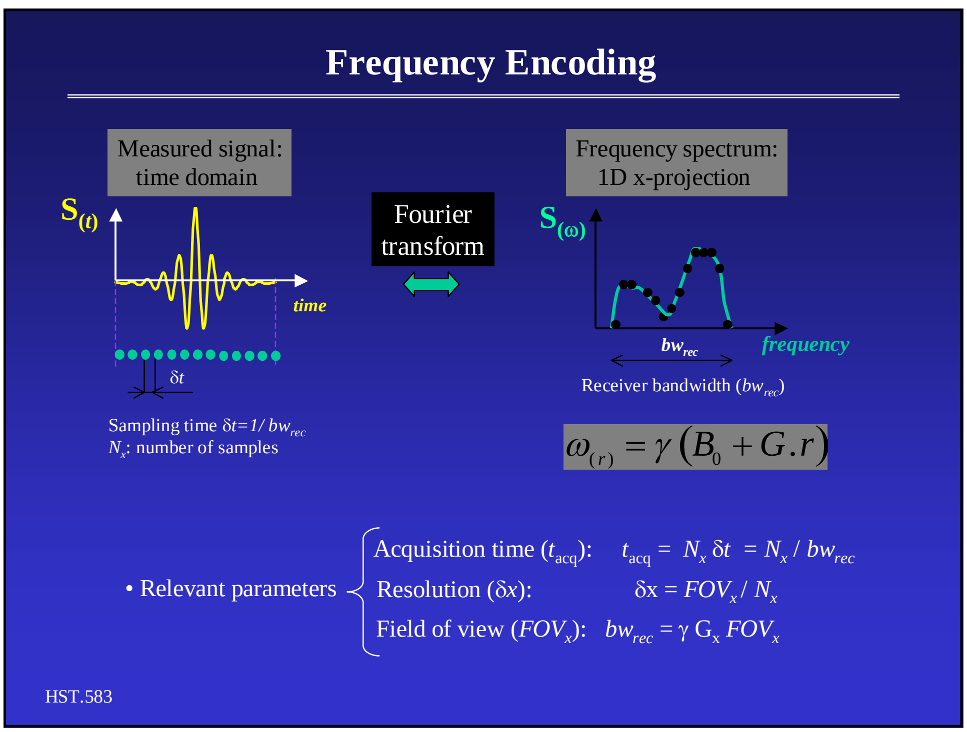
Frequency Encoding
Measured signal:
time domain
$S_{(t)}$
time
δt
Sampling time $\delta t=1/bw_{rec}$
N_x: number of samples
Fourier
transform
Frequency spectrum:
1D x-projection
$S_{(\omega)}$
bw_{rec}
frequency
Receiver bandwidth (bw_{rec})
$\omega_{(r)} = \gamma\left(B_0 + G.r\right)$
• Relevant parameters
Acquisition time (t_{acq}): $t_{acq} = N_x\,\delta t = N_x / bw_{rec}$
Resolution (δx): $\delta x = FOV_x / N_x$
Field of view (FOV_x): $bw_{rec} = \gamma\, G_x\, FOV_x$
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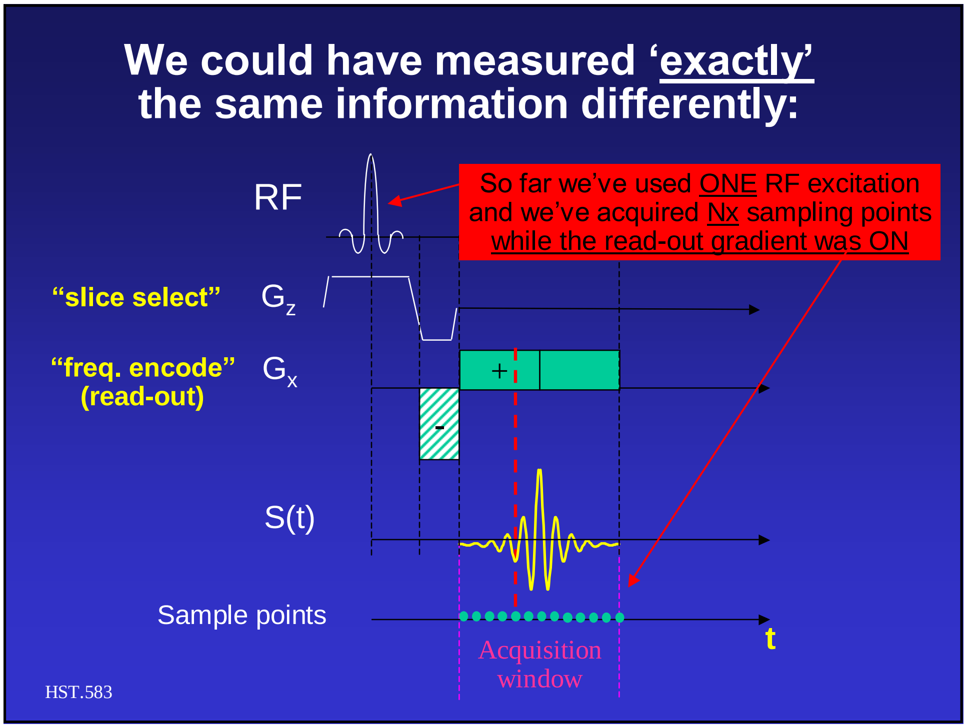
We could have measured 'exactly' the same information differently:
RF
So far we've used ONE RF excitation and we've acquired Nx sampling points while the read-out gradient was ON
"slice select"
G_z
"freq. encode" (read-out)
G_x
+
-
S(t)
Sample points
Acquisition window
t
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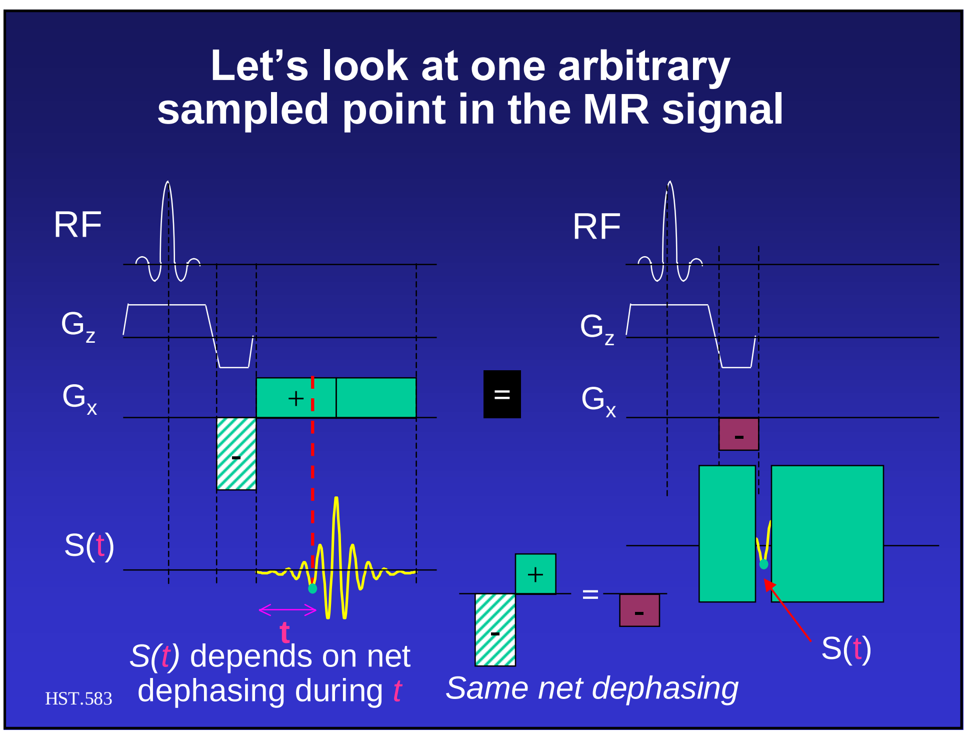
Let's look at one arbitrary sampled point in the MR signal
RF
G_z
G_x
+
-
S(t)
t
S(t) depends on net dephasing during t
=
+
-
=
-
Same net dephasing
RF
G_z
G_x
-
S(t)
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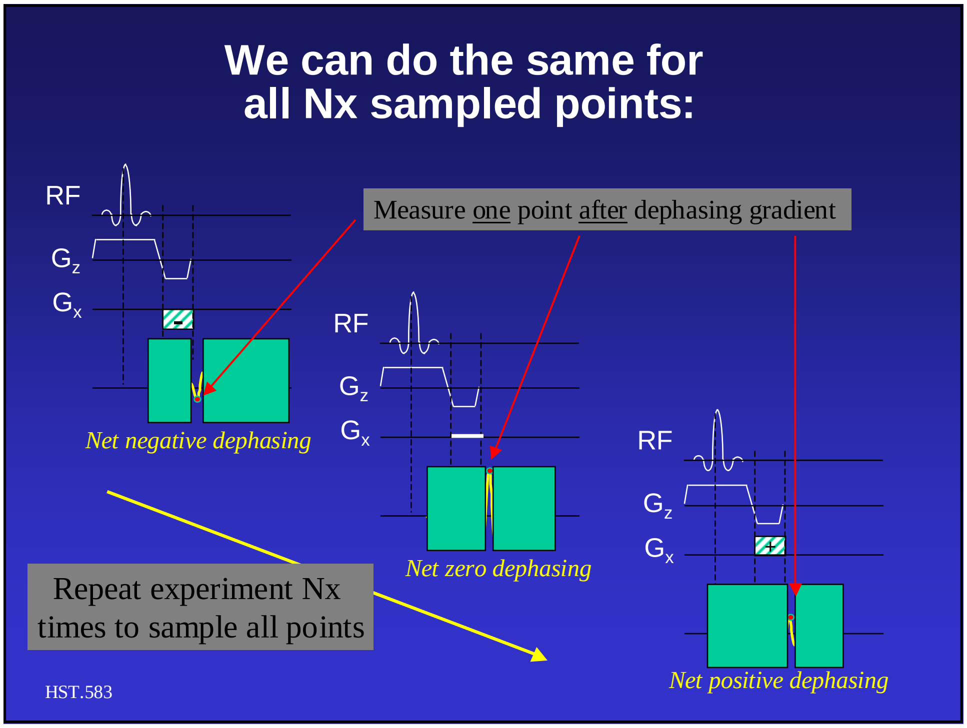
We can do the same for all Nx sampled points:
Measure one point after dephasing gradient
RF
G_z
G_x
Net negative dephasing
Net zero dephasing
Net positive dephasing
Repeat experiment Nx times to sample all points
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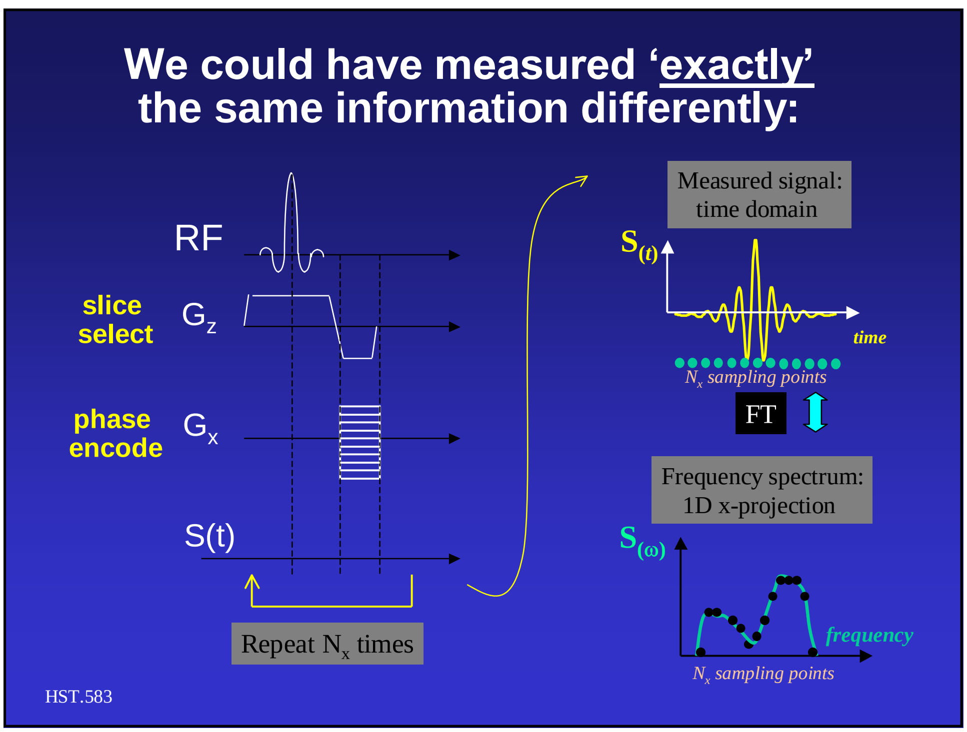
We could have measured 'exactly' the same information differently:
RF
slice select
G_z
phase encode
G_x
S(t)
Repeat N_x times
Measured signal: time domain
$S_{(t)}$
time
N_x sampling points
FT
Frequency spectrum: 1D x-projection
$S_{(\omega)}$
frequency
N_x sampling points
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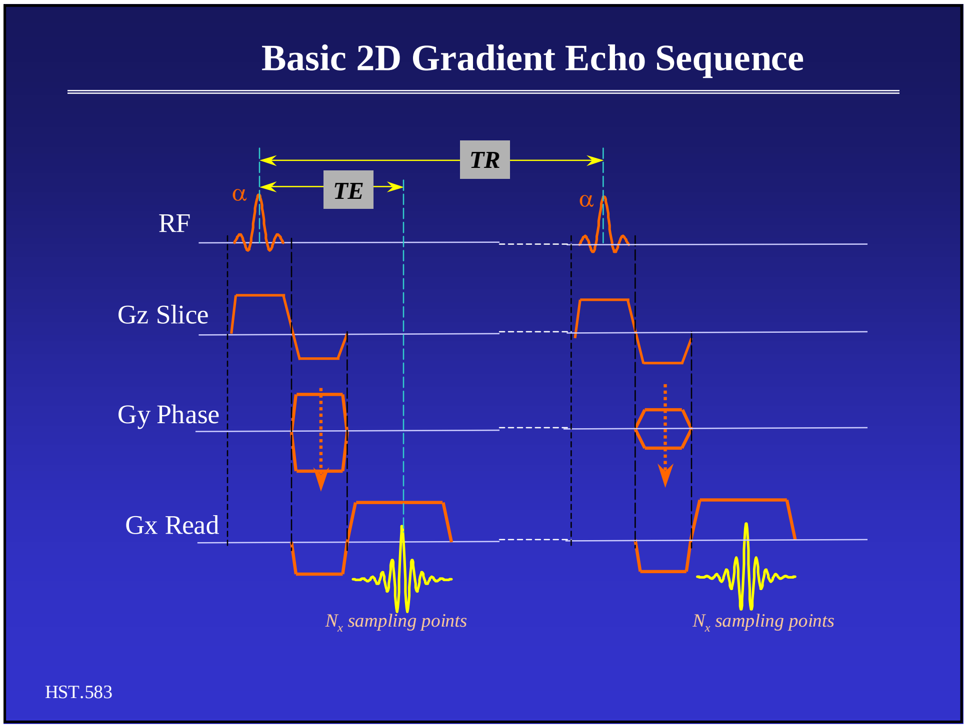
Basic 2D Gradient Echo Sequence
TR
TE
α
α
RF
Gz Slice
Gy Phase
Gx Read
N_x sampling points
N_x sampling points
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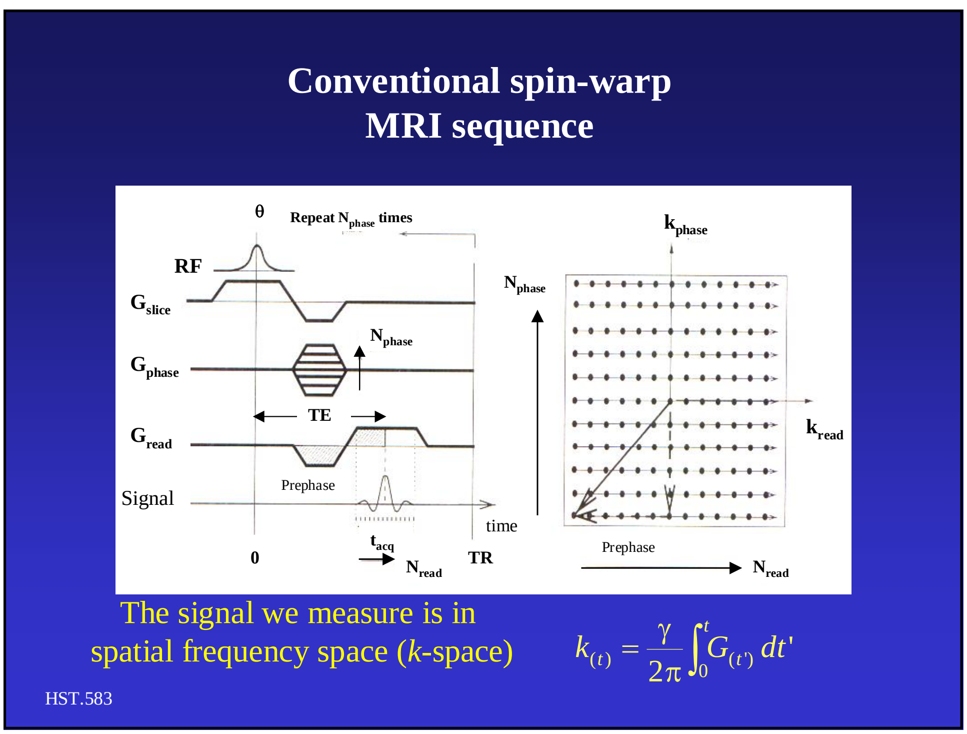
Conventional spin-warp
MRI sequence
θ
Repeat N_{phase} times
RF
G_{slice}
G_{phase}
N_{phase}
TE
G_{read}
Prephase
Signal
time
0
t_{acq}
N_{read}
TR
k_{phase}
N_{phase}
k_{read}
Prephase
N_{read}
The signal we measure is in
spatial frequency space (k-space)
$k_{(t)} = \frac{\gamma}{2\pi}\int_0^t G_{(t')}\,dt'$
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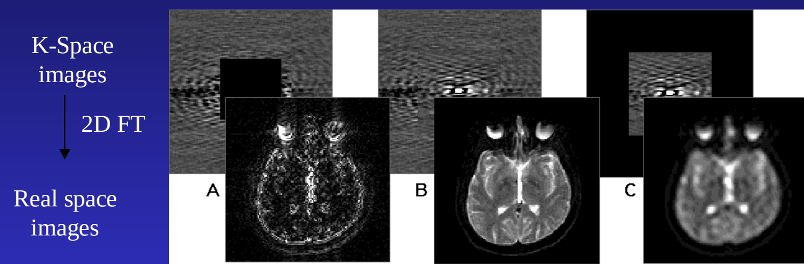
Understanding *k* - space

High frequencies only Fully sampled Low frequencies only



K-Space: spatial frequency information of the image
- Center: low frequencies ⇒ global features, image intensity (C)
- Periphery: high frequencies ⇒ sharp features, edges (A)

Images from: http://thelonius.loni.ucla.edu/AMR/EPITheory.html

HST.583 ***Review: spatial encoding***

"Spin-warp" encoding mathematics

The signal phase at time *t* depends on the accumulated phase from the read and phase encoding gradients at that time

$$w_{(t)} = \frac{d\theta}{dt} = \gamma \overrightarrow{G_{(t)}} \cdot \vec{r}$$

$$\theta_{(t)} = \gamma \int_0^t \overrightarrow{G_{(\tau)}} \cdot \vec{r}\, d\tau$$

Phase due to readout:

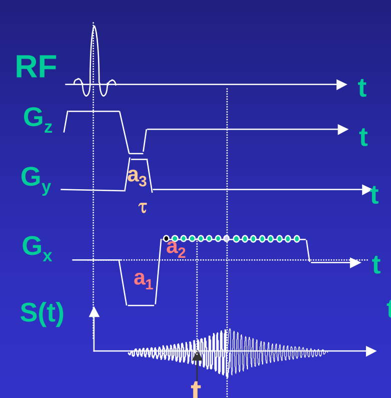
$\Delta\theta_{read}(t) = \gamma\, G_x\, x\, t = \gamma\, x\, (a_2 - a_1)$

Phase due to P.E.:

$\Delta\theta_{phase}(t) = \gamma\, G_y\, y\, \tau = \gamma\, y\, a_3$

Total phase:

$\Delta\theta(t) = \gamma\, G_x\, x\, t + \gamma\, G_y\, y\, \tau$



a_1, a_2, a_3: Areas under gradient waveforms

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"Spin-warp" encoding mathematics

Signal at time *t* from location (x,y):

$$S(t) = \rho(x,y)e^{i\gamma G_x xt + i\gamma G_y y\tau}$$

The coil integrates over object:

$$S(t) = \iint_{object} \rho(x,y)e^{i\gamma G_x xt + i\gamma G_y y\tau} dxdy$$

Substituting $k_x = -\gamma G_x t$ and $k_x = -\gamma G_x t$:

$$S(k_x,k_y) = \iint_{object} \rho(x,y)e^{-ik_x x - ik_y y} dxdy$$

HST.583 *Courtesy of L. Wald*

"Spin-warp" encoding mathematics

View signal as a matrix in k_x, k_y:

$$S(k_x,k_y) = \iint_{object} \rho(x,y)e^{-ik_x x - ik_y y} dxdy$$

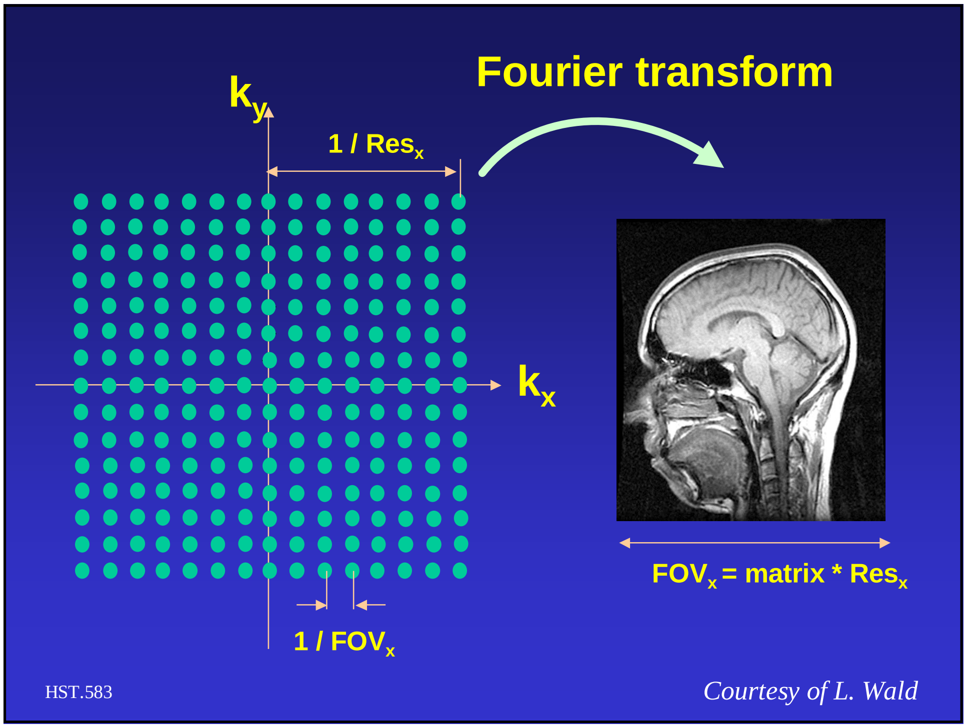
Solve for ρ(x,y,):

$$\rho(x,y) = FT^{-1}\left[S(k_x,k_y)\right]$$

$$\rho(x,y) = \iint_{kspace} S(k_x,k_y)e^{ik_x x + ik_y y} dk_x dk_y$$

The "image" is the spin density function: ρ(x)

HST.583 *Courtesy of L. Wald*


Fourier transform
ky
1 / Resx
kx
FOVx = matrix * Resx
1 / FOVx
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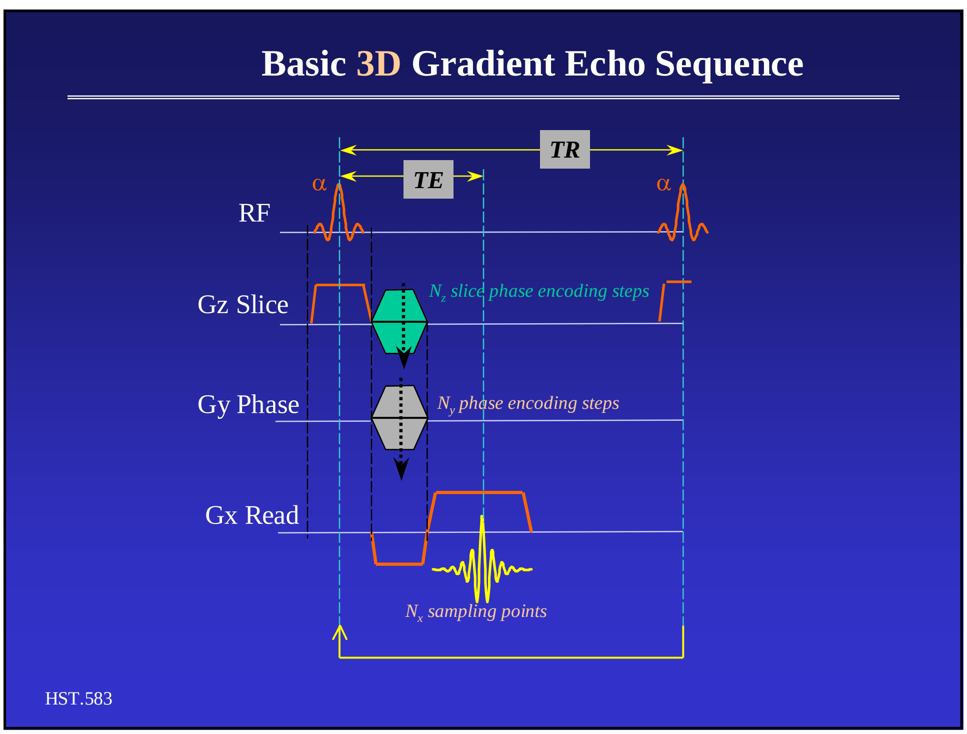
Courtesy of L. Wald


Basic 3D Gradient Echo Sequence
TR
TE
α
α
RF
Gz Slice
Nz slice phase encoding steps
Gy Phase
Ny phase encoding steps
Gx Read
Nx sampling points
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