Path-Dependence and the Dynamics of Organizational Mortality: Age-Dependence Revisited

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Abstract

This paper proposes a novel theoretical framework to model the dynamics of organizational mortality. The main theoretical contribution is a clarification of the relations between organizational fitness, endowment, organizational capital and mortality hazard. If the mortality hazard is a function of the stock of organizational capital, and the rate of accumulation of organizational capital is a function of the fitness level, organizational fitness becomes the key factor in predicting the evolution of the mortality hazard. The paper demonstrates how this new perspective can cast a new light on the much studied relation between organizational age and organizational hazard of mortality. It also introduces a novel approach to modeling organizational obsolescence as a consequence of drifting tastes and increasing organizational inertia.

1 Introduction

The dynamics of orgranizational mortality have attracted considerable theoretical and empirical attention in the organizational ecology literature. The dominant perspective has proposed that organizational mortality depends on both population-level characteristics such has the level of competition in the organizational population or the level of legitimation of the organizational form and on organizational-level characteristics such as organizational age and size. While this perspective has emphasized history-dependence at the population level of analysis, by analyzing the imprinting effects of conditions prevailing at time of founding and by considering the effect of the evolution of competition and legitimation of organizational populations, it has granted only moderate attention to path-dependence at the organizational level of analysis.

In this paper, we propose a novel theoretical framework for the analysis of the dynamics of the mortality hazard of organization. The key element in the proposed theory is the emphasis on the distinction between stocks and flows of resources. We propose that the life chances of organizations depend on their stock of organizational capital and focus on the evolution of this stock of resources. When an organization performs poorly (for example, when the routines necessary for reliable operations have not been set up), the inflow of crucial resources slows. Organizational capital tends to increase when the net flow of resources is positive, and it decreases when the net flow of resources is negative. In this framework, the dynamics of organizational mortaliy can therefore be explained by focusing on whether the net flow of resources is (and has been) positive or negative.

To illustrate the benefits of the approach developed here, we focus our attention on the much studied problem of the relation between organizational age and the hazard of organizational mortality. In the next section, we summarize some of the existing theories about age-dependence, and we identify what we believe to be the reason for the apparent conflicting predictions. We then introduce organizational capital and our approach to analyze the evolution of organizational capital. In section 4, we propose a new approach to the analysis of age-dependence that unifies previous theories. Finally, in the last section, we further develop our model to include considerations about changing organizational environments and organizational obsolescence.

2 Path-Dependence and the relation between organizational age and mortality.

2.1 Two perspectives on age-dependence

To illustrate the challenge in adopting a proper perspective on path-dependence for the study of the dynamics of organizational mortality, it is useful to re-consider a theme that has attracted considerable theoretical and empirical attention: The relation between organizational age and the hazard of organizational mortality.

Two main line of argumentation have been proposed. These make make conflicting predictions: The first perspective contends that the mortality hazard increases with organizational age (thus implying positive age-dependence) whereas the second perspective makes the opposite prediction: the mortality hazard is expected to decrease with organizational age (thus implying negative age-dependence, or lyability of newness). Empirical analyses, far from resolving this conflict, have been numerous and often contradictory. Some empirical analyses have found that liability of newness holds in a number of populations (Carroll 1983; Freeman, Carroll and Hannan 1983). Barron, West, and Hannan (1994) has found, on the contrary, that older organizations were more likely to fail (once age-varying organizational size is taken into account). Finally, others have found a liability of adolescence (Brüderl and Schüssler 1990; Fichman and Levinthal 1990).

While at least two attempts at unifying conflicting theoretical predictions and empirical findings (Hannan 1998; Hannan, Pólos, and Carroll 2007, hereafter HPC) have been proposed[1], we contend here that these previous unification efforts have overlooked an important distinction between the two existing lines of arguments. To understand what the issue is, we first provide some further details about the two main existing theories (depicted in a similar format in HPC).

[1]We postpone discussions of those until later in the paper.

2.2 Existing theories

A general line of argumentation, initially proposed by Stinchcombe (1965), implies negative age-dependence. Stinchcombe's argument hinges on the idea of capabilities and position, as explained in Hannan (1998). Here, for simplicity, we concentrate on capabilities. Stinchcombe proposed that younger organizations lack some crucial capabilities, which make them more vulnerable. More precisely, 1) they lack the technical and social skills needed for smooth functioning, 2) they must invent roles, relationships between roles, and rewards and sanctions, 3) they are subject to uncertainty pertaining to social relations with strangers, and 4) they normally lack strong social ties with external constituencies which makes it harder for them to mobilize resources and ward off attacks.

But as organizations age, they improve their capabilities, especially when the environment is stable. This is formalized in Postulate 7.1 in HPC: "An organization's expected level of capability normally rises with its age." It is also assumed that "Higher capability normally lowers the mortality hazard" (Postulate 7.2 in HPC). This reasoning leads to a negative age dependence, formalized in the following theorem: (HPC, Theorem 7.1) "Mortality hazards presumably decline monotonically with age."

The second line of argumentation predicts positive age-dependence. It assumes that, at founding, organizations would start off with an initial endowment that lasts for some time and buffers them from failure (Stinchcombe, 1965; Carroll and Hannan, 2000). Scholars have defined endowment as the initial resources such as "seed capital, credit and commitment from others, and political support" (HPC: 154) that provide some immunity to a firm after founding. Furthermore, it has been argued that there is a monotonic relationship between the level of endowment and the strength of immunity (HPC: 154). Organizations with large endowments are thus expected to have a low mortality hazard.

During this initial "endowment period", the mortality hazard increases with age, because the stock of endowment decreases as time passes. This is formalized in Postulate 7.3 in HPC that states "Expected levels of endowments normally decline monotonically with age within endowment period." This postulate (together with Postulates 7.4 and 7.5 in HPC) leads to a theorem that formalizes the relation between mortality hazard and age within periods of endowment: "Mortality hazards presumably rise with age within periods of endowments." In conclusion, the argument based on the progressive depletion of the endowment predicts a positive age-dependence within periods of endowments.

The two arguments are summarized, in a schematic way, on Figure 1.

2.3 Issues with Existing Theories

The two arguments, as summarized above, seem hardly related. The first argument uses capabilities at its core; and the second argument uses initial endowment as a central construct. We believe that the main reason why these two arguments seem unrelated, or almost contradictory, is that the two arguments treat path-dependence in substantially different ways. We see the endowment as a stock of resources whereas we see capabilities as refering to a flow of resources. Pointing out this

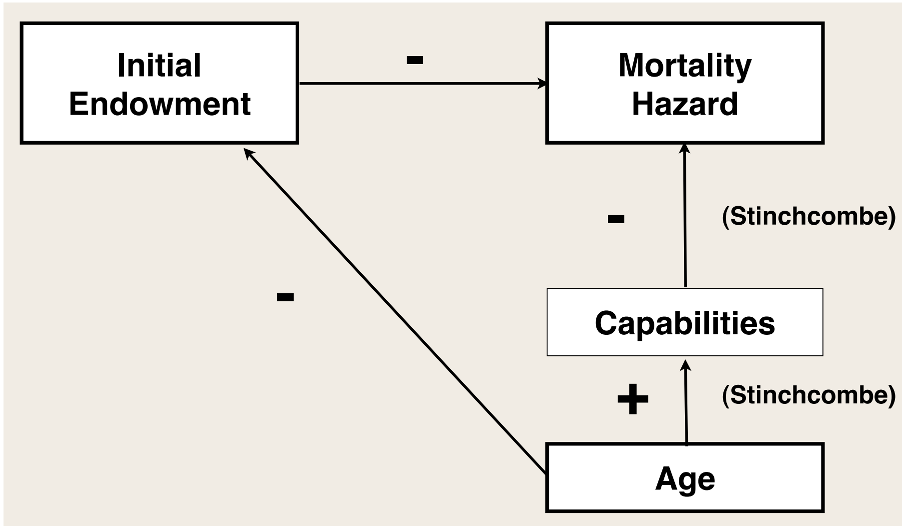

Figure 1: The key constructs used in the two perspectives on age dependence discussed in the text. There are two causal paths with conflicting predictions about the relation between age and mortality hazard.

distinction underlies a fundamental difference between the two arguments described above: The first argument, about increasing capabilities makes the implicit assumption that the past evolution of capabilities does not matter for the risk of mortality at time t. All that matters is the current level of capabilities. But if the mortality hazard is a function of a stock of resources, as suggested by the second argument about the depletion of endowment, then the past evolution of the flow of resources becomes a crucial factor: An organization with an history of low capabilities will tend to have a low stock of resources, even if its current level of capabilities is high. And an organization with an history of high capabilities will tend to have a high stock of resources, even if its current level of capabilities is low.

We believe that the risk of mortality closely depends on a stock of resources, rather than on the current flow of resources. The theoretical challenge therefore consists in integrating ideas about the evolution of capabilities with the view that the mortality hazard is primarily determined by the stock of resources.

In the following two sections, we develop undertake this endeavour and develop a novel theoretical framework that clarifies the relations between endowment and capabilities and provides a new perspective on the dynamics of the mortality hazard.

3 Theory Stage 1: Organizational Capital, Fitness Level and Mortality

Analyses of age dependence have generally considered endowment as a characteristic specific to new organizations. The most recent of such analyses (HPC, Chap. 7), for example, postulates that the expected size of endowment tends to decline with age and becomes totally depleted after some (finite) time. This assumption can be justified by the conceptual linkage of endowment to the idea of *initial* immunity. However, it does not explicitly allow for the possibility that, as an organization builds up capabilities and improves its social position, it can somehow replenish its endowment before depletion. We allow for this possibility in this new formulation.

What if we do away with the assumption that the endowment becomes completely depleted after some time and, instead, allow it to be replenished when the organization performs well? We propose the introduction of the concept of organizational capital as an extension of the concept of endowment to the entirety of the lifetime of the organization. Organizational capital is thus a measure of the stock of resources of the organization (as suggested by Levinthal [1991]). A firm possesses a high stock of organizational capital to the extent it has a large size, extensive financial assets, ample cash reserves, a good reputation, ties with powerful social actors. Consistent with the finding that firm's failure in a given year rarely stems only from poor performance or fitness in that year or the year before, but often results from sustained poor performance (Hambrick and d'Aveni 1988), we propose that ample organizational capital buffers organizations from the deleterious effect of a temporary period of poor performance.

A producer's organizational capital is, at time of founding, its initial endowment. But while initial endowment is depleted over time, organizational capital can be replenished if the organization can capture enough resources from its environment. We readily acknowledge that the nature and composition of organizational capital will generally differ from the nature and composition of the remaining endowment, but we do not see this as a concern, because we are interested in these constructs primarily to the extent that they matter to organizational immunity. Therefore, while the stock of resources associated to the initial commitment from first round investors and from entrepreneurs will generally decrease over time, and will often become completely depleted after some time, other newly acquired resources, such as those listed in the previous paragraph, might provide the producer with some immunity. And whether the current stock or organizational capital is made of resources that have been acquired at time of founding or subsequently does not matter for our theory. What matters for empirical applications of the theory, on the other hand, is that organizational capital be properly measured. A comprehensive discussion of how to properly measure the resources that make up the stock of organizational capital and their impact on immunity is far beyond the scope of this paper[2]. But let us simply note that resources acquired in the past often tend to age, and should thus be correspondingly discounted. In fact, a key feature of the stock of resources is the extent to which it is liquid and can therefore be mobilized in order to avoid organizational failure[3].

Some readers will no doubt wonder about the relation between organizational capital and organizational size. In our perspective, those two constructs are very related. The robust finding that large

[2]One could even argue that the ability to make this measurement is the core competency of credit-rating agencies. But the recent events associated to the financial crisis show that proper measurement of organizational immunity is quite a complicated task.

[3]Note also that the value of cash reserves, the most liquid for of resources, decreases with time only to the extent that there is inflation.

organizations are often less prone to failures than small organizations is consistent with this view (Baum and Mezias 1992, Ranger-Moore 1997, Banaszak-Holl 1991, Delacroix and Swaminathan 1991, and Dobrev 1998; see Carroll and Hannan, 2000 for a review).

In the following analysis of organizational aging, we use as a starting point the theoretical framework delineated by Hannan, Pólos, and Carroll (2007) in HPC. We consider a generic producer, x, that operates in an unspecified category and tries to capture resources controlled by members of the category audience. The relevant audience consists of actual and potential customers, actual and potential organizational members, and more generally, any individual, organization, or governmental agency that controls resources useful to the organization and takes an interest in the category. For simplicity, we will assume in all what follows that producer x only bears one label, the label for the focal category, and operates at a unique social position $z(x)$. Following HPC, we assume that the audience members at a social position have similar preferences.[4] We use these assumption to keep notations as simple as possible. In particular, we do not use the label and the social position as arguments of the functions we consider in the formal developments.

The organizational capital of producer x at time point t will be considered as a time-varying state variable and will be denoted by $\kappa(x,t)$. We now formulate the relation between organizational capital and mortality. To do so, we follow the spirit of HPC's Postulate 7.4 states that "During a period of endowment, a larger endowment normally yields a higher expected level of immunity." In terms of organizational capital, this postulate can be replaced by the following formulation.

Postulate 1. *[Organizational capital and the mortality hazard] A producer's mortality hazard, ω, normally decreases with its stock of organizational capital, κ.*

$$\mathfrak{N}\, x,\, \forall\, t_1, t_2\, [(\kappa(x,t_1) > \kappa(x,t_2) \rightarrow \omega(x,t_1) > \omega(x,t_2)) \\ \wedge\, (\kappa(x,t_1) = \kappa(x,t_2) \rightarrow \omega(x,t_1) = \omega(x,t_2))].$$

It is important to note that we are concerned with organizational failures. And therefore, the theory developed in this paper does not claim to account for other types of exits, such as voluntary acquisitions and mergers.

This relation between organizational capital and the mortality hazard will prove useful for the following theoretical developments. Rather than focusing on the various possible causes for organizational failure, we will concentrate on modeling the variations of organizational capital over time. Predictions about the evolution of the mortality hazard will be easily inferred from the predicted evolution of the organizational capital. The main theoretical questions that needs to be addressed at this point is therefore: How does the organizational capital vary with time?

Organizational capital is a stock of resources and, as such, *it summarizes the history of past performance* of the producer. More precisely, organizational capital increases when the net inflow of resources is positive and it decreases when the net inflow of resources is negative. Negative cash flows, the loss of key employees, or aging of technological equipment, contribute to the depletion

[4] A market segment is a prototypical example of social position.

of the stock of organizational capital. But profits, good hires, and successful branding help the build-up of the stock of organizational capital.

Looking at variations of organizational capital therefore amounts to looking at the net inflow of resources. Let $\delta(x,t)$ denote the net inflow of resources to producer x, at time t, from its environment. The relation between organizational capital and the net inflow of resources is formalized in the following meaning postulate. Let $a(x,t)$ denote the age of producer x at time t.

Definition 1. [Organizational capital and the net flow of resources] A producer's organizational capital is the integral, over time, of the net inflow of resources.

$$\kappa(x,t) = \int_{t_0}^{t} \delta(x,s)\, ds,$$

where t_0 is the time of founding of the organization, that is, $a(x,t_0) = 0$.

This relation implies that it is enough to know the history of the flow of resources to make predictions about the stock of organizational capital and, in turn, about the evolution of the mortality hazard. We thus now turn to the conceptualization of the flow of resources.

3.1 Resource Flows and Fitness Thresholds

How does an organization get resources? HPC provided an answer to this question by introducing the concept of organizational fitness as its ability to capture resources from relevant audience members. In this paper, we make the simplifying assumption that the focal producer targets one unique social position. While we recognize that offerings often simultaneously target several social positions, this assumption will help us keep the formalism as simple as possible. The following definition of the fitness of a producer is a simplified version of the definition of fitness level in HPC.

Definition 2. [Relative fitness] An organization's fitness, relative to the other members of the category, is its share of the total appeal (at the unique position it targets).

$$\phi(x,t) = \frac{Ap(x,t)}{Ap(x,t) + \overline{Ap}(x,t)},$$

where $Ap(x,t)$ denotes x's actual appeal, at time t, and $\overline{Ap}(x,t)$ denotes the total appeal of all of x's competitors. That is, $Ap(x,t) + \overline{Ap}(x,t) = \sum_{x' \in \mathbf{o}} Ap(x',t)$, where $\mathbf{o}$ is the set of producers in the category.

The offering of a producer with a high fitness level is relatively attractive to relevant audience members. Therefore, such a producer should have a high inflow of resources, provided that the demand for the offerings of the producers in the category is substantial. In the following, we denote this total demand for the offerings of the members of the (unspecified) category by $D(t)$[5]. This intuition is formulated in the following postulate.

[5] GIVE A GOOD EXAMPLE HERE OF DEMAND FOR A CATEGORICAL PRODUCT AND DEMAND FOR A SPECIFIC OFFERING

Postulate 2. *[Fitness level and net inflow of resources]. A producer's net inflow of resources is normally increasing with the product of its fitness level and the demand for the offering of producers in the category.*

$$\mathfrak{N}\, x, \forall\, t_1, t_2\, [((\phi(x,t_1)\cdot D(t_1) > \phi(x,t_2)\cdot D(t_2)) \rightarrow \mathrm{E}\,\{\delta(x,t_1)\} > \mathrm{E}\,\{\delta(x,t_2)\}) \\ \wedge ((\phi(x,t_1)\cdot D(t_1) = \phi(x,t_2)\cdot D(t_2)) \rightarrow \mathrm{E}\,\{\delta(x,t_1)\} = \mathrm{E}\,\{\delta(x,t_2)\})].$$

and, when this relation holds, the net resource flow is a continuous function of the fitness level[6].

While demand-side resources do not account for the full inflow of resources, we claim here that they constitute the most important part of the inflow. We recognize that resources can flow from other sources, such as successful law suits or settlements, or investments in the financial markets. But we see these other sources as less systematic, and more or less akin to rainfall gains. Therefore, we decided to avoid developing our theory around those. Thus, we predict that fitness generally has a positive effect on the inflow of resources to the producer.

On the other hand, no direct causal relation connects fitness, as defined above, and the outflow of resources. The systematic component of the outflow pertains to the operational costs. Less systematic components exist, such as exceptional charges for restructuration, legal costs, or bad investments in the financial markets, for example. But none of these relate directly to the appeal of the offering to audience members.

An important note on the approach adopted here is now in order. In this paper, we are interested in the effect of age on the mortality hazard of organizations, and not just on how age and mortality hazard covary. An important feature of age as a state variable is that it exactly covaries with the passage of time. From a theoretical standpoint, this implies that, in order to analyze the process of organizational aging, the investigator needs to abstract away from the influence of the passage of time that is not directly related to aging. Despite the fact that, at first sight, it seems impossible to separate the influence of these two variables, a solution exists thanks to a difference in terms of level of analyses. More precisely, the aging process affects a specific producer x, whereas the passage of time has a systematic influence on all the producers in the category. In other words, we define the effect of aging as the producer-specific effect of the passage of time. And in order to capture the effect of aging, the analyst needs to maintain constant the systematic (category-wise) effect of time.

In our model, the systematic component is captured by the variations of $D(t)$, the demand for the offering of producers in the focal category label. In the rest of the paper, we will therefore assume that this quantity is constant and will omit it from the formal specifications of the results. Variations of demand for the offering of producers in the category label affect the resource flows of all producers independently of their age, and therefore, it is necessary to "control for" variations of the category-level demand.

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[6] This part of the postulate is needed for technical reasons. It does not alter the substantive meaning of the postulate.

Auxiliary Assumption 1. *The demand for the offering of producers in the category does not change over time.*

$$\mathfrak{A}\, t\, \exists D\, [D(t) = D].$$

One of the goals of the paper is to make predictions as to when the mortality hazard increases with age or decreases with age. We will therefore focus on whether the net inflow of resources is positive of negative. The following proposition clarifies when this normally happens.

Proposition 1. *[Fitness threshold] Presumably, there exists a unique threshold $\mathfrak{f}_x$ such that producer x's expected net inflow of resources,* $\mathrm{E}\{\delta(x,t)\}$*, is positive if its fitness exceeds $\mathfrak{f}_x$, equal to 0 if its fitness equals $\mathfrak{f}_x$, and negative if its fitness falls below $\mathfrak{f}_x$.*

$$\begin{aligned}\mathfrak{P}\, x\, \exists!\, \mathfrak{f}_x \forall t \quad & [(\phi(x,t) > \mathfrak{f}_x) \rightarrow \mathrm{E}\{\delta(x,t)\} > 0] \\ \wedge \quad & [(\phi(x,t) = \mathfrak{f}_x) \rightarrow \mathrm{E}\{\delta(x,t)\} = 0] \\ \wedge \quad & [(\phi(x,t) < \mathfrak{f}_x) \rightarrow \mathrm{E}\{\delta(x,t)\} < 0].\end{aligned}$$

Proof. Establishing a proof in nonmonotonic logic requires finding all of the rule chains that link the antecedent and the consequent and determining specificity differences. If the most-specific applicable rule chain supports the claimed implication, then it is proven. This argument is entirely non-specific. At this stage of the theory, there is only one applicable rule chain. It is a direct consequence of Postulate 2, Auxiliary Assumption 1, and the intermediate value theorem. □

At this point, it seems useful to comment on the threshold $\mathfrak{f}_x$. As discussed above, fitness pertains to the systematic component of the inflow of resources. More exactly, fitness measures the efficacy of the producer at attracting resources from its environment in the face of competition. Therefore, the threshold $\mathfrak{f}_x$ has to depend on the systematic component of the outflow of resources, which we believe closely relates to the cost structure of the producer. If the producer can attract more resources than what it costs to provide its offering to the audience, this is a signal that the fitness level must be higher than the threshold. In the above formulation, we assumed that this threshold does not change over time. While we readily acknowledge that this is a simplification, we do not see any good reason to predict a systematic pattern of variation of the threshold $\mathfrak{f}_x$. One might argue that, as the producer grows in size, it might successfully target more social positions and therefore see its overall fitness level increase. But similarly, the overall operational costs increase with size as well. Absent any existing theory about the coevolution of fitness level and cost structure, we decided to opt for parsimony, at this point, and assume that the threshold $\mathfrak{f}_x$ does not change over time.

3.2 Fitness and the Mortality Hazard

It is possible to reformulate Postulate 1 in terms of organizational capital: the stock of organizational capital normally increases with age when the fitness exceeds a threshold ($\phi(x,t) > \mathfrak{f}_x$). Similarly, the stock of organizational capital decreases with age when the fitness falls below $\mathfrak{f}_x$.

We can use these relations to make some predictions regarding the relation between fitness and mortality hazard.

Proposition 2. *[Fitness level and the evolution of the mortality hazard]. A producer's mortality hazard, ω, presumably decreases with time if its fitness exceeds a threshold, $\phi > \mathfrak{f}_x$. The mortality hazard presumably remains constant if the fitness remains at the threshold $\mathfrak{f}_x$. The mortality hazard presumably increases with time if the fitness is below the threshold $\mathfrak{f}_x$.*

$$\mathfrak{P}\, x\, \exists! \mathfrak{f}_x \forall\, s, t_1, t_2\, [(s \in [t_1, t_2] \rightarrow (\phi(x, s) > \mathfrak{f}_x)) \rightarrow \omega(x, t_1) > \omega(x, t_2)]$$
$$\wedge [(s \in [t_1, t_2] \rightarrow (\phi(x, s) = \mathfrak{f}_x)) \rightarrow \omega(x, t_1) = \omega(x, t_2)]$$
$$\wedge [(s \in [t_1, t_2] \rightarrow (\phi(x, s) < \mathfrak{f}_x)) \rightarrow \omega(x, t_1) < \omega(x, t_2)].$$

Proof. The only applicable rule chain in this stage of the theory is a direct application of Postulate 1, Definition 1 and Proposition 1. □

4 Theory Stage 2: Incorporating Improving Capabilities

4.1 Background

As we noted above, a general line of argumentation, initially proposed by Stinchcombe (1965), implies negative age-dependence. This argument is agnostic about the presence of a period of initial endowment, but previous unification efforts have proposed that this theory applies without interference mainly after the end of the endowment period (Hannan 1998) or operates at all ages when it is not overridden by more specific information about endowments or obsolescence (HPC).

Is it possible to incorporate Stinchcombean thinking about increasing capabilities into the new model? We argue that it is indeed possible, but that the scope of possible empirical applicability is somewhat narrow due to the strong assumptions that we think are required.

The key feature of the following argument is that it brings in organizational fitness into the model. Organizational fitness will play a pivotal role by providing the missing link between organizational capabilities and the level of organizational capital. It is useful to read the following developments by keeping an eye on the schema of Figure 2.

Exactly what counts as a capability and what constitutes a high level of capability depends on the category in question and on the expectations of the audience for that category. There are multiple possible perspectives on the idea of capability, but we are primarily concerned by Stinchcombe's view of organizational capabilities. And we see those as closely related to the idea of efficacy at attracting resources from the relevant audience. In other words, a highly capable producer is good at addressing the demands of audience members. In terms of the constructs used in HPC, a highly capable producer is good at converting the intrinsic appeal of its offering into actual appeal[7]. The construct that captures this idea in HPC is the level of engagement of producer x at position $z(x)$.[8]

[7] Write more and better about intrinsic appeal and actual appeal

[8] This paragraph needs to be very well written.

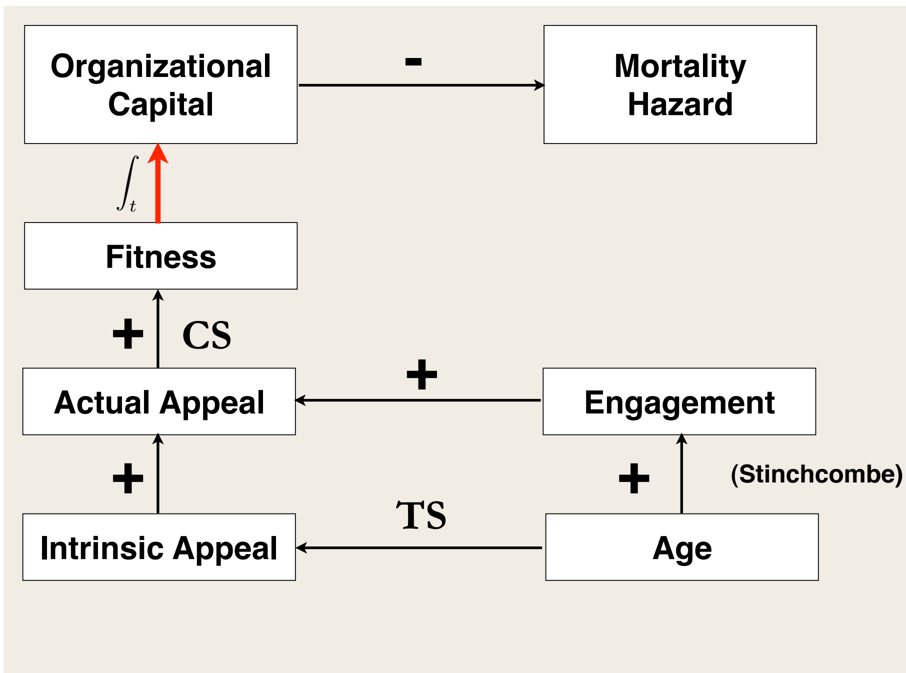

Figure 2: In the proposed theory, there is no direct relation between capabilities and mortality hazard. Rather, we see increasing capabilities as increasing engagement, which affects the accumulation of organizational capital. Actual appeal is a function of both engagement and intrinsic appeal. As the organization gains experience, engagement increases. The assumption of taste stability (TS) guarantees that actual appeal increases with engagement. The assumption of competitive stability guarantees that fitness increases with actual appeal.

Meaning Postulate 1. *[Intrinsic appeal, engagement and actual appeal] The actual appeal of an offer is equal to a portion of its intrinsic appeal. The ratio of the actual appeal over the intrinsic appeal normally is an increasing function of the engagement. In addition, for a high enough level of engagement, the actual appeal of the offering of the producer becomes arbitrarily close to its intrinsic appeal. Let g be an increasing mapping from the non-negative real numbers into the interval [0,1]*[9].

$$\mathfrak{N}\, x\, \forall\, t\, [(\tau(x,t) \neq 0) \rightarrow \frac{Ap(x,t)}{\tau(x,t)} = g(en(x,t)) \wedge \lim_{e\rightarrow\infty} g(e) = 1].$$

We can now rewrite Stinchcombe's argument about increasing capabilities by using the engagement construct.

Postulate 3. *[Age and engagement] An organization's expected level of engagement normally rises with its age.*

$$\mathfrak{N}\, x\, \exists\, \epsilon_1\, [(\epsilon_1 > 0) \wedge \forall\, t_1, t_2\, [(t_1 < t_{2)} \rightarrow \mathrm{E}\,\{en(x,t_2)\} - \mathrm{E}\,\{en(x,t_1)\} > (t_2 - t_1)\epsilon_1 > 0]].$$

We have managed to derive the Stinchcombe story only for very specialized (unrealistic) conditions. In our framework, the producer's fitness must improve over its lifetime relative to that of its competitors. (These ecological considerations are absent from the standard liability-of-newness story.) There are two issues concerning the environment: tastes and competition. We might allow the focal organization to experience secular increasing taste for its offering. But if this is the causal mechanism, then it does not really involve the effect of aging. Moreover, it cannot plausibly be the case that this kind of benefit holds generally for members of a category. So it seems most relevant to try to derive the desired theorem in a setting in which tastes are stable. We do so by invoking a predicate defined as follows.

Definition 3. [Taste stability] The taste of the audience of producer x is stable over an interval if the intrinsic appeal of x's offer does not change over the interval.

$$\mathrm{Ts}(x, t_1, t_2) \leftrightarrow \exists\, \tau_x\, \forall\, s\, [(t_1 \leq s \leq t_2) \rightarrow \tau(x,s) = \tau_x].$$

The second relevant environmental condition is the strength of competitive interactions within the category. If competition is becoming more intense, then improving capabilities do not necessarily translate to increased (relative) fitness. So we restrict the story to hold in environments in which the strength of competition is either stable or weakening over time. We do this by invoking a predicate defined as follows.

Definition 4. [Stable competition] A competitive environment for a producer x is stable over an interval if the sum of the actual appeals of its competitors (in the focal producer's category) remains constant.

[9] This formalism is need because the definition of endowment in HPC assumes that this is a non-negative quantity. An alternative approach would be to assume that endowment is between 0 and 1. In that case, this meaning postulate would become $\mathfrak{N}\, x\, \forall\, t\, [Ap(x,t) = en(x,t)\tau(x,t)]$, but some of the later develpments would need to be adapted.

$$\text{Cs}(x, t_1, t_2) \leftrightarrow \exists\, Ap_x \,\forall\, s\,[(t_1 \leq s \leq t_2) \rightarrow \overline{Ap}(x,t) = \overline{Ap_x}].$$

With these specialized conditions in place, we can relate initial fitness to the mortality hazard in a setting in which capabilities improve with age.

Proposition 3. *[Age and fitness level] In an environment characterized by taste stability and competitive stability, the following hold:*

A.) A producer's expected fitness presumably increases over time:

$$\mathfrak{P}\, x\, \forall\, t_1, t_2\, [Cs(x, t_1, t_2) \wedge Ts(x, t_1, t_2) \wedge (t_1 < t_2) \rightarrow \text{E}\,\{\phi(x, t_1)\} < \text{E}\,\{\phi(x, t_2)\}].$$

B.) A producer's fitness presumably becomes close to $\tau_x/(\tau_x + \overline{Ap_x})$:

$$\mathfrak{P}\, x\, \forall\, t_1,\ [Cs(x, t_1, \infty) \wedge Ts(x, t_1, \infty) \rightarrow \lim_{t\rightarrow\infty} \phi(x,t) = \frac{\tau_x}{\tau_x + \overline{Ap_x}}].$$

Proof. A.) follows from Postulate 3 and Meaning Postulate 1. B.) follows from postulate 3 and Definitions 1, 4 an 3. □

4.2 Fitness level and the evolution of the mortality hazard in stable environments

At this point, we have all the conceptual machinery in place to formulate predictions about the evolution of the mortality hazard with organizational age. Two dimensions will prove crucial in leading to distinct patterns of age dependence. The first dimension is whether the fitness level at founding is above or below the threshold $\mathfrak{f}_x$. The second dimension pertains to the competitive environment. Three cases can occur. If the initial fitness level is high enough, increasing capabilities imply that the fitness level will remain above the threshold. The pattern will be negative age-dependence. The second and third cases consider what happens with the fitness at founding is low. If the level of competition is so high that there is no way that the newly founded firm will be able to reach the fitness threshold even if it can put up with a high level of engagement, organizational capital will deplete monotonically over time and the pattern of age dependence will be positive age-dependence. Liability of adolescence will only emerge if the initial fitness level is below the threshold, but the producer can reach a fitness level high enough so as to start accumulating organizational capital.

Theorem 1. *[Fitness level and the evolution of the mortality hazard in stable environments] Suppose the organization's environment is characterized by taste stability and stable competition. If the fitness of producer x, at time of creation, exceeds the threshold $\mathfrak{f}_x$, the mortality hazard will presumably decrease with age (a "liability of newness"). If the fitness of producer x at time of creation (t_1) is below the threshold $\mathfrak{f}_x$, then there are two possible scenarios. If $\tau_x/(\tau_x + \overline{Ap_x}) \leq \mathfrak{f}_x$, then the mortality hazard presumably increases with age (this is a "liability of aging"). If $\tau_x/(\tau_x + \overline{Ap_x}) > \mathfrak{f}_x$,* *then the mortality hazard will presumably first increase and then decrease with age (this is a "liability of adolescence").*

$$\mathfrak{P}\, x\, \exists\, \mathfrak{f}_x, \sigma\, [\forall\, t_1, t_2, t_3, t_4, t_5\, [\{Cs\}(x, t_1, t_5) \wedge \{Ts\}(x, t_1, t_5) \wedge (a(x, t_1) = 0) \wedge (t_1 \leq t_2 < t_3 < \sigma < t_4 < t_5)$$

$$\begin{cases} (\phi(x, t_1) > \mathfrak{f}_x) & \rightarrow \omega(x, t_2) > \omega(x, t_5) \\ (\phi(x, t_1) < \mathfrak{f}_x) \wedge \frac{\tau_x}{\tau_x + \overline{Ap_x}} \leq \mathfrak{f}_x & \rightarrow \omega(x, t_2) < \omega(x, t_5) \\ (\phi(x, t_1) < \mathfrak{f}_x) \wedge \frac{\tau_x}{\tau_x + \overline{Ap_x}} > \mathfrak{f}_x & \rightarrow \omega(x, t_2) < \omega(x, t_3) < \omega(x, \sigma)) \\ & \wedge (\omega(x, \sigma) > \omega(x, t_4) > \omega(x, t_5)) \end{cases}$$

Proof. Suppose $\phi(x, t_1) > \mathfrak{f}_x$, and let t_2 and t_3 be two time points such that $t_1 \leq t_2 < t_3$. Proposition 3 implies that for all $t > t_1$, $\phi(x, t) > \mathfrak{f}_x$. Then, Proposition 2 implies that $\omega(x, t_2) > \omega(x, t_5)$.

Suppose $\phi(x, t_1) < \mathfrak{f}_x$ and $\tau_x/(\tau_x + \overline{Ap_x}) \leq \mathfrak{f}_x$. Proposition 3 implies that fitness presumably remains lower than $\mathfrak{f}_x$. Proposition 2 then implies that the mortality hazard decreases with time. Suppose $\phi(x, t_1) < \mathfrak{f}_x$ and $\tau_x/(\tau_x + \overline{Ap_x}) > \mathfrak{f}_x$. Proposition 3 implies that fitness presumably grows over time and becomes higher than $\mathfrak{f}_x$. Therefore there exists $\sigma > t_1$ such that, if $t < \sigma$, then $\phi(x, t) < \mathfrak{f}_x$ and if $t > \sigma$, then $\phi(x, t) > \mathfrak{f}_x$. Let t_2 and t_3 be two time points such that $t_1 \leq t_2 < t_3 < \sigma$. Proposition 2 implies $\omega(x, t_2) < \omega(x, t_3)$. Let t_4 and t_5 be two time points such as $\sigma < t_4 < t_5$. In this case, Proposition 2 implies $\omega(x, t_4) > \omega(x, t_5)$. □

This theorem leads to a surprising result: the shape of the relationship of age with the mortality hazard actually depends on the fitnesses of new organizations. If organizations can begin with high fitness, they can build up organizational capital, and their risk of failure will decrease with age. This result seems to contradict Stinchcombe's argument that younger organizations are vulnerable because of a deficit of capability. Theorem 1 tells the opposite: we should expect negative age-dependence (and liability of newness) when young organizations perform well. Such high-performing organizations can build up their stock of organizational capital, which should buffer them from failure.

This theorem also provides an alternate explanation for the liability of adolescence. Prior explanations have relied on population-level selection bias (Levinthal 1991) or on the ending of the endowment period (Hannan 1998; Hannan et al. 2007). According to this new theorem, the only thing that is needed for the liability of adolescence to emerge is some learning by the organization, together with low initial fitness.

While fitness and organizational capital are generally unobservable state variables, it seems possible to perform meaningful comparisons across organizational populations. In industries with great uncertainty, organizations generally start with a low fitness level, and we should expect a liability of adolescence. On the contrary, in populations where it is easier to design new organizations because competition is weak and many entrants have fitness that exceeds the threshold, one should expect negative age-dependence, because organizations can normally accumulate resources and grow their stocks of organizational capital rather quickly.

4.3 Relation to Existing Unification Attempts

As mentioned above, two prior theoretical efforts attempted to combine the improving capabilities argument and the depletion of endowment argument (Hannan 1998; HPC, Chapter 7). These have assumed that whether the first argument or the second argument should hold depends on the age under consideration. They suggested that within endowment periods, the second argument, about depleting endowment, holds. But both Hannan (1998) and HPC) proposed that, when the endowment is depleted, the dynamics of aging change considerably. The general tendency for organizations to build capabilities as time passes is no longer countered by the effect of a diminishing endowment, and therefore, the mortality hazard should then decrease with age because of these additional capabilities.

HPC identified this distinction between two periods, before and after the depletion of the endowment, with different dynamics, as problematic for true theoretical unification. Supposing that two contradictory theories (one positing positive age-dependence and another one positing negative age-dependence) apply to different periods does not really amount to theoretical unification. To address this challenge, those authors made an intense use of nonmonotonic logic to integrate the different theory fragments in a unified framework that could describe the relation between mortality hazard and age for all stages of the life of organizations. The theory developed in this paper does not really make use of specificity considerations specific to non-monotonic logic to allow for theoretical integration.

Rather, it introduced a crucial new distinction: whether the fitness level is higher or lower than a threshold. Therefore, it generates new empirical predictions about the evolution of the mortality hazard. An important aspect of the model developed here is that it allows for within-population heterogeneity. And the key instrument is the relative level of organizational fitness with respect to the fitness threshold.

5 Theory Stage 3: Organizational Obsolescence and Drifting Tastes (this section is still work-in- progress)

The previous section assumed that tastes are stable over time. But emprical evidence demonstrates that this is not the case: the organizational environment drifts. Drifting environment are at the core of theoretical thinking about organizational obsolescence. The environment of the organization changes, but aging organizations are not good at adapting to change, and thus mortality hazard increases.

Prior theorizing has focused on imprinting and impossibility for adaptation... According to this line of reasoning, organizations are selected at time of founding to fit to the prevailing environmental conditions and have an extremely limited ability to adapt to changing conditions.

In this section, we develop a model that does not make such a strong assumption about organizational inertia. Rather than assuming that organizations cannot adapt to changing environments, we propose that young organizations are adaptive, but that, as organization ages, their ability to adapt to new environmental conditions decline, because of organizational inertia.

Incorporating ideas about organizational inertia into the framework delineated in the previous sections allows us to make some predictions about the evolution of the mortality hazard for old organizations. As the first stages of the theory developed in this paper, the following developments build on the previous unification attempt reported in HPC; but keeping in the spirit of the previous sections, they does not rely on a postulate about different periods with distinct dynamics. More precisely, by contrast to the previous unification attemps (Hannan, 1998 and HPC), we do not postulate the existence of an obsolescence period. But, as in HPC, we will focus our attention onto the audience's taste, its variations over time, and the response by the organization. We will assume that audience's tastes tend to evolve over time, and that aging organizations cannot adapt their offerings to these changes in tastes. And then we will draw the implication of these basic assumptions for the evolution of the mortality hazard.

We first start by introducing a novel formalism to describe the relations between audience's tastes and a producer's offering. We define drift in terms of changes in meanings, where meanings are given by schemas. And then we explore the implications of drifting tastes and increasing organizational inertia for the evolution of the finess level and, in turn, the evolution of the hazard of mortality.

5.1 Schemas, Offerings and Tastes

As proposed in HPC, we assume that audience tastes can be modeled in terms of similarity to a schema, defined as a crisp set of feature values (see HPC, p. 63). A schema is, in a sense, a cognitive representation of an item on several feature dimensions. For example, the schema for a beer might include features such as color, transparency, one or several taste dimensions, country of origin, type of producer (e.g. large brewery, micro-brewery, ...). From the definition of the schema, we go on to define tastes, producers' offerings, and a measure of how a producer's offering fits with the taste of the audience.

5.1.1 Schemas

Let $\mathbf{f}_i = \{f_1, f_2, \ldots f_i\}$ be the set of i features that are relevant for a schema. Each feature in the set has a range of possible values. We denote the set of possible values of feature f_j by $\mathbf{r}_j$. Let $\Gamma = \mathbf{r}_1 \times \cdots \times \mathbf{r}_n$ denote the set of the ordered n-tuples of the values of the relevant feature.

An agent's schema for a label maps pairs of audience-segment members and time points to a nonempty closed and bounded subset of Γ; this subset contains exactly the schema-conforming patterns (n-tuples) of feature values. Let $\mathfrak{P}(\Gamma)$ denote a set of nonempty subsets of Γ.

$$\begin{aligned} \sigma_l : \mathbf{a} \times \mathbf{t} &\rightarrow \mathfrak{P}(\Gamma) \\ (i,t) &\mapsto \sigma_l(i,t) = \mathbf{s}_i^l \end{aligned}$$

It is important to note at this point that $\sigma_l(i,t)$ is not an element of Γ but instead is a *subset* Γ.

That is $\sigma_l(i,t)$ is not necessarily a single combination of feature values, but instead it is generally a set of combinations of feature values (see HPC, pp. 64-65 for illustrations).

We express instances of schemata with expressions of the form $\sigma_l(i,t) = \mathbf{s}^l$, which reads as "the (indexed) set of ordered n-tuples of the values of the category-relevant features, $\mathbf{s}$, conforms to the schema for the label l from agent i's perspective at time t."

5.1.2 Offerings

A producer's offering can be defined in terms of a position in the multi-dimensional feature space. More precisely, a producer's offering will be defined as a non-empty subset of Γ [10].

5.1.3 Meaning of a Label

Because memberships can be partial, it makes sense to treat an offering's fit to an agent's schema as potentially partial. Therefore, we define a grade of membership (GoM) function for an offering's fit to a schema. This function, denoted by $\mu_\sigma(x,t)$, tells the degree to which the feature values of the offering of producer x at time point t fit the (unspecified) agent's schema σ.

We think that agents typically refer to labels rather than to schemata. Therefore, we represent an entity's GoM in an agent's type as a function of the intension of the label. In formal terms, we define the function $\mu_{i(l)}(x,t)$ that tells x's GoM in the agent's meaning for the label l at time t. Because the meaning of a label is given by its associated schema, we define $\mu_{i(l)}(x,t)$ indirectly in the following meaning postulate.

Meaning Postulate 2. *[Grade of membership in the meaning of a label] An offering's grade of membership in an (unspecified) agent's meaning for a label, in notation $\mu_{i(l)}(x,t)$, is given by the offering's grade of membership in the schema associated with the label, $\mu_{\sigma(l)}(x,t)$. (adapted from HPC MP3.1)*

$$\mathfrak{N}\, l,t,x\, [\mu_{i(l)}(x,t) = \mu_{\sigma(l)}(x,t)].$$

What does it mean for an offering to have a high or low GoM in a schema? A schema, as defined above, is a set of acceptable n-tupes of feature values. Schema can thus be seen as a set of constraints on the features of an offering. An offering whose features meet all the constraints imposed by the schema will have a very high GoM, whereas an offering that meets only partially the constraints imposed by the schema will have a somewhat lower GoM. At the limit, an offering whose none of the feature values fits in the schema will have a GoM close to (or equal to 0) - see HPC, p. 65, for a discussion of these issues.

To implement these ideas in our model, we thus assume that a producer's GoM in a schema is a function of how similar to the schema audience members perceive its offering to be. But, at this point, the mathematical structure of the set of schematas is not reach enough to allow for modelling the similarity betweeen an offering and a schema. In the next subsection we add some structure to the set of schemata.

[10] It would be good to give an example here

5.1.4 Structure of the set of schematas

In the following, we develop a construction that leads to the formulation of a measure of similarity between offering and schema. But because schematas are *sets* of n-tuples of feature values and not just n-tuples of feature values, the definition of a measure of similarity necessitates several stages.

1. The first stage consists in assuming that the set of ordered n-tupes of feature values, Γ, is a metric space. That is, we assume that there exists a function d between $\Gamma \times \Gamma$ and $\mathbb{R}_+$ such that, for all $\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3$ in Γ,

- d takes non-negative values: $d(\mathbf{f_1}, \mathbf{f_2}) \geq 0$
- Two schemas are the same if and only if the distance between them is equal to zero: $d(\mathbf{f_1}, \mathbf{f_2}) = 0$ if and only if $\mathbf{f_1} = \mathbf{f_2}$,
- d is symmetric: $d(\mathbf{f_1}, \mathbf{f_2}) = d(\mathbf{f_2}, \mathbf{f_1})$,
- the following triangle inequality hold: $d(\mathbf{f_1}, \mathbf{f_3}) \leq d(\mathbf{f_1}, \mathbf{f_2}) + d(\mathbf{f_2}, \mathbf{f_3})$.

The function d is a metric that measures the distance between two n-tupes of feature values. That is, if $d(\mathbf{f}_1, \mathbf{f}_2)$ is low, then $\mathbf{f}_1$ and $\mathbf{f}_2$ are similar. But if $d(\mathbf{f}_1, \mathbf{f}_2)$ large, these two n-tuples are very dissimilar. [11]

2. We assume that the set of ordered n-tuples Γ is unbounded with respect to the metric d. That is, for any $\mathbf{f_1} \in \Gamma$, we assume that it is always possible to find a n-tuple of feature values that is extremely different from $\mathbf{f_1}$.

$$\forall\, \mathbf{f_1} \in \Gamma\, [M > 0 \rightarrow \exists \mathbf{f_2} \in \Gamma, d(\mathbf{f_1}, \mathbf{f_2}) \geq M].$$

We will define an audience member's taste as a set of feature combinations.

3. We assume that $\mathfrak{P}(\Gamma)$ contains only subsets of Γ that are closed and bounded with respect to d. These assumptions allow us to define the Hausdorff metric D_H (REF). We define D_H as a function between $\mathfrak{P}(\Gamma) \times \mathfrak{P}(\Gamma)$ and $\mathbb{R}_+$ such that for all $\mathbf{s_1}, \mathbf{s_2} \in \mathfrak{P}(\Gamma)$,

$$D_H(\mathbf{s_1}, \mathbf{s_2}) = \max \left\{ \sup_{\mathbf{f_1} \in s_1} \inf_{\mathbf{f_2} \in s_2} d(\mathbf{f_1}, \mathbf{f_2}), \sup_{\mathbf{f_2} \in s_2} \inf_{\mathbf{f_1} \in s_1} d(\mathbf{f_1}, \mathbf{f_2}) \right\}$$

It is important to note that D_H is a metric and, as such, has all the properties of any metric. Also, if $\mathbf{s_1}$ is a schema and $\mathbf{v_1}$ and offering, $D_H(\mathbf{s_1}, \mathbf{v_1})$ is a measure of the distance beween the offering and the schema.

5.1.5 Similarity between Offering and Schema

With this distance measure at hands, we can now formulate the relation between distance in the feature space and grade of membership of an offering in the meaning of a label. The following meaning postulate and assumption formalise the discussion at the end of section 5.1.3.

[11] Potentially discuss some distance metrics (include some of the elements of Mike's email about distances)...

Meaning Postulate 3. *An agent's assignment of a producer's GoM in a schema is given by the similarity of his/her (perception of) the producer's offering values on the schema-relevant features,* $\mathbf{v}(x, t)$.

$$\mu_{\sigma(l)}(x,t) = sim(\mathbf{v}^l(x,t), \mathbf{s}^l(x,t)).$$

While the exact nature of the similarity function is left unspecified because current empirical knowledge is not developed enough, we will assume that when the distance between an offering $\mathbf{v}^l(x,t)$ and a schema $\mathbf{s}^l(x,t)$ becomes large, the perceived similarity between offering and schema goes to 0. This intuition is formalized in the following meaning postulate:

Assumption 1. *The similarity function has the following property*

$$\lim_{D_H(\mathbf{v},\mathbf{s})\to\infty} sim(\mathbf{v}, \mathbf{s}) = 0$$

We have now defined a structure rich enough to model drifting tastes and how their implications for organizational mortaliry. In the next subsection, we then move onto the specifcation of drift, imprinting and alignment.

5.2 Drift, Adaptive Ability and Inertia.

We will say that the taste of an agent is drifting over time when the distance between of the her current schema to some of her past schemas becomes arbitrarily large after some time.

Definition 5. [Type drift] A type's meaning to an audience member is drifting over a time interval $[t, t']$ iff the distance between the schemas for the label at two time points is monotonically increasing in the duration between the two time points.

$$\textsc{Drift}(l, t, t') \leftrightarrow \exists\, \delta^l > 0\, [\forall\, t_1, t_2\, [t \le t_1 \le t_2 \le t' \to D_H(\mathbf{s}^l_{t_1}, \mathbf{s}^l_{t_2}) \ge \delta^l (t_2 - t_1)]].$$

We now turn to the characterization of the adaptive ability of a producer. We see a highly adaptive producer as one that requires little time to alter its offering. The adaptive ability is a time varying state variable for the producer and is defined inductively as follows:

Definition 6. [Adaptive ability]A producer's adaptive ability creates a limit on the evolution of its offering.

$$\forall\, t_1, t_2\, [t_1 \le t_2 \to D_H(\mathbf{v}^l_{t_1}, \mathbf{v}^l_{t_2}) \le \int_{t=t_1}^{t_2} \alpha^l(x,t) dt].$$

Adaptive ability can thus be seen as an upper bound on the speed of alteration of the offering in the mutli-dimensional feature space[12].

[12] THE REASON WE NEED THE INTEGRAL ISTHAT ALPHA IS NOT CONSTANT. THE REASON IS THAT WE WANT THE ADAPTIVE ABILITY TO BE HIGH IN THE EARLY STAGES OF THE LIFE OF THE ORGANIZATION BECAUSE IN ITS FORMATIVE STAGE, IT IS BEING ADAPTED TO THE ENVIRONMENT AS THE FOUNDERS LEARN ABOUT IT.

With these definitions, it is possible to characterize organizational aging. As discussed at the beginning of this section, organizations become more inert as they age. Within the framework just developped, an organization has a lot of inertia when its adaptive ability is low. The fundamental postulate that characterizes the effect of aging thus claims that adaptive ability declines over time, and ultimately becomes very low.

Postulate 4. *[Increasing Organizational Inertia] The adaptative ability normally declines with age and becomes low as the organization becomes old.*

$$\mathfrak{N}\, l, x\, \forall\, t_1, t_2 [t_1 \leq t_2 \rightarrow \alpha^l(x, t_2) \leq \alpha^l(x, t_1).$$

$$\mathfrak{N}\, l, x\, \lim_{t \rightarrow \infty} \alpha^l(x, t) = 0.$$

5.3 The Consequences of Drifting Tastes and Organizational Inertia

5.3.1 A scope condition

In this section, we focus only on positively valued types. In other words, we will assume that an offering with a high grade of membership in the meaning of the label will be attractive to audience members.

Definition 7. [Positively valued type] In a type with positive valence, an agent normally prefers (finds intrinsically more appealing) the offerings of more clear-cut members of the type, those with higher GoM. (HPC D3.10 specialized to one producer and one audience member)

$$\begin{aligned} \textsc{ptype}(l, t) \leftrightarrow \textsc{type}(l, t) \\ \wedge\, \mathfrak{N}\, x\, \forall\, t_1, t_2 [(\mu_{i(l)}(x, t_1) > \mu_{i(l)}(x, t_2)) \rightarrow \mathrm{E}(\tau^l(x, t_1) > \mathrm{E}(\tau^l(x, t_2))]. \end{aligned}$$

5.3.2 Age and Intrinsic Appeal

With the above formulation of drift and of increasing organizational inertia, it is possible to derive the consequence of a drifting environment on the intrinsic appeal of the offering.

Proposition 4. *[Age and Intrinsic Appeal] If the meaning of a label drifts, then the intrinsic appeal of a producer's offering goes to zero as the producer becomes old.*

Let $\textsc{ptype}(l, t)$ *at all times after the founding of the producer.*

$$\mathfrak{P}\, l, x, t_0\, [\textsc{Drift}(l, t_0, \infty)) \wedge a(x, t_0) = 0 \rightarrow \{\forall\, \varepsilon > 0\, \exists\, t_1 [t > t_1 \rightarrow \mathrm{E}\{\tau^l(x, t)\} < \varepsilon]\}.$$

Proof. Let us consider a producer x in the label l such as $a(x, t_0) = 0$ and its audience taste is drifting ($\textsc{Drift}(l, [t_0, \infty))$). To facilitate of the reading of the proof, we will denote $\mathbf{v}^l(x, t)$ by $\mathbf{v}(t)$ and $\mathbf{s}^l(x, t)$ by $\mathbf{s}(t)$. Let α_1 be such taht $0 < \alpha_1 < \delta^l$.

Postulate 4 implies that, normally there exists t_1 such that for all $t \geq t_1$, $\alpha^l(x,t) < \alpha_1$. This, in turn, implies that, presumably, for all $t_2, t_3 > t_1$, then

$$t_2 \leq t_3 \rightarrow D_H(\mathbf{v}(t_2), \mathbf{v}(t_3)) \leq \alpha_1(t_3 - t_2)$$

When this holds, the triangle inequality for D_H implies that for all $t > t_0$

$$\begin{aligned} D_H(\mathbf{s}(t_0), \mathbf{s}(t)) &\leq D_H(\mathbf{s}(t_0), \mathbf{v}(t_0)) + D_H(\mathbf{v}(t_0), \mathbf{v}(t_1)) + D_H(\mathbf{v}(t_1), \mathbf{v}(t)) + D_H(\mathbf{v}(t), \mathbf{s}(t)) \\ D_H(\mathbf{v}(t), \mathbf{s}(t)) &\geq D_H(\mathbf{s}(t_0), \mathbf{s}(t)) - D_H(\mathbf{s}(t_0), \mathbf{v}(t_0)) - D_H(\mathbf{v}(t_0), \mathbf{v}(t_1) - D_H(\mathbf{v}(t_1), \mathbf{v}(t))) \\ &\geq \delta^l(t - t_0) - D_H(\mathbf{s}(t_0), \mathbf{v}(t_0)) - D_H(\mathbf{v}(t_0), \mathbf{v}(t_1) - \alpha_1(t - t_1) \\ &\geq \delta(t_1 - t_0) + (\delta^l - \alpha_1)(t - t_1) - D_H(\mathbf{s}(t_0), \mathbf{v}(t_0)) - D_H(\mathbf{v}(t_0), \mathbf{v}(t_1)) \end{aligned}$$

Since $(\delta^l - \alpha_1) > 0$, the right hand side becomes arbitrarily large when t becomes large. Meaning Postulate 3 implies, in turn, that the similarity between the schema $\mathbf{s}(t)$ and the offering $\mathbf{v}(t)$ become arbitrarily low. The grade of membership of the offering in the label, $\mu_{\sigma(l)}(x,t)$, becomes close to 0. Finally, the assumption of positively valued type then implies that the intrinsic appeal of the offering becomes arbitrarily small.

□

5.3.3 Old Age and Mortality Hazard

We want to connect this result with fitness and with the hazard of mortality. It appears that we need to invoke the existence of some competition.

Definition 8. [Some Competition] The environment for a producer x is competitive if the sum of the actual appeals of the competitors is always higher than some positive lower bound.

$$\textsc{Cp}(x, t_1, t_2) \leftrightarrow \exists\, Ap_x\, \forall\, s\, [(t_1 \leq s \leq t_2) \rightarrow \Sigma Ap(x,s) > \Sigma Ap_x].$$

With this definition at hands, we can (finally) formulate our main theorem about obsolescence

Theorem 2. *[Old age and Mortality Hazard] In conditions of categorical drift, mortality hazards presumably increase beyond a certain age. Let* $\textsc{ptype}(l,t)$ *at all times after the founding of the producer and* $\textsc{Cp}(x, t_0, \infty)$.

$$\mathfrak{P}\, l, x, t_0\, [\textsc{Drift}(l, t_0, \infty) \wedge a(x, t_0) = 0 \rightarrow \{\exists\, t_1[t_1 \leq t_2 < t_3 \rightarrow \omega(x, t_2) < \omega(x, t_3)]\}.$$

Proof. Meaning Postulate 1 and Proposition 4 imply that the actual appeal Ap(x,t) becomes arbitrarily low. Definition 8 implies that, presumably, for t large enough, $\phi(x,t) < \mathfrak{f}_x$. This and Proposition 2 imply, in turn, the presumably the mortality hazard inreases over time. QED.

□

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