FRACTURE OF AHSS SHEETS

and

STATUS OF THE AHSS CONSORTIUM AT MIT

Addendum to the Research Proposal on
"Fracture of Advanced High Strength Steels"

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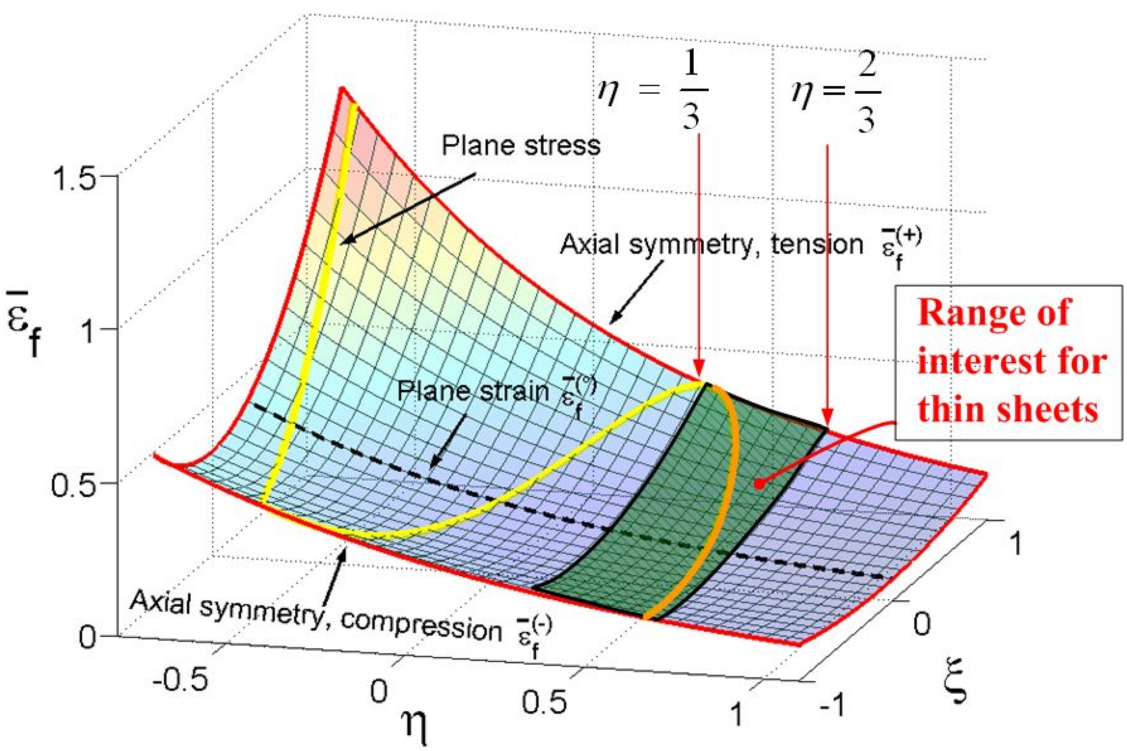

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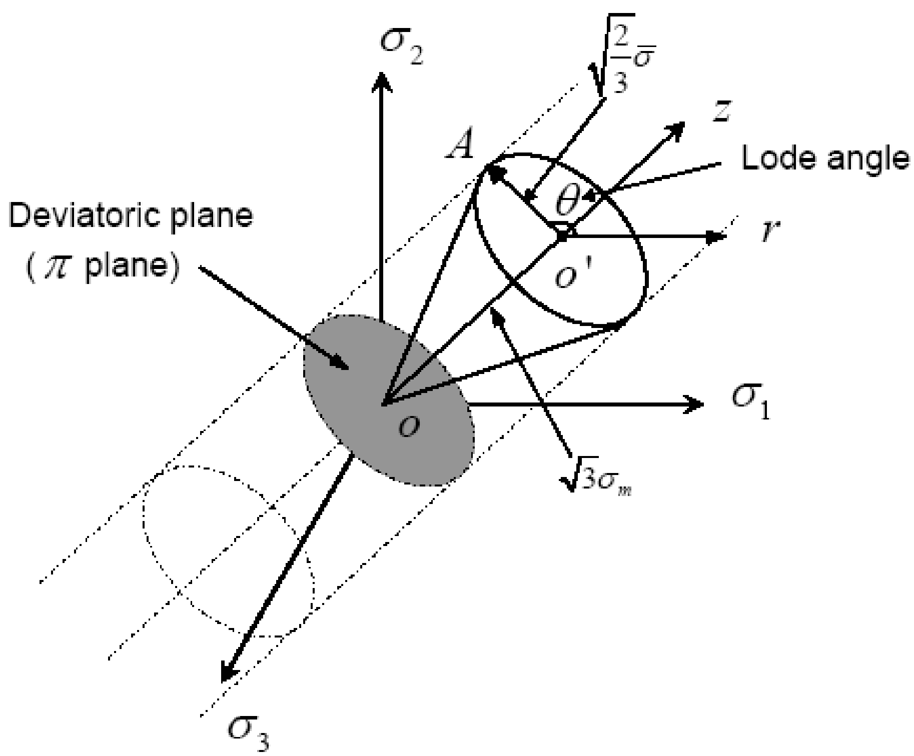
Notation:

$\sigma_1, \sigma_2, \sigma_3$	Principal stresses
s_1, s_2, s_3	Principal deviatoric stresses
$J_1 = 0$	First invariant of deviatoric stress tensor
$J_2 = s_1^2 + s_2^2 + s_3^2$	Second invariant of deviatoric stress tensor
$J_3 = s_1 s_2 s_3$	Third invariant of deviatoric stress tensor
$\bar{\varepsilon}$	Equivalent strain
$\bar{\varepsilon}_f = \hat{\varepsilon}_f(\eta, \xi)$	Fracture locus under proportional loading
$\bar{\varepsilon}_f^{(+)} = \hat{\varepsilon}_f^{(+)}(\eta, \xi = 1)$	Fracture locus under axisymmetric loading in tension
$\bar{\varepsilon}_f^{(-)} = \hat{\varepsilon}_f^{(-)}(\eta, \xi = -1)$	Fracture locus under axisymmetric loading in compression
$\bar{\varepsilon}_f^{(\circ)} = \hat{\varepsilon}_f^{(\circ)}(\eta, \xi = 0)$	Plane strain fracture locus
n	Exponent of the strain hardening
k	Shape parameter of yield locus
m	Parameter of the history effect
τ	Shear stress
σ	Normal stress
$\tau_\circ, \sigma_\circ$	Reference stresses
ε_1	Major principal strain in plane stress
ε_2	Minor principal strain in plane stress
$\alpha = \dfrac{d\varepsilon_2}{d\varepsilon_1}$	Strain increment ratio in plane stress
a	Groove radius of a flat specimen
t	Ligament thickness of the flat specimen

$\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$	Hydrostatic stress (mean stress)
$\bar{\sigma} = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}$	Equivalent stress
$\eta = \frac{\sigma_m}{\bar{\sigma}}$	Stress triaxiality
$\xi = \frac{27}{2}\frac{J_3}{\bar{\sigma}^3} = \cos(3\theta)$	Lode angle parameter
θ	Lode angle
FLD	Forming Limit Diagram
FFLD	Fracture Forming Limit Diagram



Geometrical interpretation of stress parameters

1. Introduction

In August 2006 Professor Tomasz Wierzbicki, the Director of the Impact and Crashworthiness Lab (ICL) at MIT, developed the research proposal on "Fracture of Advanced High Strength Steels". He contacted leading steel and automotive companies with a suggestion to organize necessary funding through a large international consortium. The proposal was subsequently presented to prospective sponsors at the 3rd MIT Workshop on Experimental and Computational Fracture Mechanics held in October 2006 at MIT. Since then, several pilot studies with our industrial partners have been initiated on selected representatives of the AHSS family. At the same time, modeling concepts of plasticity, necking, and fracture phenomena have been advanced at ICL. Existing experimental techniques have been improved, and new testing methods of sheet metal have been developed.

The purpose of the present document is to inform the present and prospective members of the Consortium about the progress on the fracture research and outline updated research tasks in view of new findings. While the original proposal has been endorsed by some of our professional colleagues in the industry, the proposal received critical comments from others. A part of the reason has been that the MIT team put forward a new concept of fracture and a new language which was unfamiliar to the sheet metal forming community. In response to this criticism, efforts were made in the present addendum to explain the connection between the forming limit diagrams (FLD) and traditional sheet metal testing on one hand with fracture and in particular fracture of AHSS concepts on the other. This addendum is not a stand alone document. It should be read in conjunction with the original proposal where a number of practical issues facing the industry were identified and the ICL experimental base was described.

The sheet metal industry recognized the importance of precise plasticity models to predict necking of sheets, including the effect of anisotropy. Ever more sophisticated mathematical models have been proposed, and testing techniques have been continuously improved. For a review of the advances in experimental methods and modeling concepts, the reader is referred to Kuwabara (2007) and Vegtev and Boogaard (2006).

Fracture is a local phenomenon, and there is an urgent need for more advanced plasticity models to accurately predict stress and strain fields at the potential fracture initiation sites and around the tip of a propagating crack. Such plasticity models, along with calibration procedures, have been developed at ICL using an unconventional approach. Also, the issue of neck formation in sheets under complex loading paths has been addressed in our research in order to determine if fracture occurs before or after necking for AHSS.

In summary, the MIT team is presenting a comprehensive experimental and theoretical research program on fracture, necking, and plasticity of AHSS sheets. From the present document, the reader should recognize that the ICL team is entering the first year of the Consortium with a large initial capital of knowledge and experience.

Finally, it should be clarified that the present addendum is concerned only with the Core Project, as defined in the original proposal. The Core Project encompasses the following four tasks:

CORE PROJECT: Plasticity and fracture of AHSS: Development of Reliable Biaxial Testing Techniques and Robust Computational Models

Task 1	Plasticity of AHSS – prediction of the stress and strain histories up to the point of fracture
Task 2	Fracture testing of sheets under biaxial loading
Task 3	Development of AHSS failure models and validation
Task 4	Demonstration of the new technology in real-life applications (with active participation of interested consortium members)

While the original proposal on "FRACTURE OF ADVANCED HIGH STRENGTH STEELS" describes *what will be done*, the present document entitled "FRACTURE OF AHSS SHEETS" tells *how this will be done*. Also, the original proposal was dealing more with 3D fracture while this addendum is focused on sheets for automotive applications.

The ICL has disclosed in the present text a number of new results on sheet metal plasticity, necking, and fracture, never published before. It is hoped that the expertise and leadership of ICL in the area of fracture will be recognized and appreciated by prospective sponsors.

Advances have also been made in the two Expansion Projects dealing with computational aspects of fracture (Xue, 2007) and strain rate effect on fracture (Mohr and Gary, 2007).

Because of the current level of funding, the respective research on both expansion projects is being conducted outside the AHSS Consortium (see Section 10 on the status of the Consortium).

2. New Plasticity Model for Fracture Prediction

There is growing experimental evidence that the plastic constitutive equations depend on both the hydrostatic pressure and the Lode angle parameter (Spitzig and Richmond, 1984; Wilson, 2002; Barsoum and Faleskog, 2006). Based on these observations and ICL's own test, Bai and Wierzbicki (2006) proposed the following form of the generalized yield surface (see the section on notation for the definition of various quantities).

$$\sigma_{yield} = \bar{\sigma}\left(\bar{\varepsilon}_p\right) f_p\left(\eta\right) f_\theta\left(\xi\right) \quad (1)$$

In the limiting case of $f_p\left(\eta\right) = f_\theta\left(\xi\right) = 1$, Eq. (1) reduces to the classical J2 plasticity theory. While the effect of stress triaxiality parameter η may be relatively small, the Lode angle parameter may dramatically change plastic properties. This is nicely illustrated in Fig. 1, showing that the tensile response of flat grooved specimens $\left(\xi = 0\right)$ cannot be predicted using the stress-strain curve calibrated from round bar tensile tests $\left(\xi = 1\right)$.

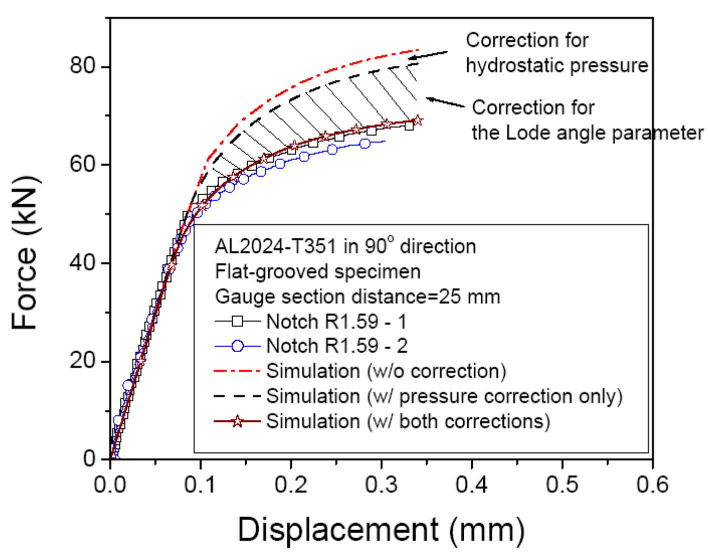


Fig. 1 A comparison of force-displacement curves between experimental results on flat grooved specimens, ligament thickness t = 2.11mm, groove radius a =1.59mm and numerical simulation.

The error is of an order of 15-20%. Introducing the correction for the Lode angle parameter rectifies the problem. Guided by the above observation, attention will then be focused at ICL to the determination of the function $f_\theta(\xi)$. It was shown by Bai and Wierzbicki (2006) that with a three-parameter representation of the function $f_\theta(\xi)$, almost any shape of isotropic yield conditions can be reproduced (refer to Fig. 2). The parameters c_θ^s and c_θ^c are responsible for the general shape of the modified ellipse, while the parameter *k* is defining the local curvature at the point of equi-biaxial tension. The concept of the Lode angle dependence of the yield condition is only one step away from incorporating the notion of material anisotropy into the present model. The von Mises yield condition can be recovered by setting either $c_\theta^s = c_\theta^c = 1$ or $k = 0$, while the case with $c_\theta^s = \sqrt{3}/2$, $c_\theta^c = 1$, $k \to \infty$ corresponds to the Tresca yield locus. A complete calibration procedure of the present plasticity model is easier for those materials from which both round and flat grooved specimens can be extracted. Determination of the parameter of the function $f_\theta(\xi)$ in the case of thin sheets poses a challenge, as explained in the next section.

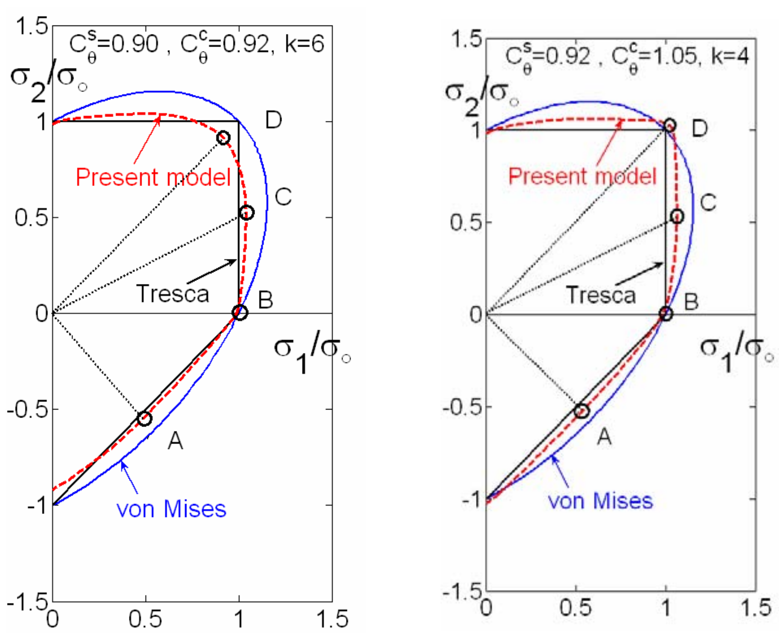


Fig. 2 By adjusting two parameters in the Lode angle dependant plasticity model, various shapes of plane stress yield locus can be generated. The parameters of the baseline von Mises ellipse are either $c_\theta^s = c_\theta^c = 1$ or k=0. The parameters of the Tresca hexagon are

$$c_\theta^s = \sqrt{3}/2, c_\theta^c = 1, k = \infty.$$

Action items to Section 2

2.1 Determine the minimum number of tests to calibrate the plasticity model with the pressure and Lode angle dependence

2.2 Extend the present theory to anisotropic solid and sheets (see a list of Ph.D. thesis at ICL)

3. A Novel Experimental Technique for Calibration of the Plasticity Model

Current industrial techniques for the determination of the initial and subsequent yield loci of metal sheets include hydraulic bulge test, biaxial compression tests on adhesively bonded sheet laminate specimens, cruciform specimen biaxial tension and thin tubes subjected to hydrostatic pressure and tension. Each of these techniques has its disadvantages, as discussed by Kuwabara (2007).

Recently, ICL has developed a new experimental method for testing thin specimens under transverse plane strain conditions (Mohr and Oswald, 2007). The specimens, design of grips and loading conditions were also described on pages 13-15 of the original AHSS proposal. From the independently measured normal and shear stresses, one can construct the entire yield surface in the plane of (σ, τ).

ICL recently completed a pilot study on plastic properties of a TRIP steel provided by ThyssenKrupp. The results are shown in Fig. 3. It is straightforward to convert the present experimental data to the space of principal stresses. This study has proven suitability of the new technique for sheet metal testing. Specimens can be easily cut by the wire EDM technique even from the hardest steels. The thinned gauge section is made by the sinker EDM. Both a wire and sinker EDM are available to ICL. Other representatives of the AHSS will be tested at ICL in the course of the present research program. The Steering Committee will select several candidate steels for comprehensive plasticity and fracture testing.

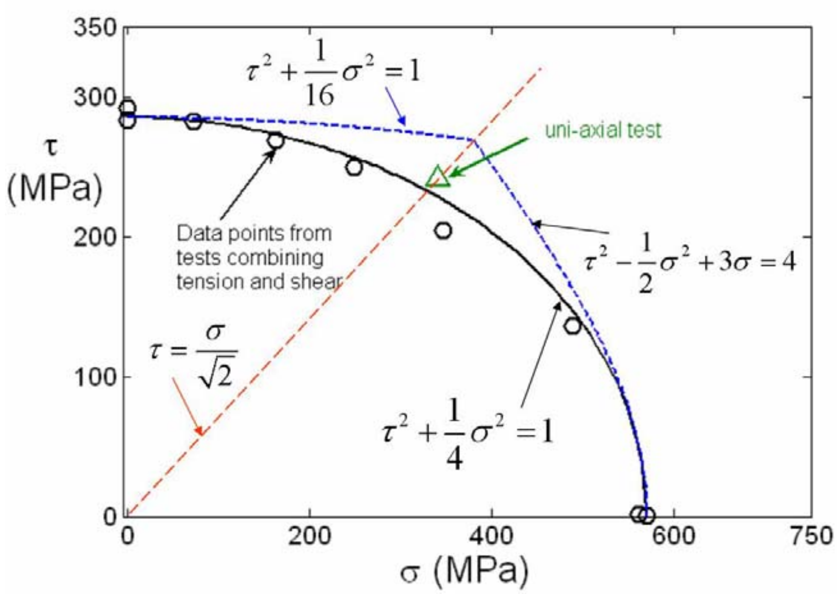


Fig 3. Test results of the pilot study conducted at MIT on a representative type of TRIP steels

Action items on Section 3

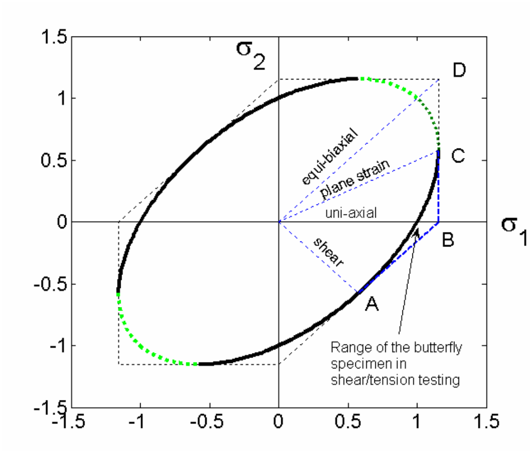
3.1 Further optimization of the shear/tension specimen (Fig. 6b)

3.2 Extend the new technique to anisotropic sheets (the TRIP steel used in the pilot study was nearly isotropic)

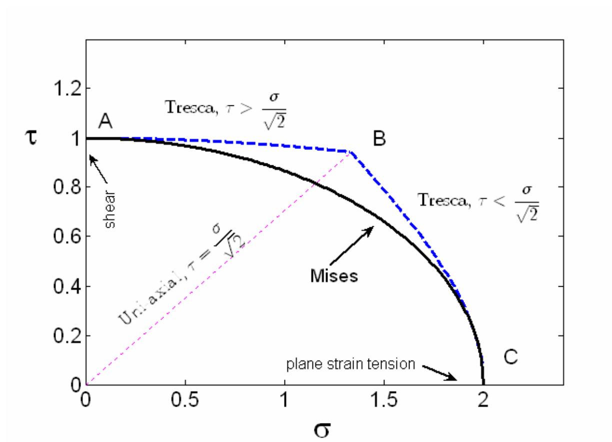
3.3 Test several representatives of the AHSS family recommended by the Steering Committee

4. Development of a Complementary Testing Method for Sheets

A further analysis of the tests described in Section 3 reveals their strengths and limitations. Using the Mohr circle method, the yield locus in the (σ, τ) space can be transformed to the space of principal stresses (σ_1, σ_2), Fig. 4. It is seen that in using the ICL technique, one can construct the entire yield locus in the second and fourth quadrants and a part of it in the first and third quadrants. The missing range is the neighborhood of the equi-biaxial tension or compression.



(a)



(b)

Fig 4. Transformation of the von-Mises and Tresca yield loci from the (τ, σ) *space to the space of principal stresses* (σ_1, σ_2)*. Note that a quarter of an ellipse on the bottom figure is mapped only into an arc (less than a quarter) on the upper figure.*

These ranges of stresses are attainable by other testing methods used by the industry. It will be up to the Steering Committee of the Consortium to decide if part of the funding should be used for the ICL to acquire alternative test equipment for biaxial testing. It is proposed to construct the Cruciform Testing Device (CTD), which will be fitted into the existing dual actuator loading frame. The cost of the CTD will not exceed $50,000 as compared to a price of $300,000 for a similar device custom made from scratch. A schematic of a CTD is shown in Fig. 5. Great attention will be paid to the design of a new generation of cruciform specimens. Uniform stress states necessary for plasticity testing will be achieved through the optimization of the cut-outs (Fig. 6d), following the concepts of Mueller and Poehlandt (1996). In addition, the gauge section will be thinned down by the already mentioned sinker EDM technique. ICL will draw expertise from other prominent research centers (Green, 2004), which successfully used the cruciforms for material calibration in the plastic range. New data points will provide information on the missing stress range between transverse plane strain and equi-biaxial tension.

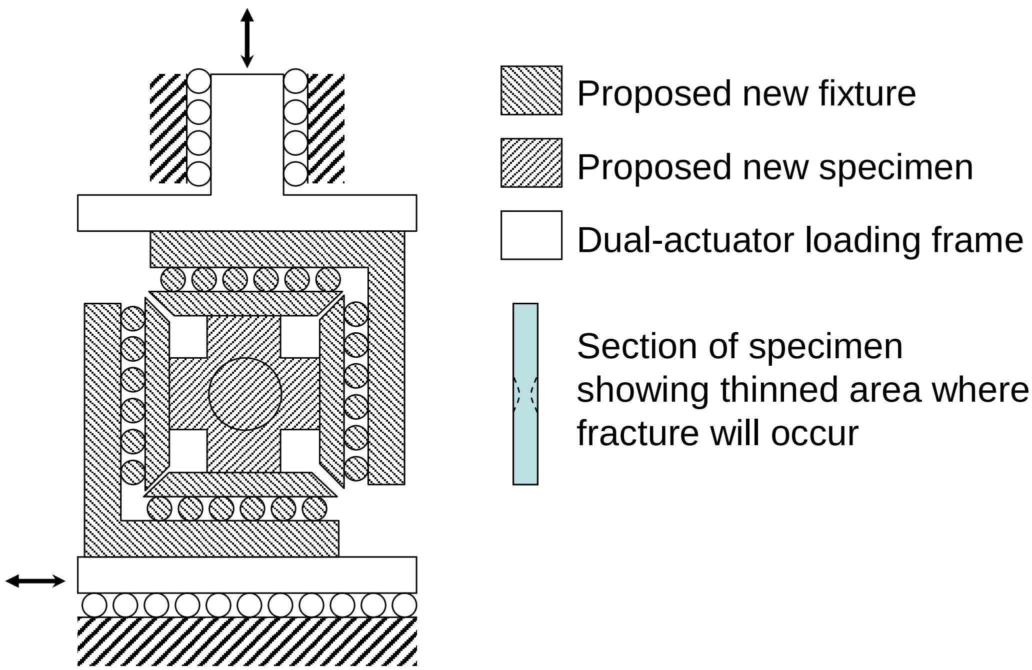


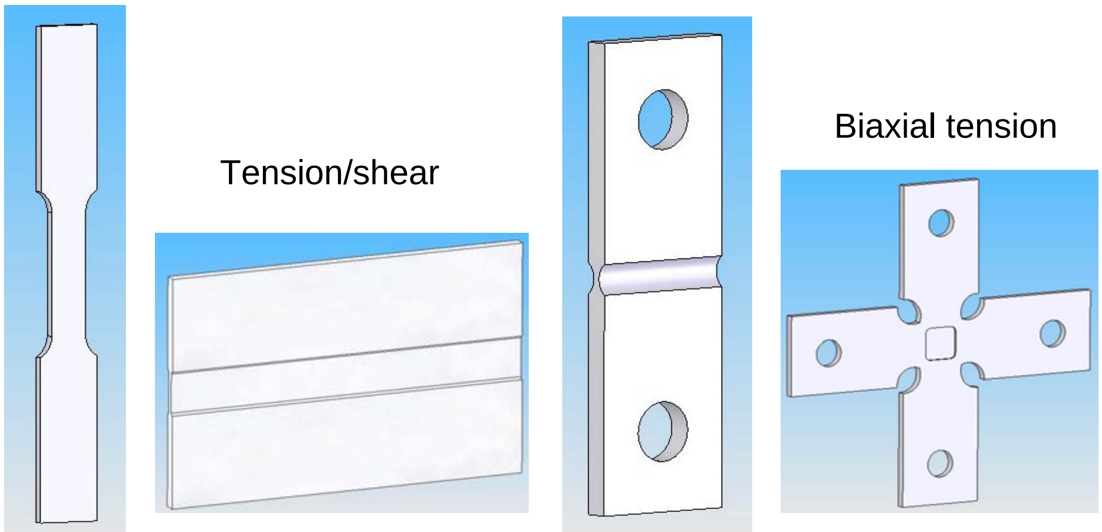
Fig. 5 Schematic of the fixture to convert the biaxial tension/shear loading into biaxial tension/tension loading to test cruciform specimens.

Action items on Section 4

4.1 Design and build the Cruciform Testing Device as an attachment to the existing dual actuator loading frame

4.2 Optimize new cruciform specimen

4.3 Test a wide range of AHSS, according to the schedule approved by the Steering Committee



(a) Dog-bone (b) Transverse plane strain (c) Flat-grooved (d) Cruciform (under development)

Fig. 6 Specimens currently used and under development at ICL for thin sheets plasticity testing

5. Effect of Loading History on FLD through a New Concept of "Forming Severity Index"

Necking behavior of sheets has been the subject of extensive theoretical, experimental, and numerical studies over the past 40 years, since the pioneering work of Keeler and Backofen (1963). The onset of necking is conveniently represented on the plane of principal strains, Fig. 7. Compared to the abundance of results from monotonic loading and straight loading paths, there have been only a handful of practical methods to predict necking under continuously or abruptly changing strain paths (Ghosh and Laukonis, 1976; Graf and Hosford, 1994; Stoughton, 2000; Yoshida et al, 2005). The effect of loading history in multi-stage stamping operations as well as subsequent crash response is of great concern to the industry as AHSS are replacing traditional deep drawn steels.

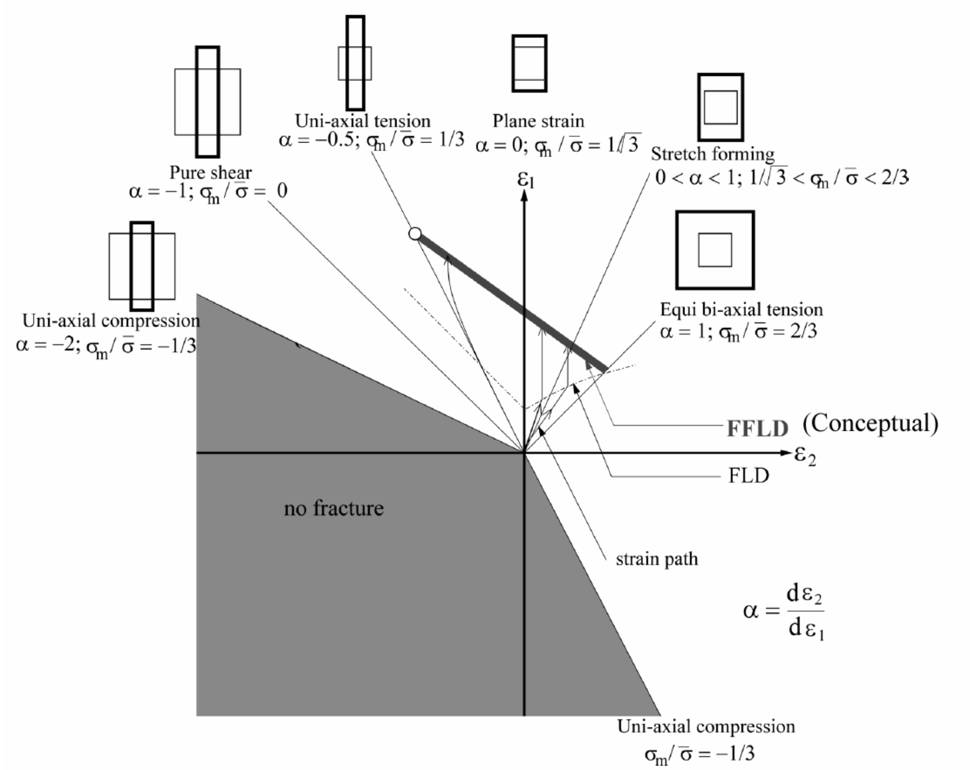


Fig. 7 Forming Limit Diagrams (FLD) and Fracture Forming Limit Diagram (FFLD) in the space of principal strains.

The MIT program is mainly concerned with the description of fracture of AHSS sheets which can occur before or after necking. Both fracture and necking events constitute an ultimate "failure" mode in the press shop, and both must be correctly predicted and dealt with in crash simulation. Industry has the necessary equipment and experience in predicting necking under constant strain ratio using the hydraulic bulge tests, Nakazima hemispherical dome tests, or Marciniak tests. There is no need for MIT to enter this territory. What ICL can offer to the Consortium members is a relatively simple method of predicting the effect of loading history on necking. The concept was originally developed for fracture applications (Bai et. al, 2006; Xue, 2007; Bai, 2008), but the pilot study conducted at ICL has shown its applicability to sheet metal forming as well.

Following the idea of the "damage indicator", one can define a new functional, the "forming severity index":

$$FSI = \int_0^{\bar{\varepsilon}_n} f\left(\frac{\bar{\varepsilon}}{\hat{\varepsilon}_{nec}}\right)\frac{d\bar{\varepsilon}}{\hat{\varepsilon}_{neck}} \tag{2}$$

where $f\left(\bar{\varepsilon} = \bar{\varepsilon}_{neck}\right) = 1$. Equation (2) postulates that the FSI (which is a scalar measure of sheet resistance to necking) accumulates in a nonlinear way. The function $f(\lambda)$ can be taken as either power type or exponential

$$f\left(\lambda\right) = \begin{cases} \left(m+1\right)\lambda^m \\ \dfrac{m}{e^m - 1} e^{m\lambda} \end{cases} \tag{3}$$

where $\lambda = \dfrac{\bar{\varepsilon}\left(\alpha\right)}{\hat{\varepsilon}_{neck}\left(\alpha\right)}$ and $\alpha = \dfrac{d\varepsilon_2}{d\varepsilon_1}$. Both formulas involve only one calibration parameter *m*. Necking is said to occur when *FSI=1*. The predictive power of the above formulation is amazing and is illustrated in Fig. 8. Graf and Hosford (1994) conducted a comprehensive series of necking tests on 6111-T4 aluminum alloy under proportional and complex loading histories. Their experimental forming limit curves under proportional loading are indicated by circles fitted by a solid line. Then the sheets were subjected to equi-biaxial stretching followed by radial loading (see the arrows in Fig. 8). A set of Hosford's experimental forming limit curves, corresponding to a different level of pre-straining, are denoted by dashed lines. Clearly, they were shifted and underwent shape distortion.

The input data for the present predictive method is the experimental forming limit curve $\varepsilon_1\left(\alpha\right)$ or $\bar{\varepsilon}\left(\alpha\right)$ under proportional loading. This curve contains all information about the material. Then, information on the loading path (sequence of α) is introduced into Eq. (2). The unknown parameter *m* is then calibrated from one piecewise linear loading path and applied to predict necking in the remaining complex loading path. The predicted FLDs are denoted in Fig. 8 by dotted lines. The correlation between the analysis and measurements is seen to be very good. Compared to other computationally intensive methods (Dell et al, 2001; Werner et al 2003; Yoshida et al, 2005; Yoshida and Kuwabara, 2006), the MIT approach is very simple and thus promising for direct press shop applications.

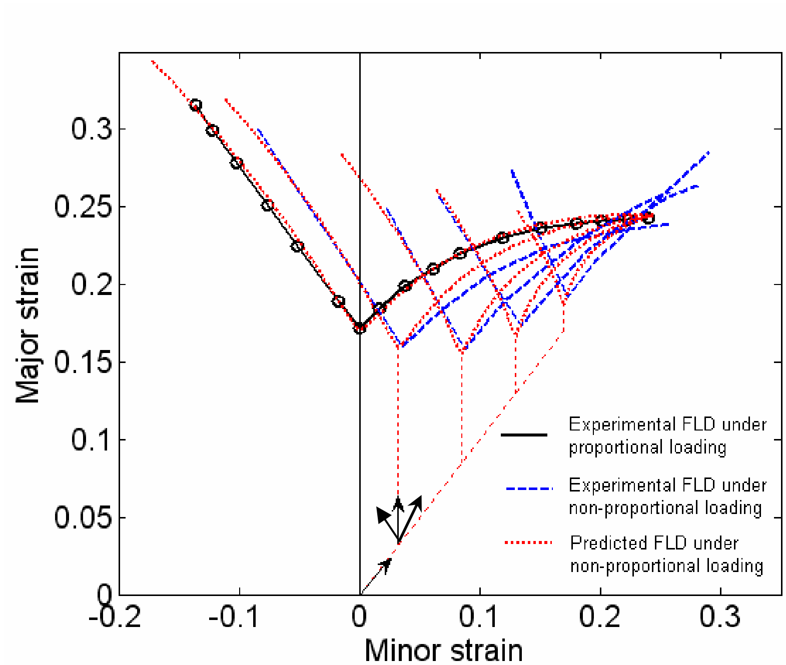


Fig. 8 ICL developed a simple and powerful tool for predicting plane stress necking under complex loading paths (Experimental data after Graf and Hosford, 1994).

Action items on Section 5

5.1 Investigate other, more recent case studies of nonlinear loading to fracture published in the open literature

5.2 Develop a simple PC-based software for FLD prediction under complex loading paths

5.3 Work closely with industrial partners on further validations and applications

5.4 Seek advice of the Steering Committee regarding the incorporation of this topic into the Consortium research plan

6. The New MIT Three-dimensional Macroscopic Fracture Theory

As opposed to necking, which is a plane stress phenomenon, fracture of sheets must be approached from the standpoint of a 3-D formulation. For ductile sheets, fracture occurs after necking, and the stress state inside the neck is no longer plane. Also, blanking and cutting operations introduce through thickness shear stresses and strains. These strains may substantially reduce the remaining material ductility in the subsequent flanging operation, thus causing premature edge fracture. The present section gives a short summary of the macroscopic formulation of ductile fracture, which is necessary to understand subsequent sections of plane stress sheet metal fracture.

Fracture of a unit material volume is exclusively controlled by current stresses and their histories. In the principal coordinate system, this means that fracture is governed by a certain functional of the history of principal stresses $F\left(\sigma_1, \sigma_2, \sigma_3\right) = 0$. There is a unique relation between the principal stresses and three invariants of the stress tensor (see the notation). For ductile material obeying the hardening rule $\bar{\sigma} = f_h(\bar{\varepsilon})$, a small increment of the equivalent stress $d\bar{\sigma}$ correspond to a larger increment of equivalent strain $d\bar{\varepsilon}$. Therefore, the resolution of detecting fracture is better if the second invariant of the strain tensor $\left(\text{or } \bar{\varepsilon}\right)$ is used instead of the second invariant of the stress tensor $\left(\text{or } \bar{\sigma}\right)$. Furthermore, it is more convenient to work with the dimensionless first and third stress invariants, normalized by $\bar{\sigma}$. They are denoted respectively by η and ξ. In the most general case, the onset of fracture is determined by the functional $F(\bar{\varepsilon}, \eta, \xi) = 0$. For proportional loading fracture (as well as necking for that matter) is governed by the finite values of the process parameters. The functional F is then reduced to a function which is referred to as a "quasi-static fracture locus."

$$\bar{\varepsilon}_f = \hat{\varepsilon}_f(\eta, \xi) \tag{4}$$

Extensive research has been conducted at ICL to determine the shape of the function $\hat{\varepsilon}_f(\eta, \xi)$. According to the most recent study (Bai and Wierzbicki, 2006), the fracture locus is a monotonic function of the stress triaxiality parameter η, and an asymmetric function of the Lode angle parameter ξ, see Fig. 9. One convenient representation of Eq. (4) is given by

$$\begin{aligned}\bar{\varepsilon}_f = \hat{\varepsilon}_f\left(\eta, \xi\right) &= \left[\frac{1}{2}\left(\hat{\varepsilon}_f^{(+)} + \hat{\varepsilon}_f^{(-)}\right) - \hat{\varepsilon}_f^{(\circ)}\right]\xi^2 + \frac{1}{2}\left(\hat{\varepsilon}_f^{(+)} - \hat{\varepsilon}_f^{(-)}\right)\xi + \hat{\varepsilon}_f^{(\circ)} \\ &= \left[\frac{1}{2}\left(D_1 e^{-D_2\eta} + D_5 e^{-D_6\eta}\right) - D_3 e^{-D_4\eta}\right]\xi^2 + \frac{1}{2}\left(D_1 e^{-D_2\eta} - D_5 e^{-D_6\eta}\right)\xi + D_3 e^{-D_4\eta}\end{aligned} \tag{5}$$

where the physical meaning of the function $\hat{\varepsilon}_f^{(+)}$, $\hat{\varepsilon}_f^{(-)}$ and $\hat{\varepsilon}_f^{(\circ)}$ are clearly marked in Fig. 9. The present fracture locus is more general than the Johnson and Cook (1985) function that is independent of ξ as well as the Wilkins et. al. (1980), Kamoulakos et al (2003), and Xue (2007) concepts, which assumed symmetry with respect to ξ. The representation given by Eq. (5) involves six material constants.

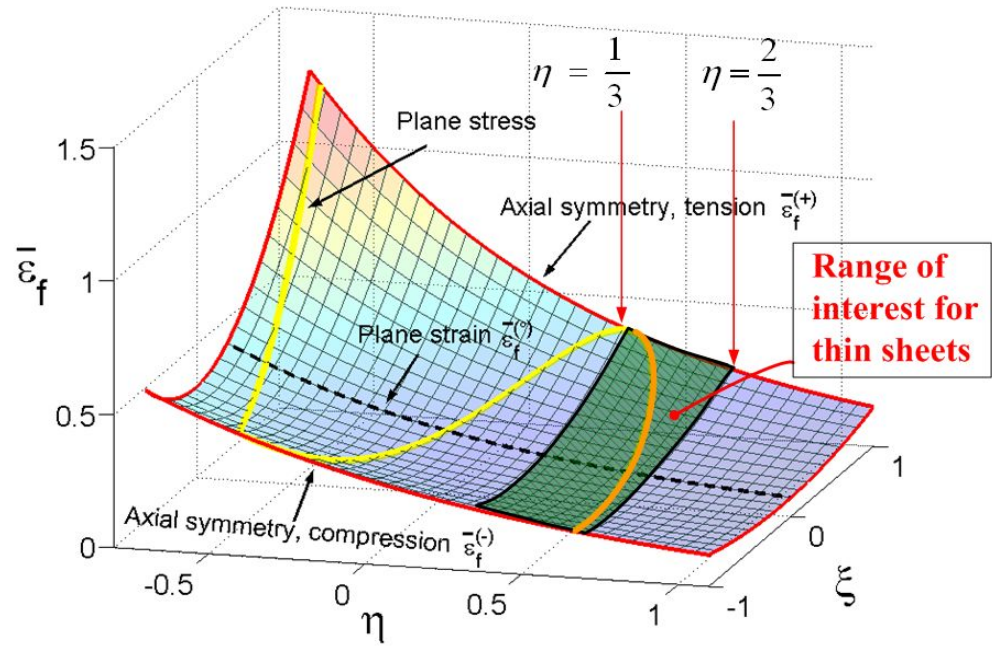


Fig. 9 A new asymmetric 3D fracture locus. The shaded area denotes the range of interest for thin sheets.

In the general case of non-proportional loading, the fracture functional takes the following form

$$D_c = \int_0^{\bar{\varepsilon}_f} g(\lambda) h(\mu) \frac{d\bar{\varepsilon}}{\hat{\varepsilon}_f(\eta,\xi)} = 1 \qquad (6)$$

where $\hat{\varepsilon}_f(\eta,\xi)$ is given by Eq. (4) or Eq. (5), and the parameter μ is a measure of a departure of the loading trajectory from a proportional path. A distinctive feature of the present formulation is that damage accumulates in a nonlinear way. Fracture occurs at $D_c = 1$. Specific forms of the function have been proposed (Bai, 2007). The above formulation is valid for arbitrary loading paths, including strain reversal.

Action items to Section 6

6.1 Extend the present isotropic fracture to the case of material anisotropy

6.2 Introduce the strain rate effect into fracture formulation (Expansion Project 2)

6.3 Further study of history effect and validation of Eq. (6).

7. Experimental Study of Multi-axial Fracture

The backbone of the fracture research at ICL has always been a strong experimental program. In addition to classical round bar and dog-bone specimens, as well as torsion and upsetting tests, two new experimental techniques have been developed: bi-axial butterfly specimen testing and flat grooved plane strain tests. The range of stress states attainable by various types of specimens and various testing methods is shown in Fig. 10. The combined tension/shear and compression/shear tests on butterfly specimens are most universal because they cover a broad range of stress triaxialities. A detailed FE analysis is needed to interpret the experimental results and to determine the history of stress parameters (η, ξ) and the magnitude of the equivalent strain to fracture. The necessary procedure has been described in several ICL publications (Bao and Wierzbicki 2004, Wierzbicki et al 2005).

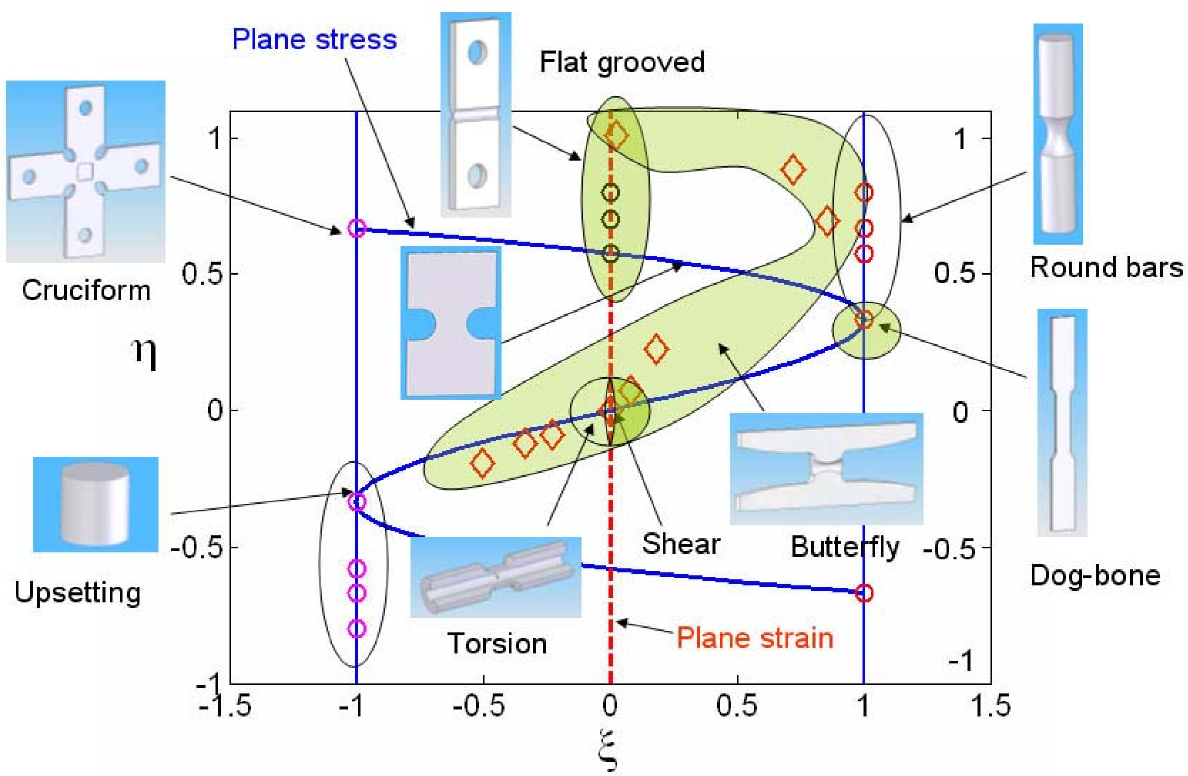


Fig. 10 Ranges of stress state attainable by various types of tests used at ICL. The shaded area corresponds to specimens extracted from thin sheets.

By contrast, all fracture parameters $\left(\bar{\varepsilon}_f, \eta, \xi\right)$ in flat grooved specimens can be determined directly from tests without making reference to numerical simulation (Bai et al, 2007). In the above paper, the classical Bridgman correction for round bar with notches has been extended to the case of plain strain. The initial stress triaxiality was shown to be a log function of the radius of the groove and the ligament thickness $\eta = \hat{\eta}(a,t)$. The significance of this result is that sheet metal ductility can be quickly assessed by flat groove tests because fracture in sheets is almost exclusively a plane strain fracture.

We were asked by one of our industrial partners to look into an applicability of the flat specimen with semicircular cutout for fracture calibration. The stress state determined by one of our FE simulation is shown in Fig. 10 by an arrow. Generally, the triaxiality parameter changes between uni-axial and plane strain ($1/3 \le \eta \le 1/\sqrt{3}$). However, the Lode angle parameter changes dramatically in the loading process. It can be concluded from this preliminary study that in general the range of stress state attainable by MIT butterfly specimen and the flat specimen with cutouts are different. For a certain geometry of the flat specimen, both ranges may overlap.

A minimum of six representative tests are needed to determine six unknown parameters of the 3-D fracture locus, given by Eq. (5). The equi-biaxial tests by means of cruciforms are not generally necessary because other experimental techniques could be sufficient to calibrate the 3-D fracture locus. However, there are fewer options for fracture testing of sheets, and the cruciforms test could provide a critical data point. Here, the unique experience of ICL in designing and testing double curvature butterfly specimens will be used. The gauge section of the cruciform will have a shallow semi-spherical profile, thus ensuring that fracture always occurs at the center rather than at the edge. In this way, points on the fracture locus at and around the equi-biaxial tension $\eta = 2/3$ will become available for calibration.

Action items on Section 7

7.1 Select a minimum number and most convenient type of fracture tests to calibrate the fracture locus

7.2 Design a comprehensive series of tests to assess the effect of loading history on fracture and determine the parameters of the nonlinear damage accumulation rule (Eq. 5).

8. Specification of the Fracture Model to Sheets

Plane stress represents an important condition in sheet metal forming and crash analysis. Necking of sheets as well as fracture of more brittle materials (such as AHSS) occurs under plane stress. Fracture following a shallow neck also develops under nearly plane stress. The range of stress triaxiality that can be achieved in thin sheets usually falls between the uniaxial and equi-biaxial tension ($1/3 \le \eta \le 2/3$). This range is indicated in Fig. 9 by a shaded area, which is then enlarged in Figs. 11 and 12. It was shown by Wierzbicki and Xue (2006) that the requirements of plane stress $\sigma_{33} = 0$ impose the following relationship between the two stress parameters

$$\xi = -\frac{27}{2}\eta\left(\eta^2 - \frac{1}{3}\right). \qquad (7)$$

The above third order parabola is depicted in Fig. 9 by a yellow (in color print) line resembling a trajectory of a “snowboarder” in a half pipe. The plane stress fracture envelope then reduces from the 3-D surface to the 2-D curve. Furthermore, the independent variables could be either η or ξ or the strain rate ratio α. The projections of the 3-D fracture locus on the $\left(\bar{\varepsilon}_f, \xi\right)$ and $\left(\bar{\varepsilon}_f, \eta\right)$ planes are shown respectively in Figs. 13 and 14. The present result points out difficulties in describing fracture of sheets by means of the original GTN model (generalized Gurson model), Feucht et al (2006). Such a model predicts a monotonic decrease of material ductility with the triaxiality parameter η, while in actuality there may be a local minimum in the function $\hat{\varepsilon}_f\left(\eta\right)$. This shortcoming has been recognized by the BMW/MatFEM team who introduces a two-term function which describes a non-monotonic dependence of $\bar{\varepsilon}_f$ on η (Dell et al., 2001). Finally, using the J2 plasticity model, the line representing the onset of fracture can be mapped into the plane of principal strains, Fig. 15. Such a line on the $\left(\varepsilon_1, \varepsilon_2\right)$ plane is often referred to as a Fracture Forming Limit Diagram (FFLD).

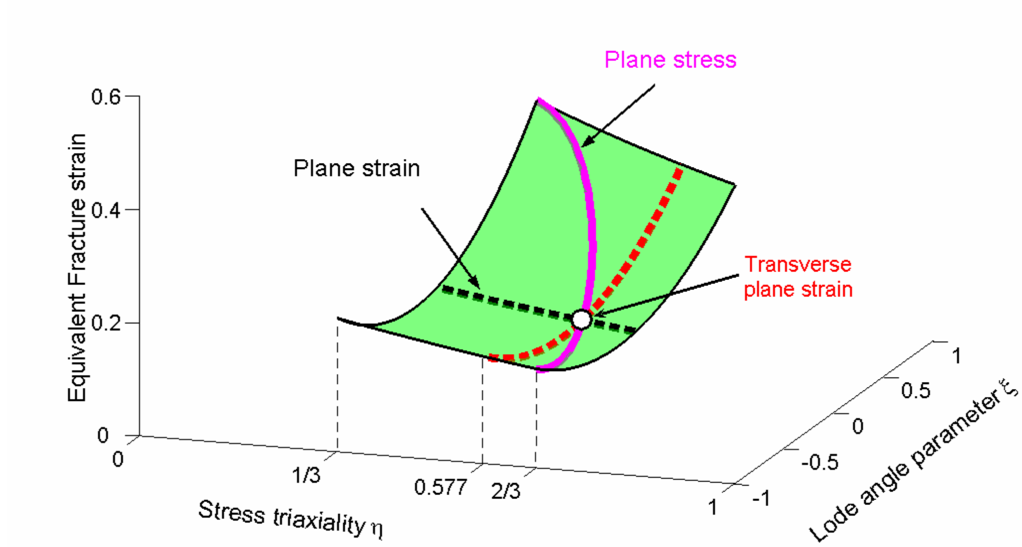


Fig 11. A 3D fracture locus for thin sheets. The point with coordinates $\eta = 1/\sqrt{3} = 0.577$ *and* $\xi = 0$ *corresponds to transverse plane strain.*

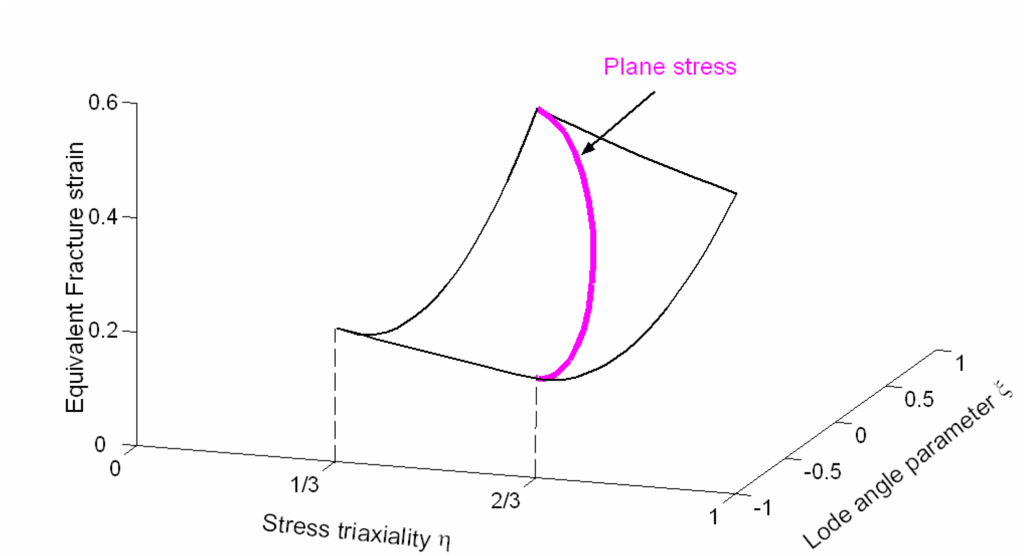


Fig 12. When fracture occurs before necking, it is a plane stress fracture. The fracture envelope reduces then to a segment of a spiral curve.

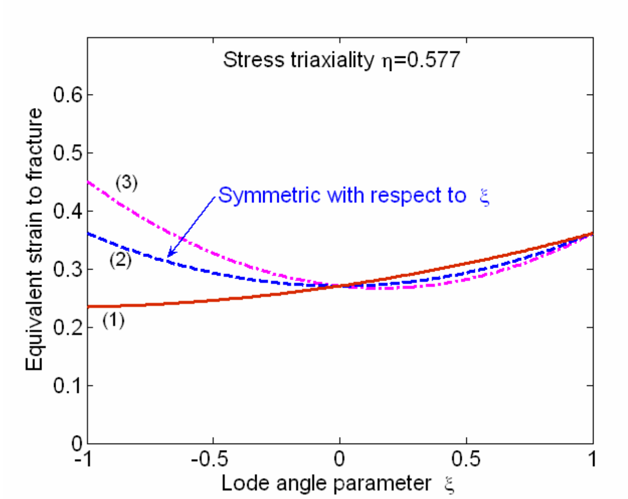


Fig 13. A conceptual graph showing three possible cases of the dependence of material ductility (equivalent strain to fracture) on Lode angle parameter.

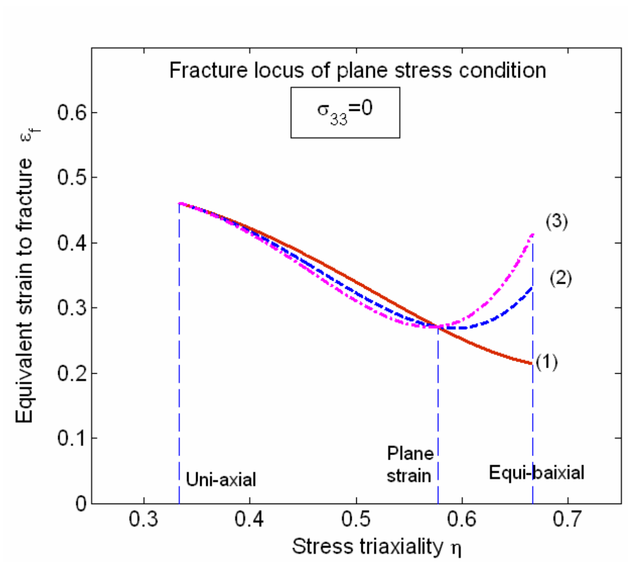


Fig. 14 The projection of a 3D plane stress fracture locus into the plane of the equivalent strain to fracture and the stress triaxiality.

It should be recognized from the above analysis that, in the plane stress condition, dependence of material ductility on the Lode angle parameter, ξ, is much stronger than on the triaxiality parameter, η. A family of functions representing the dependence of $\bar{\varepsilon}_f$ on the parameter ξ is shown in Fig. 13. The curve labeled "2" corresponds to the symmetric fracture locus (Wilkins et al. 1980; Xue 2007). The same curves are then mapped into the $(\bar{\varepsilon}_f, \eta)$ plane, Fig. 14 and the $(\varepsilon_1, \varepsilon_2)$ plane, Fig. 15. The experimentally measured fracture points collected from various sources by Lee (2005) and the straight line FFLD fit are reproduced in Fig. 16.

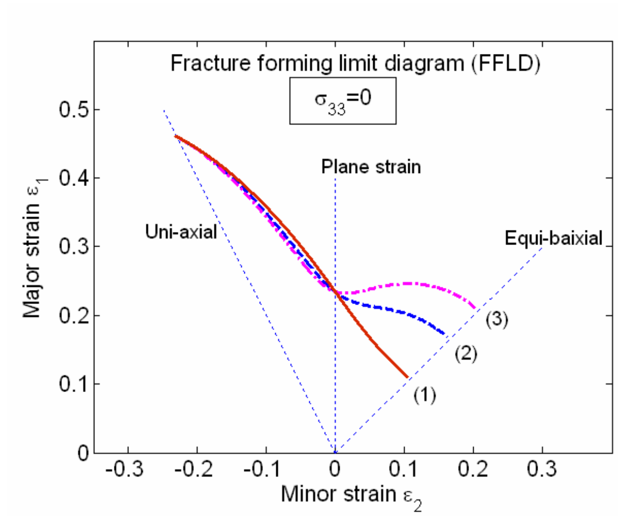


Fig. 15 Transformation of the 2D fracture locus shown in Fig. 14 to the space of principal strains. The curve (2) corresponds to the symmetric fracture locus, while the other curves show the effect of asymmetry with respect to the coordinate ξ.

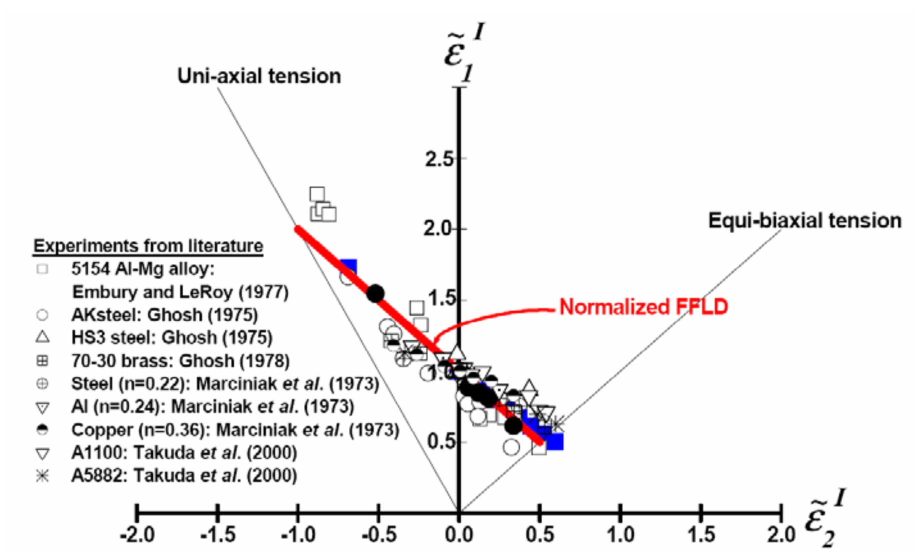


Fig. 16 Normalized FFLDs for eleven different materials in classical punch indentation problems. Note that all curves are made to pass through the same point $(\tilde{\varepsilon}_2^I = 0, \tilde{\varepsilon}_1^I = 1.0)$.

It is seen that the full line, labeled "1" corresponds most closely to the empirical FFLD. This observation provides indirect proof that the 3-D fracture locus must be an asymmetric function of the Lode angle parameter. If fracture occurs soon after necking, then the stress state is strictly not plane stress, but the 2D representation could offer a good engineering approximation. At the same time, sheet fracture that develops after deep necks must be treated by a full 3-D formulation.

Action items on Section 8

8.1 Establish the range of validity of the plane stress fracture formulation for several AHSS sheets

8.2 Determine errors in using 2-D fracture formulation for ductile-to-brittle sheets

9. Fracture Prediction using Butterfly Specimens vs. Flat Notched Specimens

The flat specimens with two semi-circular cutouts are often used by the industry to study fracture of sheets. Such specimens are easy to manufacture and easy to test. Depending on the radius of cutout, fracture could initiate at the center of the specimen (Point A) or at the edge (Point C), see Fig. 17. Also, different stress states are generated for different radii. At the request of one Consortium member, ICL conducted a comparative study of usefulness of both types of specimens for fracture calibration of sheets.

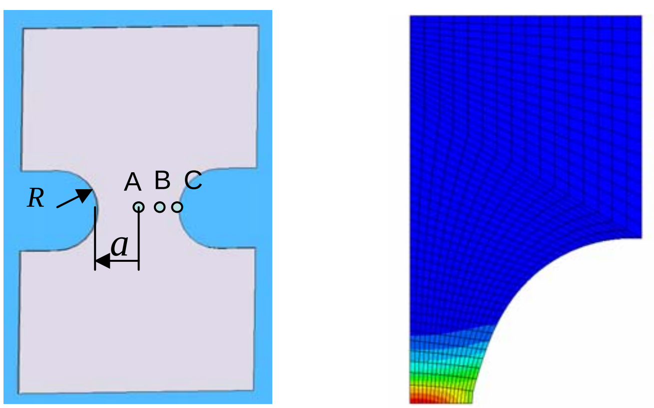


Fig. 17 Flat notched specimens (left) and the plane stress FE quarter model showing contour plot of equivalent plastic strain.

Two plane stress FE models were developed for 1mm thick sheet with a ligament width $a = 8mm$ and two cut-off radii of $R = 8mm$ and $R = 16mm$. The evolution of the stress tiaxiality and the Lode angle parameter at the three chosen locations is shown in Fig. 18. Note that traditional flat specimens can only be loaded in tension. The range of stress triaxiality attainable by the flat notched specimens is confined to a narrow range of $1/3 \le \eta \le 1/\sqrt{3}$ and $0 \le \xi \le 1$ (see shaded area in Fig. 19). One can observe that the triaxiality parameter varies little in the loading process. At the same time, the Lode angle parameter undergoes large changes through the whole deformation process at the potential fracture location. The average values of η and ξ calculated for the two specimen geometries are indicted by triangles in Fig. 19.

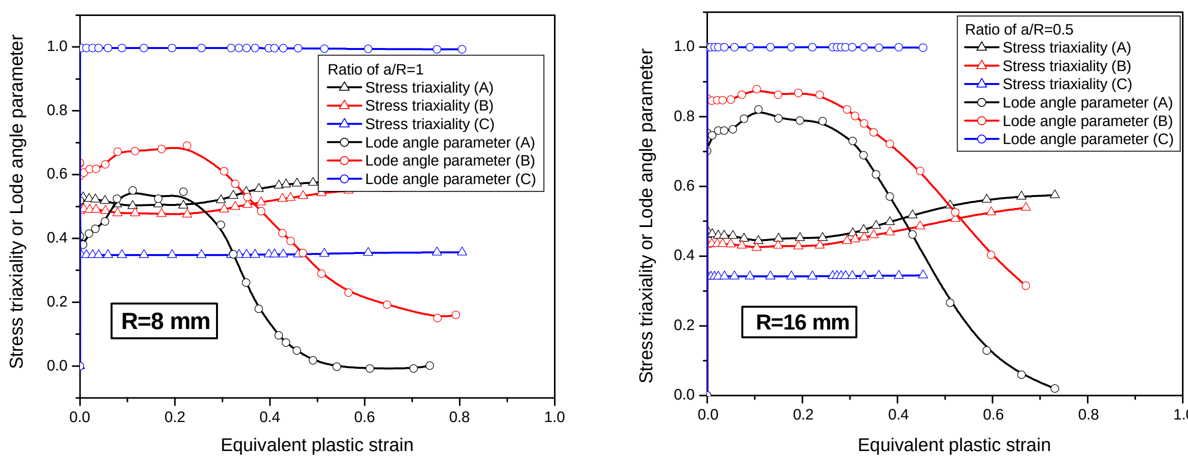


Fig. 18 History of stress triaxiality and Lode angle parameter at point A, B, C at minimal cross-section for two cutout radii.

ICL has performed numerous simulations of butterfly specimens. A typical variation of stress triaxiality and the Lode angle parameter are shown in Fig. 20. Note that the relative contribution of tension and shear is defined by the loading angle. The following conclusions can be drawn from the above comparison:

1. The range of the stress triaxiality parameter covered by our butterfly specimen is from -1/3 to 1 (we have results of simulations of other cases).

2. The stress triaxiality for the butterfly specimen is almost constant all the way to fracture.

3. The Lode angle parameter for the same specimen after an initial transient phase settles on a more or less constant value.

4. The butterfly specimen is able to cover almost the entire range of stress state.

It can be concluded that both types of specimens have their advantages and disadvantages. The Lode angle parameter of butterfly specimen stays almost constant, while it undergoes large changes in flat specimens. Also, the butterfly specimen can provide much more flexibility by covering a wider range of stress state.

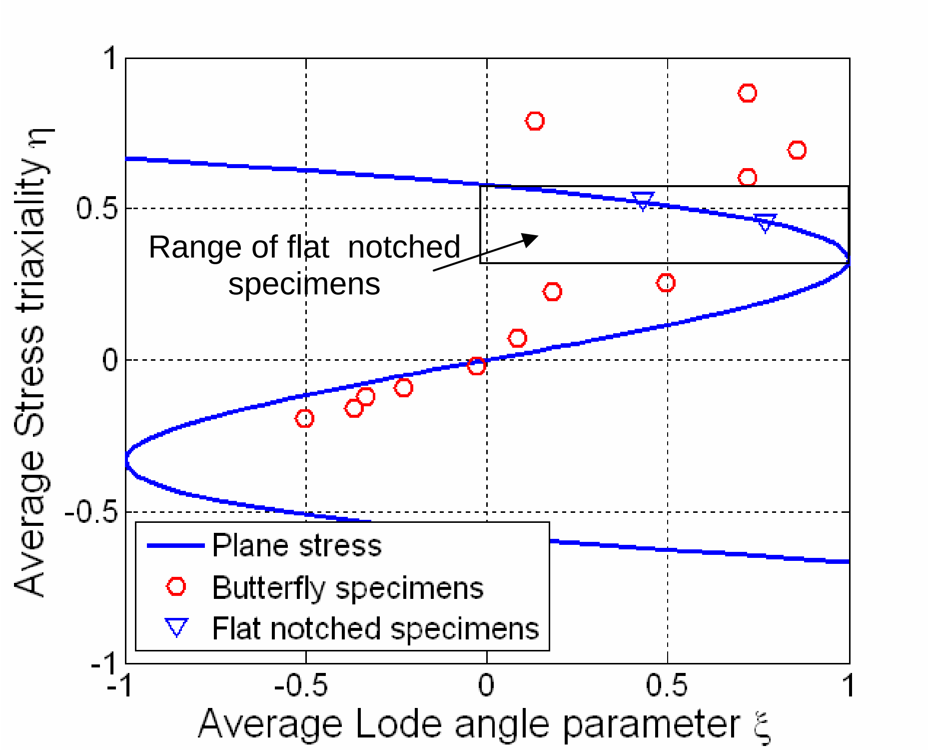


Fig. 19 Range of stress state parameter attainable by the flat specimens with cutouts and the butterfly specimens.

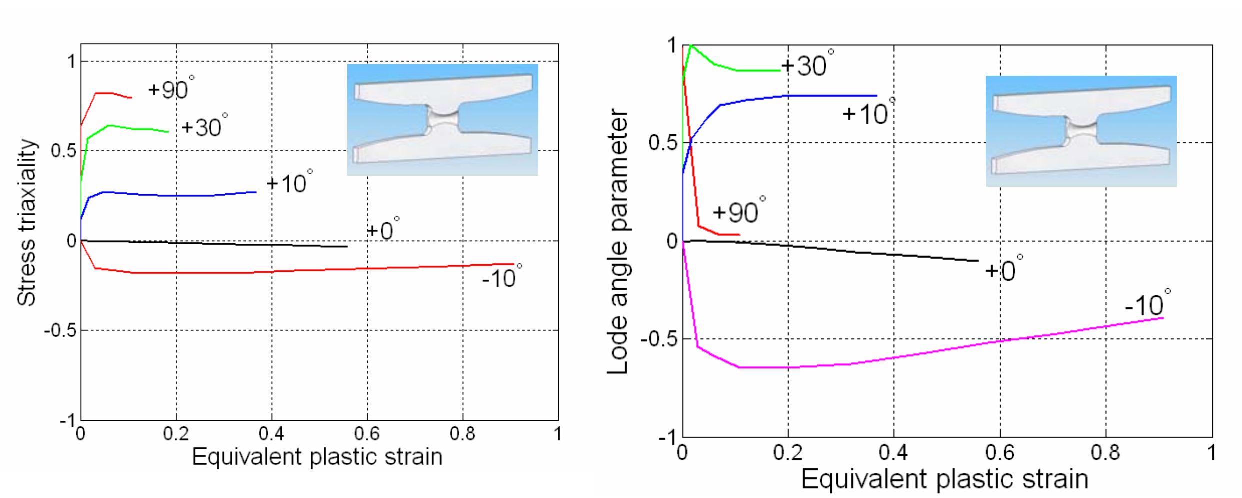


Fig. 20 Evolution of stress state parameters for a butterfly specimens in the loading processes all way to fracture.

Action items on Section 9

9.1 Include flat notched specimens into a standard testing program for all AHSS sheets.

9.2 Develop automated surface strain measurement procedure for flat notched specimen.

9.3 Develop Bridgman-like formula for initial stress triaxiality and Lode angle parameters.

10. Experimental investigation of the effect of microstructure on fracture

It is envisaged to perform a series of in-situ tensile tests on selected samples of the sheet material which will be studied within the consortium. Small tensile specimens (of an estimated cross section of 3x10mm) will be extracted from the sheet metal and prepared for in-situ testing. The size of the specimen is restricted by the maximal 5kN capacity of the loading frame. The complex preparation procedure comprises electrolytic and chemical polishing. It is noted that a special polishing procedure needs to be developed in accordance with the microstructural composition of each specimen. The polished specimens will be tested inside the chamber of a scanning electron microscope (SEM, available at Dr. Mohr's lab in Paris), see Fig. 21.

Fig. 21 Portable SEM at Ecole Polytechnique in France to be used in conjunction with a tensile loading frame for in-situ testing.

High resolution micrographs will be taken at various stages of deformation all the way up to fracture. The overall objective is to identify the governing deformation and fracture mechanisms at the microstructural level. The ICL team will make use of this information to develop macroscopic fracture criteria that are consistent with the microstructural deformation mechanisms.

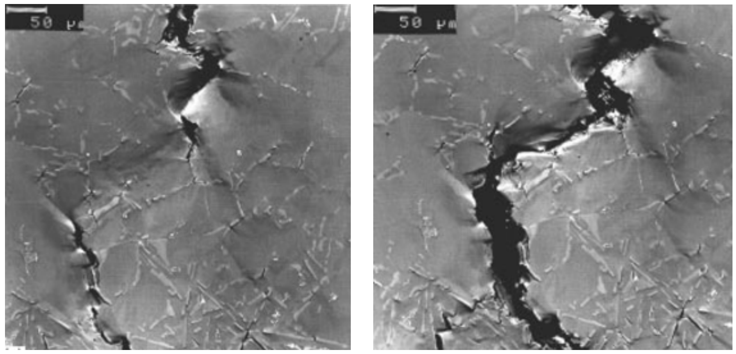


Fig. 22. Example micrographs taken during the in-situ testing of an aluminum alloy (after Mishnaevsky et al., 1999)

As an alternative to in-situ testing, it is proposed to study the microstructural deformation and fracture mechanisms using conventional optical microscopy. The ICL team recently purchased a high quality compound ZEISS optical microscope with an integrated digital camera. The magnification of this microscope is up to 100X. The idea is to interrupt selected experiments (including the multi-axial tests of butterfly specimens) and to polish a characteristic cross-section of the specimen for optical microscopy. Furthermore, SEM pictures may be taken post-mortem of the fractured specimen surfaces.

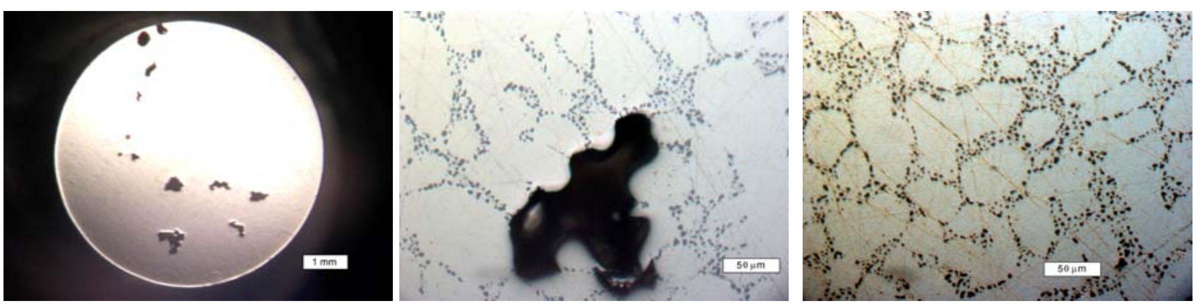


Fig. 23. Examples of pictures taken from cast aluminum specimens before fracture with various magnifications (Joint project between ICL and Honda)

It should be understood that the study of the evolution of microstructure will be complementary to our main program on macroscopic testing and modeling. At the same time, the ICL team will not be able to perform a full micro evaluation of all steels. In particular, the present study will not tell what particular type of microstructure is responsible for the strength and ductility of a given steel.

Action items on Section 10

10.1 Perform ZEISS microscopic study of selected specimens from the steels recommended by the Steering Committee.

10.2 Carefully chose candidate sheets for SEM testing. The cost of such tests will be of an order of $5,000 per material. This work will be subcontracted to the Solid Mechanics Lab at Ecole Polytechnique in Paris.

11. Status of the AHSS Consortium

Members of AHSS Consortium (as of February 19, 2007): The following companies have already subscribed to the Consortium:

American Iron and Steel Institute

Arcelor/Mittal

Hyundai

Nissan

POSCO

ThyssenKrupp

Volkswagen/Audi

It was anticipated that all of the administrative and legal matters related to signing the Consortium Agreement at MIT would be completed by the end of December 2006. However, the process is still underway. The MIT Office of Sponsored Research has already received fully executed agreements from three companies, and the remaining four companies are expected to send the signed copies very soon. The Principal Investigator would like to thank all current members of the Consortium for their encouragement and support.

Related projects: ICL has recently completed a joint study with Honda R&D on fracture of aluminum castings. This study involved the testing of dozens of dog bone specimens. A great deal of knowledge has been acquired in that project. In a separate development, Ferrari has just signed a contract with ICL for a similar study and is providing support comparable to an annual standard Consortium participation fee. It was found that the scatter of experimental data of cast aluminum alloys is larger than similar data for various steels. The Principal Investigator would like to treat Ferrari as an Associate Member of the Consortium and provide this company with some data generated in the first year of the project. A dozen other companies in the U.S., Europe, and Japan are still reviewing the proposal. The initial goal of ICL was to attract 10 Consortium members in order to reach the estimated half a million annual budget needed to conduct the promised research. The consortium has now reached a critical mass, and it is very close to meeting the above goal.

Pilot project in cooperation with Consortium members. In anticipation of industrial funding, ICL initiated a pilot study on plasticity and fracture of two types of AHSS together with Volkswagen and ThyssenKrupp in August 2006. These projects are almost completed now, and they provided ICL with a wealth of knowledge on the type of steel sheets of interest to the Consortium. At the same time, a number of present and potential Consortium members sent us very valuable comments on our original proposal, which has helped us to put together the present addendum. The continuous correspondence with almost all current supporting institutions gave us a good idea about industrial needs and also about the way in which fracture should be treated in the industrial environment. Again, the PI would like to thank those who contributed to that exchange of thoughts and ideas.

Leverage funding. ICL is presently engaged in several research projects funded by federal institutions in the U.S. All those projects are related directly or indirectly to fracture, and it is clear that the present Consortium members are greatly benefiting from those additional sources of funding. The results of the government sponsored research are published as ICL technical reports, which later appear as journal publications. Also, the annual Fracture Workshop at MIT constitutes a forum for an exchange of information between industry, academia, and federal sponsoring institutions.

Program on the exchange of personnel and industrial internships. As a part of its obligation to the Consortium members, ICL is arranging for visiting engineer positions at MIT for our industrial sponsors. In the fall of 2006, ICL hosted a BMW engineer who worked with us on strength and fracture of aluminum weldments. Starting in April 2007, ICL will host a one-year stay of Dr. Kim of POSCO. He will conduct research on the topics of interest to the Consortium. Later in the year, Ferrari is planning to send a representative to ICL for a shorter 1-2 month visit. In exchange, Ferrari invited Mr. Carey Walters, who is a third-year doctoral student at ICL, for a summer internship in Ferrari's factory in Maranello. Similar internships were organized earlier for our students at BMW R&D Center in Munich. Other member companies are encouraged to arrange for a similar exchange of personnel. This should be planned ahead of time since ICL can host a maximum of two visitors at a time.

12. Proposed Sub-tasks for the First Year

(i) **Choice of representative AHSS Sheets.** It is proposed that three representative materials for initial plasticity and fracture testing are selected:

TRIP

DUAL PHASE

STAINLESS STEEL

During the first Steering Committee meeting, it was requested that MIT develop procedures to assess fracture performance of AHSS rather than establish a data bank for many sheets. The decision on the choice of a particular material will be greatly facilitated if steel companies team up with those automotive companies to which they are supplying sheets. For example, POSCO could decide jointly with Hyundai on their preference. Volkswagen is proposing to use ThyssenKrupp DP steel. The U.S. Consortium members should come up with their material of choice. On the request of Arcelor, ICL has agreed to include a stainless steel into the pool. In this way, the MIT project could attract more attention of French automotive industry and the NGV program on stainless steel sheets, directed by Roland Gustafsson at Volvo. At this point, the PI is waiting for specific recommendations from the Consortium members. One important restriction is that the selected material should exhibit only a small amount of in-plane anisotropy because the extension of the fracture theory to anisotropic sheets will be the subject of 2^{nd} and 3^{rd} year research.

(ii) **Cut and machine various types** of specimens from sheets. The number of specimens of each type should be sufficient to determine the amount of anisotropy and inhomogeneity within a sheet. **Perform all types of tests** shown schematically in Fig. 10 and analyze the loads and optical measurements.

(iii) **Perform numerical simulation of the tests** using plasticity models obtained from measurements in sub-task (ii). The purpose of tests and FE calculations will be to get a fundamental understanding of two issues outlined in sub-tasks (iv) and (v).

(iv) **Determine of a minimum number of tests for fracture characterization.** The general 3-D fracture locus, Eq. (5), involves six free parameters. Therefore, at least six independent tests should be run to determine the locus and another six to further verify it. It is not unusual for the industry to run 12 or more experiments in order to establish the fracture forming curve. The difficulty within the Consortium is incompatibility of the testing equipment in the industrial labs with the unique MIT dual actuator testing rig. Therefore, a great deal of attention will be given to establishing a minimum number of tests and to choosing representative types of tests needed to predict fracture of sheets in other loading situations. For example, the plane stress fracture is defined as a curve and perhaps only three test points will be sufficient to characterize fracture. However, this would necessitate performing bulging or cruciform equi-biaxial tests.

Another fundamental issue to be resolved is the relationship between measurements and FE simulation. The analysis of MIT butterfly specimens, based on the inverse methods, requires performing detailed calculations for each test. This is inconvenient and impractical for industrial use. **A detailed comparison will be made between the fracture locus determined from the inverse method and direct surface strain measurements combined with plane stress transformation from one space to the other.**

(v) **Effect of loading paths on necking and fracture.** The effect of loading history on necking will be studied by analyzing data available from the open literature. At the same time, the Consortium members are encouraged to provide ICL with their own data to further validate the very promising theory outlined in Section 5. History effect on fracture of sheets will be studied experimentally and theoretically to discover trends and determine parameters and functions in the damage evolution law, Eq. (6).

(vi) Document results of first year research in ICL Technical Reports to be presented at **the 4th MIT Workshop on Experimental and Computational Fracture Mechanics, which will be organized together with the 2nd meeting of the Steering Committee**. Both events will be held in the first week of October 2007. The PI could organize an interim meeting in the spring of 2007 at MIT if this should turn out to be absolutely necessary. In making their respective decision, the Steering Committee should take into account that many of the Consortium members come from either Europe or the Far East, and a trip to Boston could bring unnecessary time constraints for technical representatives of Consortium companies. The PI's suggestion is to come up with a consensus on the choice of steels and the first year research tasks through correspondence.

13. Long-range Program on Ductile Fracture at ICL

The present funding received from the Office of Naval Research, NSF/Sandia alliance, and Army Research Office has been supporting a long-range research program on fracture conducted at ICL for six years now. A list of completed Masters and Ph.D. theses in the area of fracture is listed below. One can see that each Ph.D. thesis is looking at one specific topic in fracture, and with the passage of time, research conducted by ICL is becoming more and more relevant to the steel and automotive industries. At the same time, all Masters theses that have been completed since 2000 have focused on more specific projects dealing with theoretical formulation, computational mechanics, and component testing.

Ph.D Theses in the Area of Fracture Completed at ICL

1. Osamu Muragishi, Premature Cleavage of Ship Plating under Reversed Bending, September 2000
2. Yingbin Bao, Prediction of Ductile Crack Formation in Uncracked Bodies, July 2003
3. Young-woong Lee, Fracture Prediction in Metal Sheets, December 2004
4. Xiaoqing Teng, High Velocity Impact Fracture, December 2004
5. Li Zheng, Fracture of Welded Aluminum Thin-Walled Structures, June 2005
6. Liang Xue, Ductile Fracture Modeling – Theory, Experimental Investigation and Numerical Verification, February 2007
7. Yuanli Bai, Effect of Loading History on Necking and Fracture, (expected completion 2008)
8. Carey Walters, Effect of Strain Rate on Fracture, (expected completion 2008)
9. Allison Beese, Ductile Fracture of Anisotropic Sheets, (expected completion 2010)

Master’s Theses in the Area of Fracture Completed at ICL

1. Robert Bebermeyer, Effect of Geometrical Misfit on Ductile Fracture of Butt Welds, May 2002.
2. Jeffrey Woertz, Quasi-Static Tearing Test of Metal Plating, August 2002.

3. Wolfgang Guenthner, Finite Element Modeling Techniques for the Failure Prediction of Aluminum Overlap Fillet Welds, March, 2004.
4. Mike Roach, Experimental Investigation of Tearing Fracture in Sheets under Quasi-static Loading, May, 2004.
5. Hagbart Alsos, A Comparative Study of Shell Element Deletion and Element Split, June, 2004.
6. Christian Roth, Characterization of Plastic and Fracture Properties of Aluminum Overlap Fillet Welds, April 2006.
7. Martin Oswald, A new experimental technique for the mulit-axial testing of advanced high strength steel sheets, February 2007.

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