Modeling, Testing and Calibration of Ductile Fracture
Lab facility

- **Custom-made biaxial testing machine**
  - Hydraulic
    - Vertical: 50 kN
    - Horizontal: 25 kN

- **MTS universal testing machine**
  - Electro-mechanical
    - 200 kN, 10 kN

- **SHPB systems in France**
  - Max. vel: 20 m/s

- **Three-point bending of a hat assembly**

- **INSTRON 9250HV**
  - 2 kN, 100 N

- **INSTRON 5944**

Vic 2D and Vic 3D software, two digital cameras, high speed cameras, Abaqus, LS-DYNA, Hypermesh, workstations

Industrial Fracture Consortium
Plasticity
Conventional metal plasticity

* Yield surface
- Von Mises yield criterion: \( f(\sigma, \bar{\varepsilon}_p) = \sqrt{\frac{3}{2}} \sigma' \cdot \sigma' - k(\bar{\varepsilon}_p) \leq 0 \)

* Flow rule
- Associated flow rule: \( d\bar{\varepsilon}_p = d\bar{\varepsilon}_p \frac{\partial f}{\partial \sigma} \)
  (Plastic potential=yield function)

* Hardening law
- Swift law: \( k(\bar{\varepsilon}_p) = A(\varepsilon_0 + \bar{\varepsilon}_p)^n \)
- Voce law: \( k(\bar{\varepsilon}_p) = k_0 + Q(1 - \exp(-\beta\bar{\varepsilon}_p)) \)
- Extrapolation up to a large strain
- Isotropic hardening
Anisotropy and non-associated flow rule*

* Different anisotropy between yield stress and Lankford ratio (r-value), \( r_\alpha = \frac{d\varepsilon_w}{d\varepsilon}\).  

Non-associated flow rule

\[
f(\sigma, \overline{\varepsilon}_p) = \sqrt{(P\sigma)\cdot \sigma - k(\overline{\varepsilon}_p)} \leq 0
\]

\[
g(\sigma) = \sqrt{(G\sigma)\cdot \sigma} \Rightarrow d\varepsilon_p = d\lambda \frac{\partial g}{\partial \sigma}
\]

Associated flow rule

\[
f(\sigma, \overline{\varepsilon}_p) = \sqrt{(P\sigma)\cdot \sigma - k(\overline{\varepsilon}_p)} \leq 0
\]

\[
d\varepsilon_p = d\overline{\varepsilon}_p \frac{\partial f}{\partial \sigma}
\]

Validation of non-associated flow rule

* Validation with DP590 and TRIP780

** DP 590 **

** TRIP 780 **
Hardening curve after necking

* Fracture occurs after a significant amount of necking
  - Need for the stress–strain curve after necking
  - Non-uniform deformation in the gauge section after necking
    → hard to obtain hardening curve after necking experimentally

* ‘Inverse method’ using finite element simulation
  - Use of so-called ‘Notch tension test’ with R20 mm
  - Adjustment to the stress–strain curve in the post necking area until force–displacement curve from simulation agrees well with the one from the experiment
  - It is unavoidable to repeat simulation

* Why ‘Notch tension test’?
  - Its geometry always generates necking exactly in the middle of specimen because of the minimum gauge width there
  - Robustness and repeatability of tests
Procedure for inverse method

\[ k_{\text{Swift}}(\varepsilon_p) = A(\varepsilon_0 + \varepsilon_p)^n \]
\[ k_{\text{Voce}}(\varepsilon_p) = k_0 + Q(1 - \exp(-\beta \varepsilon_p)) \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1501</td>
</tr>
<tr>
<td>(\varepsilon_0)</td>
<td>0.0002</td>
</tr>
<tr>
<td>n</td>
<td>0.123</td>
</tr>
<tr>
<td>(k_0)</td>
<td>715</td>
</tr>
<tr>
<td>Q</td>
<td>400</td>
</tr>
<tr>
<td>(\beta)</td>
<td>37.9</td>
</tr>
</tbody>
</table>

Mixed Swift–Voce law

\[ k(\varepsilon_p) = \alpha k_{\text{Swift}}(\varepsilon_p) + (1 - \alpha) k_{\text{Voce}}(\varepsilon_p) \]

\(\alpha\) : weighting factor
Strain rate dependency*

* Rate dependent hardening model

\[ k[\dot{\varepsilon}_p, \dot{\varepsilon}_p, T] = k_\varepsilon[\dot{\varepsilon}_p] k_\eta[\dot{\varepsilon}_p] k_r[T] \]
\[ k(\dot{\varepsilon}_p) = \alpha k_{Swift}(\dot{\varepsilon}_p) + (1 - \alpha)k_{Voce}(\dot{\varepsilon}_p) \]
\[ k_\varepsilon[\dot{\varepsilon}_p] = \begin{cases} 1 & \text{for } \dot{\varepsilon}_p < \dot{\varepsilon}_0 \\ 1 + C \ln \left( \frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0} \right) & \text{for } \dot{\varepsilon}_p \geq \dot{\varepsilon}_0 \end{cases} \]
\[ k_r[T] = \begin{cases} 1 & \text{for } T < T_r \\ 1 - \left( \frac{T - T_r}{T_m - T_r} \right)^m & \text{for } T_r \leq T \leq T_m \end{cases} \]

\[ dT = w[\dot{\varepsilon}_p] \frac{\eta_k}{\rho C_p} \bar{\sigma} d\bar{\varepsilon}_p \quad (\eta_k : \text{Taylor-Quinney coefficient}) \]
\[ w[\dot{\varepsilon}_p] = \begin{cases} 0 & \text{for } \dot{\varepsilon}_p < \dot{\varepsilon}_i \\ \left( \frac{\dot{\varepsilon}_p - \dot{\varepsilon}_i}{2(\dot{\varepsilon}_a - \dot{\varepsilon}_i)} \right)^2 & \text{for } \dot{\varepsilon}_i \leq \dot{\varepsilon}_p \leq \dot{\varepsilon}_a \\ 1 & \text{for } \dot{\varepsilon}_a < \dot{\varepsilon}_p \end{cases} \]

Dependence of weighting factor on strain rate

Plasticity model for reverse loading (MYU)

* Plasticity model for reverse loading *(by Stephane Marcadet)*
- Compression followed by tension or tension followed by compression
- New behaviors should be incorporated into plasticity model
  - Bauschinger effect
  - Work hardening stagnation
  - Permanent softening
- Kinematic hardening is used instead of isotropic hardening
- Four parameters \([\gamma, \eta, \phi, \beta]\)
- Calibration of the model based on uniaxial tension followed by compression
- Effect of four parameters

**Graphs**

- Bauschinger transition
- Stress level at stagnation
- Duration of stagnation
- Slope at post stagnation

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Novel experiment technique for plasticity
Fracture model
* Formulation of stress triaxiality $\eta$ and Lode angle $\bar{\theta}$

$\overline{OP} = (\sigma_1, \sigma_2, \sigma_3)$ Load vector in Cartesian coord.

$\overline{OO'} = (\sigma_{m}, \sigma_{m}, \sigma_{m})$ Hydrostatic pressure vector

$\overline{O'P} = (\sigma_1 - \sigma_{m}, \sigma_2 - \sigma_{m}, \sigma_3 - \sigma_{m}) = (s_1, s_2, s_3)$

$|\overline{O'P}| = \sqrt{s_1^2 + s_2^2 + s_3^2} = \sqrt{2J_2} = \sqrt{\frac{2}{3} \sigma_{Mises}}$

$\theta = \frac{1}{3} \cos^{-1} \left( \frac{27 \det[s_{ij}],}{2 \sigma_{Mises}^3} \right)$

$\therefore$ Cartesian coord. $\rightarrow$ Cylindrical coord. (Haigh-Westergaard coord.)

$\eta = \frac{-p}{\sigma} = \frac{\sigma_{m}}{\sigma} = \frac{\sqrt{2}}{3} \cot \varphi$, ($\eta$ : Stress triaxiality)

$\bar{\theta} = 1 - \frac{6\theta}{\pi}$, ($\bar{\theta}$ : Normalized Lode angle, $0^\circ \leq \theta \leq 60^\circ$)

($\eta, \bar{\theta}$) : Representing specific loading path
History of fracture models at ICL

Three-branch fracture model\(^{(1)}\)

Modified Mohr–Coulomb model (2010)\(^{(2)}\)

Extended Mohr–Coulomb model (2014)\(^{(3)}, (4)\)

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\[ \hat{\varepsilon}_p' = \left( \frac{A}{c_1} \right) c_3 \left( \frac{1 + c_2}{c_1} \cos \left( \frac{\bar{\theta}_\pi}{6} \right) + c_3 \right) \left( \frac{1}{3} \sin \left( \frac{\bar{\theta}_\pi}{6} \right) \right) \]

\[ D = \int_0^{\bar{\varepsilon}_p'} \frac{d\bar{\varepsilon}_p}{\bar{\varepsilon}_p' (\eta, \bar{\theta})} \]

\[ \bar{\varepsilon}_p' (\eta, \bar{\theta}) = b(1+c)^n \left[ \frac{1}{2} \left( (f_1-f_2)^{n_2} + (f_2-f_3)^{n_2} + (f_3-f_1)^{n_2} \right) \right]^{\frac{1}{n}} + c(2\eta + f_1 + f_3) ]^{\frac{1}{n}} \]

\[ b = \begin{cases} \frac{b_0}{1 + \gamma \ln \left( \frac{\hat{\varepsilon}_p'}{\hat{\varepsilon}_0} \right)} & \text{for } \hat{\varepsilon}_p' < \hat{\varepsilon}_0 \\ b_0 & \text{for } \hat{\varepsilon}_p' \geq \hat{\varepsilon}_0 \end{cases} \]

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\(^{(3)}\) Mohr and Marcadet, “Hosford–Coulomb model for predicting the onset of ductile fracture at low stress triaxialities”, submitted for the publication.

Calibration of fracture model

* Fracture tests
5 types of tests characterizing a different combination of $\bar{\theta}$ and $\eta$
- NTR 20: notch tension with R20
- NTR 6: notch tension with R6
- CHD4: central hole with D4
- Butterfly: tension & shear (plane strain tension & pure shear)
- Punch: biaxial tension
Identification of loading path

Experiment

Displacement measured using DIC (VIC2D)

Two red points are symmetric

FE analysis

1/8 model C3D8R

Comparison

Force [kN] vs. Engineering Strain

Optimization of fracture parameters

Extract loading path

Fracture initiation point
Extreme bending of riser with fracture