

# Price-Cap Regulation for Transmission: Objectives and Tariffs

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## Abstract—

In this paper we construct a mathematical metric for measuring the performance of the transmission provider (TP) under the newly proposed price cap regulation scheme. The heart of the problem lies in developing the systemwide social welfare function which captures the unique role of the TP in the new industry environment where the electricity is provided through the market mechanism.

The restructuring of the electric power industry is still a relatively recent event at the time of this writing, and there is yet to be a consensus on the actual implementation scheme for regulating the TP based on the guaranteed rate-of-return. In this paper, one of the implementation schemes referred to as *ex ante* flow tax scheme is described. Starting from this implementation scheme the price cap regulation (PCR) is proposed as a possible alternative regulation scheme to be imposed on the TP. Then, we develop the systemwide social welfare function associated with the PCR.

## I. INTRODUCTION

In this paper we construct a mathematical metric for measuring the performance of the transmission provider (TP) under the newly proposed price cap regulation scheme. The heart of the problem lies in developing the systemwide social welfare function which captures the unique role of the TP in the new industry environment where the electricity is provided through the market mechanism.

The paper is organized as follows: First, we describe two possible regulation schemes to be imposed on the TP, namely the rate-of-return regulation and the price-cap-regulation (PCR). The TP remains a monopoly through the restructuring process due to the assumption that there exists a high degree of economies of scale and economies of scope for the network. The main function of the TP is to provide adequate transmission capacity necessary for participants to trade electricity in the electric energy market.

Then, we consider what we refer to as *ex ante* flow tax scheme. Starting from this scheme the PCR is proposed as a possible alternative regulation scheme to be imposed on the TP. Finally, we develop the systemwide social welfare function associated with the PCR and illustrate the concept through an example.

The concluding remarks are made at the end.

## II. ROLE OF REGULATOR OVERSEEING THE TP

After the restructuring process, the operation and the planning of an electric power network consist of four entities as shown in Figure 1. The transmission provider is a monopolistic entity whose responsibility is to design the

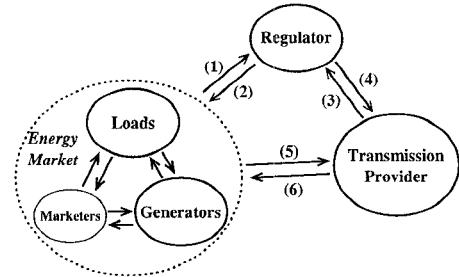


Fig. 1. Composition of the electric power network economics after the restructuring process

transmission network and to operate the electric power system consisting of generation and transmission by virtue of controlling the allocation of the existing transmission capacity. The energy market is a generic term used to refer to a place for trading the energy portion of electricity (rather than limiting its use to refer only to the spot market where the centralized auctioning process takes place), and is composed of loads, generators and marketers. The loads are the consumers of various electric services (generation and transmission) while the generators are the suppliers of the energy portion of electric services. The marketers participate in trading of electric services often on behalf of loads or generators and typically do not own or operate generation, transmission or distribution systems. The function of marketers is largely ignored in this paper.

The regulator is typically a government agency whose responsibility is to oversee the operation and the planning of the network by the transmission provider directly and/or indirectly. The regulation by the regulator is necessary even after the restructuring process since the TP provider remains as a monopoly largely due to the economies of scale. As a monopoly the TP charges for the transmission portion of electric services above the marginal cost of network capacity so that the TP may continue to support the network as a viable business while ensuring a reasonable return on her investment. The regulation determines what the degree of reasonable return is and limits the TP from charging more than the reasonable. The rate-of-return regulation is one form of the cost-of-service regulations which guarantees the return on all of the investments that are made with an approval, up to the amount allowed by the regulator.

With the introduction of competition the function of the regulator may, at first glance, seem reduced in terms of the direct influence it imposes on the operation and the planning of a regional electric power network since the energy

portion of the electricity is provided through the market mechanism. Only the transmission portion of electric service is under the direct control of the regulator through the rate approval. However, there is a significant expansion of the regulator's function in terms of the indirect control over the electric power network economics. This is due to that fact that the particular form of market mechanism governing the energy market is required to be approved by the regulator before implementation. The role of regulator is two fold, (1) designing the market mechanism for energy market (2) prescribing the rational rates for transmission capacity, so that the overall operation and planning of electric power network approaches the systemwide social welfare optimization.

### III. TRANSMISSION CHARGE UNDER THE COST-OF-SERVICE REGULATION

In order to ensure the TP continues to support the energy market as a viable business with a reasonable expected return on her investment, the high degree of economies of scale needs to be addressed through the transmission charge for the investment into network.

Under the rate-of-return regulation (as a particular form of the cost-of-service regulation) imposed on the TP, the regulator guarantees a reasonable rate of return on all of the approved investment into transmission made by the TP. Let  $\Upsilon[n]$  be the allowed revenue of the TP for year  $n$  determined by the regulator based on the total investment cost given by:

$$\Upsilon[n] = (1 + r_{cos}) \sum_{k=(n-1)T_T+1}^{nT_T} \sum_l (1 - \xi)^k C_l^T(K_l^T[k], I_l^T[k], k) \quad (1)$$

where  $r_{cos}$  is the rate of return on investment allowed by the regulator.  $C_l^T$  denotes the cost of investment given the current capacity  $K_l^T[k]$  and the newly expanded capacity  $I_l^T[k]$ . In Eq. (1) we use the fact that typically the time scale for investment into transmission is a year, i.e.,  $T_T = 1$  year. From the perspective of the TP, the profit is, then, determined by:

$$\begin{aligned} \Pi_{TP}[n] &= \Upsilon[n] - \sum_{k=(n-1)T_T+1}^{nT_T} (1 - \xi)^k \left( \sum_l C_l^T(K_l^T[k], I_l^T[k], k) \right. \\ &\quad \left. + v_{tech}(e_{tech}[k]) + v_m(e_m[k]) \right) \\ &= \sum_{k=(n-1)T_T+1}^{nT_T} (1 - \xi)^k \left( r_{cos} \sum_l C_l^T(K_l^T[k], I_l^T[k], k) \right. \\ &\quad \left. - v_{tech}(e_{tech}[k]) - v_m(e_m[k]) \right) \end{aligned} \quad (2)$$

where the expression in Eq. (1) is substituted for  $\Upsilon[n]$ . The decision for dispensing the efforts into control and into maintenance,  $e_{tech}$  and  $e_m$  respectively, are assumed to be made only once at the beginning of each year for simplicity.  $v_{(.)}$  denotes the corresponding cost. Accordingly the profit maximization of the TP under the rate-of-return regulation is given as the following:

$$[\overline{\mathbf{I}_T}, \overline{e_{tech}}, \overline{e_m}]^* = \arg \max_{\mathbf{I}_T[n], e_{tech}[n], e_m[n]} \sum_{n=1}^{T_I/T_T} \mathcal{E}\{\Pi_{TP}[n]\} \quad (3)$$

where we make another simplifying assumption that the investment decision is made not over the infinite time horizon but over the time scale of  $T_I$ .

From the perspective of the regulator, the associated cost,  $TC_{reg}[n]$  for year  $n$  encloses the expense arising from compensating the difference between the revenue collected from the loads and generators and the revenue guaranteed to the TP. By again employing the modeling simplification in [4] of treating the process of making up the difference in the revenue collected and allowed as an exclusive process between regulator and loads, the expression of this cost is given as the following:

$$TC_{reg} = (1 + \lambda_f)(\Upsilon[n] - TR[n]) \quad (4)$$

where  $\lambda_f$  is the shadow cost of public funds (the key concept in the simplification step), and  $TR[n]$  is the total revenue collected over the entire year  $n$ , i.e.,

$$TR[n] = \sum_{k=(n-1)T_T+1}^{nT_T} (1 - \xi)^k \mathcal{E}\{TR[k]\} \quad (5)$$

derived from

$$\begin{aligned} TR[k] &= \sum_{d_j} \rho_{t,d_j}(\mathbf{Q}_D[k], \mathbf{Q}_G[k], k) \cdot Q_{d_j}[k] \\ &\quad + \sum_{g_i} \rho_{t,g_i}(\mathbf{Q}_D[k], \mathbf{Q}_G[k], k) \cdot Q_{g_i}[k] + \sum_l \mu_l[k] \cdot F_l^{max}[k] \end{aligned} \quad (6)$$

where  $\rho_{t,(.)}(\mathbf{Q}_D[k], \mathbf{Q}_G[k], k)$  is the price for transmission portions of electric services.  $Q_i[k]$  denotes the injection at bus  $i$  and  $\mu_l[k]$  is the congestion charge on line  $l$ . The congestion refers to

$$F_l(\mathbf{Q}_G[k], \mathbf{Q}_D[k]) = F_l^{max}(\mathbf{F}[k], K_l[k], e_{tech}[k], e_m[k]) \quad (7)$$

Based on the cost associated with the regulator in Eq. (4) and/or in Eq. (5), it is evident that there is a significant weight placed on the transmission charge levied on the loads other than the shadow cost related to the transmission congestion. This is due to the high degree of economies of scale assumed for the investment into transmission as before, which without the transmission charge leads to a considerable difference in the revenue between the collected and the allowed.

The concept of basic importance linked to the assigning of the transmission charge is three fold, namely (1) sufficient revenue collection, (2) the distortion introduced by the charge and (3) fairness to the parties being levied considering their individual characteristics. The notion of the *optimal transmission pricing* lies with the scheme that allows sufficient revenue collection while minimizing the distortion introduced by the charge and appearing fair to those who pay for the charge. It turns out that the first criterion may be the easiest to comply with assuming that the investment into transmission is made with prudence, although not necessarily the optimal possible, and the relative price for the transmission portion of electricity services are much lower than that for the energy services, i.e.,

$$\sum_{k=1}^{T_T} \sum_{d_j} \rho_{t,d_j} \cdot Q_{d_j}[k] \gg \sum_{k=1}^{T_T} \sum_{d_j} \rho_{t,d_j} \cdot Q_{d_j}[k] \quad (8)$$

where  $\rho_{e,d_j}$  denotes the energy price, which is usually satisfied for many regions in US. Almost any reasonable transmission charging scheme satisfies this criterion. In comparison, the second criterion may be the hardest to comply with because the degree to which distortion is introduced in behavior of parties affected by the transmission charge depends on their respective utility functions, and thus may be quite system specific. At the time of writing, no generalized result exists for quantifying the effect of transmission charge. The third criterion is a delicate standard by which different schemes are judged since it tends to be highly subjective. Here we consider only what we refer to as *ex ante* flow tax on load and *ex post* settlement scheme [5] [7].

#### A. Ex ante flow tax on load and ex post settlement scheme

Under the *ex ante* flow tax on load and *ex post* settlement scheme, first, the tax rate for allowing flow through the network,  $\hat{\rho}_t[n]$ , is determined. Then, the transmission charge is levied on the load in the form of flow tax proportional to total electric power flow throughout the network caused by the load satisfying her demand at each hour. If there exists a difference in revenue between the amount collected through *ex ante* injection tax and that allowed by the regulator at the end of the year, *ex post* charges are imposed on the loads. The *ex post* charge can again take on various forms as discussed earlier. We make the simplifying assumption that from the sense of expected value, the adequate *ex ante* flow tax rate can be determined so that no *ex post* charge becomes necessary at the end of the year.

The apparent electric power flow through transmission line  $l$  at hour  $k$ ,  $F_l[k]$ , is a function of the total injection into each bus in the system, i.e.,

$$F_l[k] = F_l(\mathbf{Q}_G[k], \mathbf{Q}_D[k]) \quad (9)$$

for an existing network. The vectors,  $\mathbf{Q}_G[k]$  and  $\mathbf{Q}_D[k]$ , designate the amount of electricity injected into the network by generators and the amount of electricity withdrawn from the network by load respectively, *determined through the market clearing process in the spot market under the ex ante flow tax scheme*. Let  $f_{l,d_j}$  denote the flow on line  $l$  related to load  $d_j$  derived by decomposing the apparent flow  $F_l[k]$  into the flow corresponding to supplying the demand at the same load,  $Q_{d_j}[k]$ . Then,  $f_{l,d_j}$  can be computed using the following expression:

$$f_{l,d_j}[k] = F_l(\mathbf{Q}_{G,d_j}[k], \mathbf{Q}_{D,d_j}[k]) \quad (10)$$

where  $\mathbf{Q}_{G,d_j}[k]$  and  $\mathbf{Q}_{D,d_j}[k]$  are given by:

$$\mathbf{Q}_{G,d_j}[k] = \left( \frac{Q_{d_j}[k]}{\sum_{d_j} Q_{d_j}[k]} \right) \cdot \mathbf{Q}_G[k] \quad (11)$$

$$\mathbf{Q}_{D,d_j}[k] = [0, \dots, Q_{d_j}[k], 0, \dots, 0]^T \quad (12)$$

Typically, for notational convenience, given a transmission line  $l$  connecting buses  $i$  and  $j$ , an arbitrary direction  $ij$  is defined. According to this direction the computed flow is either positive if the flow is from bus  $i$  to bus  $j$ , or negative otherwise. Let  $q_{l,d_j}^+[k]$  and  $q_{l,d_j}^-[k]$  denote the positive and the negative directional flow of  $f_{l,d_j}[k]$ , i.e.,

$$q_{l,d_j}^+[k] = \begin{cases} f_{l,d_j}[k] & \text{if } f_{l,d_j}[k] \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

$$q_{l,d_j}^-[k] = \begin{cases} -f_{l,d_j}[k] & \text{if } f_{l,d_j}[k] \leq 0 \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

For example, the apparent flow through transmission line  $l$ ,  $F_l[k]$ , is the difference between the positive directional flow,  $q_{l,d_j}^+[k]$ , and the negative directional flow,  $q_{l,d_j}^-[k]$ , caused by supplying the individual demand  $Q_{d_j}$ , summed over all loads given by:

$$F_l[k] = \sum_{d_j} (q_{l,d_j}^+[k] - q_{l,d_j}^-[k]) \quad (15)$$

The implied reasoning for choosing this particular method of decomposing the apparent flow is that in the spot market, the demand at each load is being supplied by every generator participating in the market proportional to the total demand throughout the network. For other interesting decomposition methods, we refer to [8].

Using the decomposition method in Eq. (10) and accounting for the profit of the TP and the cost of the regulator given in Eqs. (2) and (5) the systemwide social welfare defined under the scheme may be computed by solving the optimization problem given as the following:

$$\left[ \frac{\overline{I_T}^*}{\overline{e_m}^*} \right]' = \arg \max_{\overline{I_T}^*[n], \overline{e_m}^*[n], e_{tech}[n]} \sum_{n=1}^{T_I/T_T} (1-\xi)^{n T_T} \varepsilon \left\{ r_{cos} \sum_l C_l^T(K_l^T[n], I_l^T[n], n) \right. \\ \left. - v_{tech}(e_{tech}[n]) - v_m(e_m[n]) - (1 + \lambda_f) \left[ \sum_l C_l^T(K_l^T[n], I_l^T[n], n) \right. \right. \\ \left. - \sum_{k=(n-1)T_T+1}^{T_T} \sum_l (1-\xi)^{k-T_T} \left( \hat{\rho}_t[n] \sum_{d_j} (q_{l,d_j}^+[k] + q_{l,d_j}^-[k]) \right. \right. \\ \left. \left. + \mu_l[k] \sum_{d_j} (q_{l,d_j}^+[k] - q_{l,d_j}^-[k]) \right) \right] \right\} \quad (16)$$

where  $\mu_l[k]$  denotes the Lagrangian multiplier corresponding to solving the following optimization problem:

$$\left[ \mathbf{Q}_G^*[k], \mathbf{Q}_D^*[k] \right]' = \arg \max_{\mathbf{Q}_G[k], \mathbf{Q}_D[k]} \varepsilon \left\{ \sum_{d_j} \left( \int_{Q_{d_j}^-[k]=0}^{Q_{d_j}[k]} D_{d_j}(Q_{d_j}^-[k], k) dQ_{d_j}^-[k] \right. \right. \\ \left. \left. - \sum_l \hat{\rho}_t[n] (q_{l,d_j}^+[k] + q_{l,d_j}^-[k]) \right) - \sum_{g_i} \int_{Q_{g_i}^-[k]=0}^{Q_{g_i}[k]} S_{g_i}(Q_{g_i}^-[k], k) dQ_{g_i}^-[k] \right\}$$

subject to

$$\sum_{g_i} Q_{g_i}[k] = \sum_{d_j} Q_{d_j}[k] : \quad \lambda[k] \quad (18)$$

$$Q_{g_i}^{min}[k] \leq Q_{g_i}[k] \leq Q_{g_i}^{max}[k] : \quad \eta_{g_i}[k] \quad (19)$$

$$F_l(\mathbf{Q}_G[k], \mathbf{Q}_D[k]) \leq F_l^{\max}[k] : \quad \mu_l[k] \quad (20)$$

#### IV. PERFORMANCE-BASED-REGULATION (PBR)

Under the cost-of-service regulation a close link is made between the cost of providing the service and the price charged for the service by the regulated firm. In the context of the electric power industry after the restructuring process, this means the price charged for providing transmission capacity by the TP is strictly based on the cost of investment into the transmission network. As it is pointed out in earlier discussions, in this environment there is little or no incentive for the TP to reduce costs by improving productivity.

The performance-based-regulation (PBR) is a regulatory structure where this linkage between the cost and the price of the service is broken by offering financial incentives to the regulated firm, the TP, to lower the cost instead. One such PBR scheme is the PCR approach.

Under the price cap approach, first the regulator determines an appropriate price for providing the service and sets the initial ceiling price. This first step of setting the initial price is *similar to that under the cost-of-service regulation*. Once the initial price is set, then the regulator decides on various indices to be used to compute the ceiling prices for the specified period into the future. These indices include the changes in productivity and unanticipated changes in costs not under the control of the regulated firm. The change in productivity is often referred to as the *X* factor and prescribes the targeted improvement in efficiency to be achieved by the firm. The unanticipated changes are called the exogenous factor or the *Z* factor and include such elements as low-income program expenditures and sometimes research and development (R&D) costs [6] [3].

The firm's incentives to reduce costs comes from the higher profit expected under this approach. Any reduction in costs increases the profit of the firm given the price ceilings for the specified period into the future. It is interesting to note that the period over which the price ceilings (typically 5 years) are determined is usually much longer than the price review by the regulator under the rate-of-return regulation (1 year). Such stability in regulation also adds to induce higher efficiency since the firm is assured by keeping the additional profits realized from cost reduction without causing regulatory interference.

##### A. Price-cap regulation applied to the transmission provider (TP)

Consider the *ex ante* flow tax scheme discussed earlier. From Eq. (16) it is evident that the TP's revenue for the year  $n$  is given by:

$$TR[n] = \mathcal{E} \left\{ \sum_{k=(n-1)T_T+1}^{nT_T} \sum_l \left( \hat{\rho}_t[n] \sum_{d_j} (q_{l,d_j}^+[k] + q_{l,d_j}^-[k]) + \mu_l[k] \sum_{d_j} (q_{l,d_j}^+[k] - q_{l,d_j}^-[k]) \right) \right\} \quad (21)$$

where  $\mu_l[k]$  denotes the Lagrangian multiplier corresponding to solving the following optimization problem:

$$\left[ \begin{array}{c} \mathbf{Q}_{\mathbf{G}}^* \\ \mathbf{Q}_{\mathbf{D}}^*[k] \end{array} \right]' = \arg \max_{\mathbf{Q}_{\mathbf{G}}[k], \mathbf{Q}_{\mathbf{D}}[k]} \mathcal{E} \left\{ \sum_{d_j} \left( \int_{Q_{d_j}[k]=0}^{Q_{d_j}[k]} D_{d_j}(Q_{d_j}[k], k) dQ_{d_j}[k] \right) - \sum_l \int_{Q_{g_i}[k]=0}^{Q_{g_i}[k]} S_{g_i}(Q_{g_i}[k], k) dQ_{g_i}[k] \right\} \quad (22)$$

subject to the constraints in Eq. (18) and Ineqs. (19) and (20). Suppose the rate of the flow tax,  $\hat{\rho}_t[n]$ , is allowed to vary hour-by-hour denoted as  $\hat{\rho}_t[k]$ . By rearranging the expression inside  $(\cdot)$  on the right-hand-side (RHS) of Eq. (21) and substituting  $\hat{\rho}_t[k]$  for  $\hat{\rho}_t[n]$  we have

$$TR[k] = \mathcal{E} \left\{ \sum_l \sum_{d_j} \left[ (\hat{\rho}_t[k] + \mu_l[k]) q_{l,d_j}^+[k] + (\hat{\rho}_t[k] - \mu_l[k]) q_{l,d_j}^-[k] \right] \right\} \quad (23)$$

where

$$TR[n] = \sum_{k=(n-1)T_T+1}^{nT_T} (1 - \xi)^k TR[k] \quad (24)$$

From Eq. (23) it is clear what service the TP provides and what price is charged for the service, namely the transmission capacity in the positive direction and in the negative direction,  $q_{l,d_j}^+[k]$  and  $q_{l,d_j}^-[k]$ , and the transmission rent,  $\hat{\rho}_t[k] + \mu_l[k]$  and  $\hat{\rho}_t[k] - \mu_l[k]$ , respectively.

The newly proposed PCR mechanism consists of regulating the price elements,  $\hat{\rho}_t[k]$  and  $\mu_l[k]$ , for providing the transmission capacity service with the ceiling prices determined by the regulator,  $\hat{\rho}_t[n]$  and  $\mu_l[n]$ , respectively.

First, the regulator defines the initial ceiling prices,  $\hat{\rho}_t[1]$  and  $\mu_l[1]$ . Following the initial prices, the regulator sets the appropriate indices for price adjustment including the inflation  $i$  factor and the *X* factor. Suppose the period of the price review by the regulator is set to be 5 years. Then, the ceiling prices for the subsequent years up to the year 5 are determined by:

$$\hat{\rho}_t[n+1] = \hat{\rho}_t[n] (1 + i_\rho - X_\rho) + Z_\rho \quad (25)$$

$$\mu_l[n+1] = \mu_l[n] (1 + i_\mu - X_\mu) + Z_\mu \quad (26)$$

for  $n = 1, 2, \dots, 4$ . In case there is a significant effect from exogenous factor, which requires an adjustment to the price before the end of the review period, the *Z* factor is defined for each price element.

Having defined the price cap for each year until the end of the review period the conventional application of PCR means *transferring* the operation and planning authority completely from the regulator to the regulated firm, in this case the TP, so long as the following constraints are met:

$$\hat{\rho}_t[k] \leq \hat{\rho}_t[n] \quad (27)$$

$$\mu_l[k] \leq \mu_l[n] \quad (28)$$

where  $k = (n-1)T_T+1, (n-1)T_T+1, \dots, nT_T$ . However,  $\mu_l[k]$  reflects the value of scarcity in transmission capacity and is determined exogenously through solving the optimization problem in Eq. (22). Thus, some modifications

are necessary in enforcing the PCR on the TP. In the following section the necessary modifications are described, and thus, the complete PCR structure of the newly proposed scheme for regulating the TP is presented.

## V. ILLUSTRATIVE EXAMPLES

We illustrate some of the ideas presented in this paper through a numerical example using the 5-bus electric power network shown in Figure 2. Table I summarizes the initial

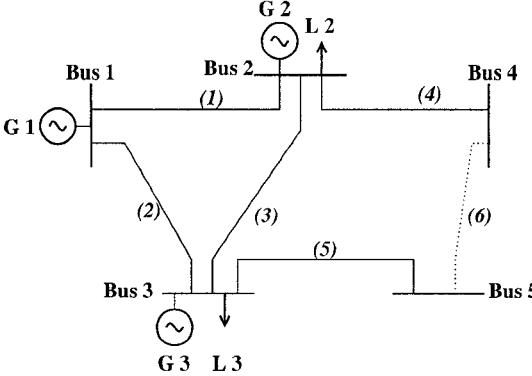


Fig. 2. One-line diagram of the 5-bus electric power network

capacity of each transmission line in the network. The

Line #	(1)	(2)	(3)	(4)	(5)	(6)
Initial capacity (MW)	300	300	120	300	300	0

TABLE I

INITIAL CAPACITY OF EACH TRANSMISSION LINE IN THE 5-BUS ELECTRIC POWER NETWORK EXAMPLE

network capacity of zero between bus 4 and bus 5 indicates that currently no line exists between those buses. It is assumed that the small network capacity on the transmission line between bus 2 and bus 3 relative to the other lines is a result of physical restriction for network expansion in the area, such as zoning limits. Thus, no additional transmission capacity is allowed on that line. No other restriction exists for expanding other lines including between bus 4 and bus 5. As before, there are 4 thermal units at bus 1, 7 hydro units at bus 2, and 1 gas-turbine unit and 1 nuclear unit at bus 3, for the total of 13 units. This time, however, the capacity of each generating unit in the network is assumed to be infinite. Table II summarizes the characteristics of these units for the first year ( $n = 1$ ), including their marginal costs of the form,  $S_{g_i}(Q_{g_i}[k]) = 2a_{g_i}Q_{g_i}[k] + b_{g_i}$ .

These marginal cost functions are useful in computing the systemwide generation cost in this example according to the perfect market assumptions. In this example, a year is composed of 2 seasons, each having 3 days. Depending on the demand of the loads, the seasons and the days are differentiated as peak, shoulder and off-peak. Figure 3 shows the load characteristics at each bus for the first year. For simplicity we assume that the demand is inelastic throughout the year. The expected system conditions for the next few years are, then given as follows: At the beginning of

Unit #	Type	Bus # ( $g_i$ )	$a_{g_i}$	$b_{g_i}$
1	thermal	1	60	0
2	thermal	1	60	0
3	thermal	1	250	0
4	thermal	1	122.5	0
5	hydro	2	25	0
6	hydro	2	2.5	0
7	hydro	2	70	0
8	hydro	2	70	0
9	hydro	2	80	0
10	hydro	2	80	0
11	hydro	2	80	0
12	gas-turbine	3	1000	100
13	nuclear	3	3	0

TABLE II  
CHARACTERISTICS OF GENERATING UNITS IN THE 5-BUS NETWORK

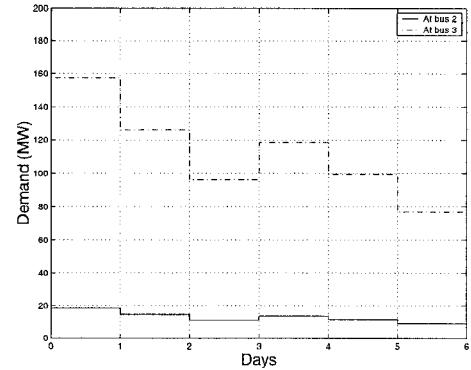


Fig. 3. Load characteristics in years  $n = 1, 2$

the second year ( $n = 2$ ) the nuclear unit at bus 3 is taken out of service for maintenance and is not expected to come on line until the beginning of year 4. The expected demand of loads in this year is same as shown in Figure 3. In year 3 ( $n = 3$ ), the projected demand of loads increases by 5% from the previous year throughout the network while no change is expected to take place in the generation. At the beginning of year 4, the nuclear plant is expected to come back on line while the projected demand stays the same from the previous year.

At the beginning of each year the TP decides the amount of investment into transmission,  $I_l^T[k]$ , and determines the size of expenses for the control effort,  $e_{tech}[k]$ , and the maintenance effort,  $v_m(e_m[k], k)$ . For simplicity we assume that the marginal cost of investment into transmission is piecewise constant: \$30.4616 for 0 to 20MW, \$60.9233 for 20 to 40MW, \$91.3849 for 40 to 60MW, \$121.8465 for 60 to 80MW and \$152.3082 for 80 to 100MW while there is a fixed cost of \$2,000. Consequently the actual cost of investment is piece-wise linear. As discussed earlier, it is evident from the figures that the marginal cost of the investment into transmission is much smaller than the average cost for the ranges of investment being considered. In addition, it is assumed that the cost function associated with the maintenance effort is \$0, while the cost function associated with the control effort into transmission is given by \$180 if the control effort is made, or \$0 otherwise, i.e.,

$$v_m(e_m[k], k) = 0 \quad (29)$$

$$v_{tech}(e_{tech}[k], k) = \begin{cases} 180 & \text{if } e_{tech}[k] = 1 \\ 0 & \text{otherwise} \end{cases} \quad (30)$$

Suppose the operational limit on power transfer through line  $l$  is given as either a half of the line capacity if no control effort is made or additional 5MW otherwise, i.e.,

$$F_l^{\max}(\mathbf{F}[k], K_l[k], e_{tech}[k], e_m[k]) = \begin{cases} 0.5K_l[k], & \text{if } e_{tech}[k] = 0 \\ 0.5K_l[k] + 5, & \text{otherwise} \end{cases} \quad (31)$$

Further, suppose that we apply the so-called DC load flow assumption. Then, the expression for the flow on transmission line  $l$ ,  $F_l(\mathbf{Q}_G[k], \mathbf{Q}_D[k])$  is given by:

$$F_l(\mathbf{Q}_G[k], \mathbf{Q}_D[k]) = \sum_{g_i} H_{lg_i} Q_{g_i}[k] - \sum_{d_j} H_{ld_j} Q_{d_j}[k] \quad (32)$$

where  $H_{li}$  denotes the power transfer distribution factor (PTDF) of line  $l$  with respect to bus  $i$ .

Then, the systemwide optimal solution can be established as:

$$\begin{aligned} I_{(l)}^T[n] &= \begin{cases} 20 & \text{if } l = 6, n = 2 \\ 0 & \text{otherwise} \end{cases} \\ e_{tech}[n] &= \begin{cases} 1 & \text{if } n = 2, 3 \\ 0 & \text{otherwise} \end{cases} \\ e_m[n] &= 1 \end{aligned} \quad (33)$$

The significance of the investment into transmission and the expense in control effort is the savings in overall cost for meeting the demand in years 2 and 3. For example, without the network reinforcement, the total generation cost is \$7,527.86. With the reinforcement, whose cost amounts to \$2969.23, the total generation cost is reduced to \$2,491.72. This is a savings of \$2,066.90.

In comparison, either 40MW or 60MW of the investment into transmission alone amounts to \$2,609.23 or \$3,370.77 while the total generation cost corresponding to the investment reduces to \$2,945.08 or \$2,164.62. This results in savings of \$1,973.54 and \$1,992.46, respectively. The combined 20MW investment and the control effort in only year 2 or year 3 amounts to the network cost of \$2,789.23, and the corresponding total cost of generation is given by \$2754.72 or \$2,682.08. This is savings of \$1,983.91 and \$2,056.54, respectively.

Under the rate-of-return regulation scheme, it is likely that the TP would prefer 30MW of the investment into generation alone since while the systemwide savings is still at a reasonable level, in this case the rate base is higher than when combined with the expense in control effort. Therefore, unless a regulator is aware of this particular advantage of the control effort, the network is likely to be operated at a suboptimal level, i.e. Averch-Johnson effect [2].

By contrast, suppose the ceiling prices on congestion charge are set to be \$45 for years 2 and 3 on the transmission line between bus 4 and bus 5 with no additional penalty for exceeding this limit (i.e.,  $r_{penalty} = 0$ ) under the proposed PCR scheme. Then, the decision for the 40MW investment followed by the control effort made only in year 3 is clearly favorable to the TP compared to other decisions. This is because the maximum profit is obtained while staying within the ceiling prices set by the regulator. This solution is very close to the optimal solution.

Finally, it is noted that the cost of the investment into transmission is not recovered solely based on the congestion

charge. This is because of the high fixed cost element of \$2,000 for transmission investment. For example, for the optimal decision by TP for the ceiling price of \$45 for years 2 and 3, the total investment cost is over \$2,600 yet the congestion charge collected is only \$1,963. Therefore, some form of supplemental charge is required in order to induce the optimal decision by the TP. In the case of the proposed PCR scheme, this results in 6.92 (\$/MW) to be imposed as one possible *ex ante* flow tax.

## VI. CONCLUSION

The importance of a properly functioning forward market for energy has been well understood including from the perspective of solving the unit commitment problem [1]. It is also well recognized that it is practically impossible to have a liquid forward market for energy without well-thought through delivery (transmission) provision.

In order to create a long term transmission market, the ability of the TP to take on the financial risks is also very important when implementing the longer term transmission rights under uncertainty. In this paper we have shown that the propose price-cap regulation provides a possible framework for performance based regulation necessary for such undertaking of financial risks.

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