Mapping Political Communities: A Statistical Analysis of Lobbying Networks in Legislative Politics*†

In Song Kim‡ Dmitriy Kunisky§

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Abstract

Relationships between special interest groups and politicians play a significant role in policymaking. Yet empirical studies of interest group politics have been limited by the difficulty of observing these ties directly. We address this by constructing an original dataset that combines the politicians who sponsor congressional bills with the interest groups that lobby on those bills. We then develop a methodological framework to examine the political networks these data describe. Unlike networks in electoral politics, whose structure has been found to reflect ideology, we find distinct political communities in the lobbying network that are organized according to industry interests and jurisdictional committee memberships. Furthermore, we observe that important actors belong to multiple political communities, capturing their simultaneous commitments to diverse political interests. Our findings provide evidence for the existence of powerful networks in U.S. legislative politics that are distinct from electoral networks and that exhibit numerous unique structural features.

Keywords: Network analysis, lobbying, ideal point estimation, scaling, stochastic block model, link community model, community detection

Word Count: 11,806 (abstract: 149)

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‡Associate Professor, Department of Political Science, Massachusetts Institute of Technology, Cambridge, MA, 02139. Email: insong@mit.EDU URL: http://web.mit.edu/insong/www/
§Ph.D. Student, Department of Mathematics, Courant Institute of Mathematical Sciences, New York University, New York, NY, 10012. Email: kunisky@cims.nyu.edu URL: http://www.kunisky.com/
1 Introduction

Special interest groups engage in lobbying to promote their political objectives (e.g., Wright 1996; Grossman and Helpman 2001). The nature of the connection between politicians and interest groups, however, is much debated by political scientists. One popular view holds that interest groups take part in such costly political activity in order to buy votes or otherwise persuade policymakers, and in return policymakers gain an effective means of informing their legislative decisions (Bauer, Dexter, and Poll 1972; Potters and Van Winden 1992; Austen-Smith and Wright 1992; Wright 1990). Others interpret lobbying as a “legislative subsidy” through which interest groups help allied politicians pursue common objectives (Hall and Deardorff 2006). To assess these possibilities, researchers are often interested not only in the question of how interest groups decide which legislators to lobby, but also in relationships and patterns among the political connections driven by lobbying.

Empirical works such as Wright (1990) and Heinz et al. (1993) have investigated these regularities by analyzing interviews with lobbyists, interest groups, and politicians involved in specific domains of legislation such as agriculture, energy, and healthcare. Though these inquiries have yielded substantial insights, they are limited in scale and treat only a few policy domains in isolation. Important questions therefore remain: how do various domains of lobbying overlap (as do agriculture and energy, for instance) and how is lobbying activity distributed in such situations? How do prominent legislators and interest groups allocate their attention among their many interests? Despite the significance of such behaviors and of the policy contexts that give rise to them, large-scale empirical studies of interest group politics have been limited by the difficulty of making direct quantitative observations of the ties between interest groups and politicians.

1 We use the term special interest group to refer to any political actor who has a specific policy objective, including firms, trade associations, labor unions, business associations, and professional associations. See https://lobbyingdisclosure.house.gov/amended_lda_guide.html for the definition of lobbying.
Our first contribution in this paper is to construct a large political network database tracing the universe of lobbying activities related to 108,086 congressional bills introduced between the 106th and the 114th Congress. We apply natural language processing techniques to overcome the challenge of correctly identifying the bills mentioned in more than two million textual descriptions of lobbying activities (Section 2). We then identify a type of political connection between members of the U.S. Congress and the interest groups that engage in lobbying by combining two types of political behavior related to each bill: (1) sponsorship by a politician and (2) lobbying by interest groups. Although lobbying on a single bill does not necessarily imply political ties to its sponsor, recurring instances of lobbying that involve the same interest group and sponsor across several bills do reliably indicate close political relationships. Therefore, analyzing how often various politicians and interest groups interact in this way lets us infer the structure of political networks underlying the legislative process.

Our second contribution is to develop statistical network models to infer the unobserved underlying factors that might drive political connections between interest groups and politicians. We begin by applying a latent space model to estimate a spatial location summarizing each political actor’s behavior (Section 3). This approach has been widely used to study various legislative bodies, including state legislatures (Shor and McCarty 2011), the U.S. Congress (Fowler 2006), and the United Nations (Bailey, Strezhnev, and Voeten 2017). Unlike existing studies that rely on roll call votes for the few bills that are voted on the floor (Poole and Rosenthal 2011; Clinton, Jackman, and Rivers 2004), however, we analyze all congressional bills in order to infer the latent structure at the time of their introduction.

Our model locates legislators and interest groups in a common space, in which proximity implies a closer alignment of interests. We find that, in this space, there is clustering of interest

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2While our analysis focuses on this subset of the data, the full network dataset includes not only politicians and interest groups but also lobbying firms, lobbyists, and government organizations, which will be made publicly available at [https://www.lobbyview.org](https://www.lobbyview.org).
groups and legislators according to their industry affiliations and memberships in committees with jurisdiction over those industries, respectively. This finding differs from those based on similar models that analyze other political contexts, such as campaign contributions and social media following (Bonica, 2013; Barberá, 2014; Desmarais, La Raja, and Kowal, 2015; Bond and Messing, 2015), where political actors’ revealed underlying preferences tend to align strongly with ideology. Instead, our finding is consistent with the thesis of Ansolabehere, Snyder, and Tripathi (2002), who argue that interest groups that act mainly through lobbying are more bipartisan and less ideological than those that act mainly through campaign contributions. It also shows that interest groups make use of the power that committees have over policy outcomes during the legislative process, as described by Shepsle and Weingast (1987) and Fournaies and Andrew (2017). In short, we provide quantitative evidence that interest groups target legislators with influence on relevant committees in interactions localized to specific policy domains, forming communities in the lobbying network.

We then propose a new methodological framework that incorporates the community structure we identify in the spatial model (Section 4). We begin by applying the stochastic block model, a sociological tool (Snijders and Nowicki, 1997) more recently studied in the physics and machine learning literature (Fortunato, 2010). We verify that the communities identified by the stochastic block model largely agree with those identified ex post in the latent space model, confirming that the lobbying network exhibits significant community structure. However, we also observe a novel and important qualitative pattern in the lobbying network, namely that many interest groups and politicians tend to represent varied interests across several political communities.

Motivated by this empirical regularity, we develop a new statistical model, the Bipartite Link Community Model (biLCM), that allows actors not only to belong to multiple political communities. 

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3We use the term “community” in the technical sense common in network science: a community is a densely connected subset of actors in the network described by our interaction data. Two actors belonging to the same community are interested in affecting the same legislation, but need not both support or both oppose that legislation.
(so-called “mixed membership”), but also to interact with a given partner simultaneously in multiple policy domains. The model finds that interest groups such as the Chamber of Commerce that represent the heterogeneous interests of many members engage in aggregate lobbying by targeting politicians who are members of broad “power committees” such as the House Committee on Ways and Means and the Senate Committee on Appropriations (Fenno 1973). This is consistent with Wright (1990) who, based on numerous personal interviews and a survey of lobbyists, found that lobbying by powerful interest groups affected politicians’ voting especially strongly within the House Committee on Ways and Means. Our model also captures the focused lobbying of industry-specific organizations and firms that we observed previously with the spatial model. Indeed, for each interest group and politician, our model provides a quantitative measurement of their participation in a variety of legislative domains. In this regard, the proposed methodology improves our understanding of the organization of lobbying in the U.S. Congress, accurately capturing the full spectrum of behaviors from very targeted to very broad lobbying. The proposed methods for network analysis will be implemented in the open-source software package “polnet: Statistical Analysis of Political Networks.”

Taken together, our results illustrate a nuanced community structure in the lobbying network. First, we find that the key to understanding interactions in the lobbying network is actors’ overlapping topical interests rather than their shared ideology. The community models we propose are also well-suited to identifying both direct and indirect political connections (both “friends” and “friends of friends”), the latter of which have been notoriously difficult to examine in interest group politics previously. Our models accomplish this task by inferring actors’ shared community memberships even when researchers do not observe their direct interactions. Second, our results call for more careful consideration of the ways in which interest groups’ policy preferences are

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4 For instance, if Interest Group A frequently lobbies on bills that Politician 1 and Politician 2 sponsor, while Interest Group B lobbies on bills that Politician 2 and Politician 3 sponsor, then our models would find it likely that Interest Group A belongs to the same community as Politician 3, though they do not interact directly.
expressed through committee action (Shepsle and Weingast, 1987; Schickler, 2001; Sinclair, 2016). In particular, these mechanisms cannot be understood merely by analogy with how politicians represent the preferences of the public at large, a process driven mainly by electoral motivations (Mayhew, 1974) and ideological affiliation. Finally, based on mixed membership modeling, we find that “specialist” and “generalist” interest groups, respectively representing narrow and aggregate political interests, coexist in U.S. legislative politics.

Our analyses also provide technical guidance for further research on political networks. To the best of our knowledge, ours is the first statistical study of lobbying networks in legislative politics that examines both politicians and interest groups using micro data. More broadly, the proposed community detection approach is a novel empirical framework for applied political scientists to systematically identify political communities and characterize their members, which will help leverage the rich network data that have become increasingly available in various fields of political science (Keck and Sikkink, 1998; Hoff, Raftery, and Handcock, 2002; Hafner-Burton, Kahler, and Montgomery, 2009; Clark and Lauderdale, 2010; Ward, Stovel, and Sacks, 2011).

The network data, all pairwise measurements of political interactions between politicians and interest groups, the estimated latent spatial positions and their posterior distributions, the estimated community memberships of political actors, and the visualization tools used in preparing this paper will also be made publicly available at https://www.lobbyview.org.

2 The Lobbying Network Database

The Lobbying Disclosure Act of 1995 as amended by the Honest Leadership and Open Government Act of 2007 requires mandatory quarterly electronic filing for any organization (“client”) that “actively participates” in lobbying. Filings must disclose the general lobbying issue area, the chamber of Congress and federal agencies contacted, and the specific issues on which lobbyists (“registrants”) have engaged in political activities. Based on the information available from
these disclosures, researchers have studied various aspects of the politics of lobbying, including the sources of and returns on decisions to lobby (Richter, Samphantharak, and Timmons 2009; de Figueiredo and Richter 2014; Kang 2015; You 2017), the similarities and differences between campaign contributions and lobbying (Ansolabehere, Snyder, and Tripathi 2002), characteristics of lobbyists (Baumgartner et al. 2009; Bertrand, Bombardini, and Trebbi 2014), and the implications of lobbying for trade politics (Bombardini and Trebbi 2012; Kim 2017). However, the available lobbying data has the important limitation that interest groups are not required to report the identities of their political contacts. This is unfortunate, given that almost 90% of lobbying reports indicate that at least one member of Congress or a member of their staff was contacted.

In order to study the connections between interest groups and politicians, there is therefore no choice but to infer those unobserved connections statistically from the available data. To that end, we construct an original database of the congressional *lobbying network* that captures the connections among lobbyists, interest groups, bills, and politicians. The database is generated from lobbying reports filed between 1999 and 2017, which are available from the Senate Office of Public Records. To build the database, we use a suite of automated systems to (1) identify congressional bills reported to have been lobbied, (2) identify the session of Congress those bills were introduced in, and (3) identify each bill’s sponsor. Below, we give a brief outline of the process used to construct the lobbying database.

First, we identify all lobbying activities related to congressional bills, which is possible due to 2 U.S.C. S. 1604(b)(2)(A) a law requiring interest groups to disclose the list of bills on which they have lobbied. For example, a report filed by Intel Corporation in 2013 mentions H.R. 289, “National STEM Education Tax Incentive or Teachers Act of 2011—Science and Math Education Legislation.” We analyze more than two million textual corpora, the “specific lobbying issues,”

found in each lobbying report, in which lobbied bills are referenced by either their number or their title.\footnote{Each report may contain several issues lobbied with separate accompanying descriptions, resulting in a set of 2,396,693 corpora to parse.} Second, we identify the session of Congress that each bill belongs to, which the report often does not mention explicitly, as in the above example. This empirical challenge has prevented prior works from analyzing Congressional lobbying accurately and systematically without manually inspecting lobbying reports or other related texts. To overcome this difficulty, we predict the session number based on a combination of numerous signals mined from the report text, including mentions of bill titles and phrases like “Act of [YEAR],” other bills identified in the same report, and quantitative measurements of similarity between bill texts and the report text where a bill is mentioned. Our text analysis algorithm, described in greater detail in Appendix A.1, for instance correctly determines that the above bill, H.R. 289, is from the 112th Congress. If we had instead used only the year that the lobbying report was filed to determine the Congress, then we would have incorrectly identified this bill as H.R. 289, “Value Our Time Elections Act” from the 113th Congress, since the report was filed in 2013.\footnote{In fact, the OpenSecrets.org database from the Center for Responsive Politics, which is often used in academic research, erroneously states that Intel lobbied on the “Value Our Time Elections Act.” Such examples call into question the validity of previous findings based on these data. See \url{http://www.opensecrets.org/lobby/?billsum.php?id=hr289-113}.} Finally, we identify the sponsor of each bill, completing the connection between interest groups and politicians. We repeat this process for each of 1,111,859 lobbying reports, which in total link 20,092 special interest groups with 1,164 (current and former) members of Congress.

The left panel of Figure 1 shows that about 12,000 bills are introduced in each Congress and the majority of them are lobbied by at least one interest group. Very few of these bills make it to a vote as written, however. Instead, they tend to be merged into larger bills that are voted on, following a complex process of amendment and political compromise. Our dataset allows us to observe interactions before this noisy process takes place, and thus highlights especially close political connections: there is ample empirical evidence that lobbyists help to draft bills or even
Figure 1: **Descriptive Statistics of Lobbying on Congressional Bills.** The left panel compares the numbers of bills introduced, lobbied, and voted on in each congressional session from the 106th through the 114th Congress. The number of bills that are identified as lobbied increases significantly beginning in the 110th Congress, when lobbying reports became digitized per the Honest Leadership and Open Government Act of 2007. The right panel shows the distribution of the number of interest groups that lobbied on each bill introduced in the 113th Congress. The distribution is highly skewed: the median is three, the maximum is 946 (on 113th H.R. 1, “Tax Reform Act of 2014”), and 22.9% of the lobbied bills are lobbied by only one interest group.

...write entire bills on behalf of legislators with whom they have connections...[Nourse and Schacter 2002; Hertel-Fernandez 2014]. Indeed, as the right panel shows, most bills are lobbied by very few interest groups, implying that each individual instance of lobbying tends to reflect narrow interests, which we furthermore find to often be in favor of a bill early in the legislative process.

A typical example: on “A bill to exempt the aging process of distilled spirits from the production period for purposes of capitalization of interest costs” (113th S. 1457), sponsored by Senator Mitch McConnell (R-KY), the Distilled Spirits Council of the U.S. was the only interest group to lobby. This bill has never been voted on in the Senate, but has been reintroduced twice in subsequent sessions of Congress.

Both lobbying and bill sponsorship are relatively rare activities: most interest groups lobby on only a small number of bills and most members of Congress introduce only a small number of bills, while a very small number of actors from both categories are highly active. Not surprisingly, then, see Kim [2017] for examples of similar patterns in trade bills. See Figure E.1 in the Appendix for more precise statistics.
most politician-interest group pairs experience no lobbying interactions on sponsored bills while a small number interact frequently. For instance, 97.96% of pairs of actors in the 113th Congress do not interact on any bills, while the pair with the greatest number of interactions is Senator John “Jay” Rockefeller (D-WV) and the National Cable Telecommunications Association, which interacted on 26 bills. More details on our datasets are given in Appendix A.

Notations Before describing our models, we introduce some mathematical notation and shorthand terminology that will be useful. We index a set of interest groups by \([m] = \{1, \ldots, m\}\) and a set of politicians by \([n]\). When a bill is reported as lobbied on by interest group \(i\) and sponsored by politician \(j\), we say that the interest group and politician interact on that bill (interacting pairs will also sometimes be called “partners”), and each lobbying report interest group \(i\) writes that mentions the bill is a political interaction. We denote the number of interactions (over some period of time, usually a single session of Congress) between the pair \(i, j\) by \(A_{i,j}\), and organize these numbers as the entries of the interaction matrix \(A \in \mathbb{R}^{m \times n}\). This matrix may be viewed as the adjacency matrix of a bipartite graph \(G\) with weighted edges, where the politicians and interest groups lie on opposite sides of the partition. The degree (a term from network theory) of an interest group or politician is the total number of interactions they have with every actor of the other type. Therefore, high-degree politicians are those who sponsor bills that are lobbied often, and high-degree interest groups are those who lobby many bills.

3 Latent Space Modeling of the Lobbying Network

In this section, we develop a Bayesian Latent Space Network Model (LSNM). Following the introduction of this class of model by Hoff, Raftery, and Handcock (2002), there has been a proliferation of variants, of which we mention just a few that are useful for understanding our methods. Our

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10Senator Rockefeller’s ties to the telecommunications industry are well known; see e.g., https://www.nytimes.com/2007/10/23/washington/23nsa.html.
11We will use this terminology throughout, though the Lobbying Disclosure Act calls the organizations that file lobbying reports “lobbying clients.”
model is formally similar to the “Wordfish” model of Slapin and Proksch (2008) for text analysis, which estimates ideal points of a single group of political agents based on text data, whereas we estimate the underlying preference structure of two distinct groups of interacting political actors. Our model is also closely related to that of Barberá (2014), which relates ideal points of politicians and social media users, but instead of modeling whether a binary political interaction (e.g., a Twitter follow) exists, we consider how frequently the behaviors indicating a possible political contact (lobbying on and sponsorship of the same bill) occur. Thus our interaction measurements have an associated magnitude.

The Model  We model the connection between interest group $i$ and legislator $j$ as a function of the two actors’ latent spatial positions in a $d$-dimensional Euclidean space, $\theta_i \in \mathbb{R}^d$ and $\psi_j \in \mathbb{R}^d$ respectively, and assume that the number of their interactions $A_{i,j}$ has a Poisson distribution, with its mean parametrized by the Euclidean distance $\| \theta_i - \psi_j \|_2$.

To account for the differences in agents’ baseline propensities to sponsor or lobby (see Figure E.1), we also include interest group- and legislator-specific “popularity” terms, $\alpha_i$ and $\beta_j$ respectively, in the mean of the modeled $A_{i,j}$.

We then take the probabilistic model

$$A_{i,j} \sim \text{Poisson}(\exp(\alpha_i + \beta_j - \| \theta_i - \psi_j \|_2^2))$$ (1)

for the interaction matrix entries. To implement inference, it is more convenient to reparametrize in terms of $\bar{\alpha}_i = \alpha_i - \| \theta_i \|_2^2$ and $\bar{\beta}_j = \beta_j - \| \psi_j \|_2^2$, which yields the equivalent model

$$A_{i,j} \sim \text{Poisson} \left( \exp \left( \bar{\alpha}_i + \bar{\beta}_j + 2\theta_i^\top \psi_j \right) \right).$$ (2)

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12We do not impose an ex ante interpretation of the latent spatial locations; similar analyses of political actors other than interest groups sometimes call the locations “ideal points” or “preferred policies.”

13In the literature, popularity terms are also called “gregariousness” or “idiosyncratic factors.” A similar construction to ours was tested on synthetic datasets by Krivitsky et al. (2009).
We then model the entries as independent conditional on the parameters, to obtain the following joint distribution:

\[
P(A | \alpha, \beta, \theta, \psi) = \prod_{i=1}^{m} \prod_{j=1}^{n} \text{Poisson} \left( A_{i,j} \mid \exp \left( \tilde{\alpha}_i + \tilde{\beta}_j + 2\theta_i^T \psi_j \right) \right).
\]

(3)

To impose parameter identification and improve the numerical behavior of Markov Chain Monte Carlo (MCMC) sampling, we take a further hierarchical extension of this model where the latent space positions and modified popularity factors are distributed with normal population distributions. We take interest group and politician latent positions to both be distributed as \(N(0, \text{diag}(\tau))\), and interest group and politician modified popularity factors to be distributed as \(N(0, \sigma^2_I)\) and \(N(\nu, \sigma^2_P)\) respectively. The posterior distribution under this model is then given by:

\[
P(\alpha, \beta, \theta, \psi | A) \propto \prod_{i=1}^{m} \prod_{j=1}^{n} \text{Poisson} \left( A_{i,j} \mid \exp \left( \tilde{\alpha}_i + \tilde{\beta}_j + 2\theta_i^T \psi_j \right) \right) \times
\]

\[
\prod_{i=1}^{m} N(\theta_i | 0, \text{diag}(\tau)) \times \prod_{j=1}^{n} N(\psi_j | 0, \text{diag}(\tau)) \times
\]

\[
\prod_{i=1}^{m} N(\tilde{\alpha}_i | 0, \sigma^2_I) \times \prod_{j=1}^{n} N(\tilde{\beta}_j | \nu, \sigma^2_P).
\]

(4)

**Computation** We perform inference for our model with the Hamiltonian Monte Carlo No-U-Turn Sampler and Automatic Differentiation Variational Inference implementations in the Stan software package [Carpenter et al. 2017]. In the results presented here, we use empirical posterior means of our parameters taken as point estimates, and we perform further analysis of these point estimates so as to focus on politically meaningful findings. We include the Stan code describing our model and provide a more technical evaluation of our implementation, discussing how parameter identification is achieved, the choice of dimensionality, the concentration of the posterior distribution, and MCMC sampling diagnostics, in Appendix B.
Empirical Findings  We apply the proposed model to the lobbying network dataset for the 113th Congress. We first consider a one-dimensional LSNM, the simplest model our framework admits. To compare with the situation in electoral politics, we compare estimated latent spatial positions of politicians ($\psi_j \in \mathbb{R}$) with DW-NOMINATE ideology measures \cite{Poole2011}. As the left panel of Figure 2 shows, we do not find evidence that shared ideology drives the interaction between interest groups and politicians (Pearson’s $\rho = 0.15$ between our latent dimension and the DW-NOMINATE ideology dimension). This lack of correlation also holds for restrictions to only Democratic or Republican members of Congress. Our finding is consistent when we increase the dimensionality of the latent positions as well, as the right panel shows.

Increasing the dimensionality of the LSNM facilitates the interpretation of the estimated latent space positions.\footnote{For more technical analysis of choice of dimensionality for the LSNM based on heuristics from spectral methods, the reader may consult Appendix B.2.} For the two-dimensional LSNM, Figure 3 presents the posterior means of the estimated latent spatial positions, $\theta_i$ and $\psi_j$, for all interest groups $i$ and politicians $j$.\footnote{We analyze the posterior distributions further in Appendix B with particular attention to the uncertainties of the point estimates given by the posterior means to illustrate the stability of these results.} Each
Figure 3: Estimated LSNM Positions: Full 113th Congress. This figure presents the two-dimensional latent space positions and popularity factors inferred from the 113th Congress dataset. We indicate several significant clusters corresponding to specific industries and issue areas. Interest groups are represented with black dots while politicians are represented with colored dots according to their ideological score on the DW-NOMINATE scale. The size of each dot is proportional to the exponential of the agent’s popularity factor ($\exp(\alpha_i)$ or $\exp(\beta_j)$). We annotate the clusters with some representative members.

*One actor with outlying mean latent positions (right of the region shown), DIRECTTV, Inc., is omitted for the sake of visual clarity.*
Figure 4: **Committee Membership of Politicians by Cluster.** Histograms of the top 10 committee memberships for politicians in the larger clusters highlighted in Figure 3. Bars are divided by political party (red for Republican and blue for Democrat), and the horizontal bars at the top show the overall party distribution of each cluster. Senate committee and House committee labels are prefixed with (S) and (H), respectively.

interest group is represented by a black circle and each politician is represented by a colored circle according to their DW-NOMINATE ideology dimension. The size of each circle is proportional to the exponential of their popularity factor ($\alpha_i$ and $\beta_j$), so that the mean of the Poisson interaction between two agents at fixed distance is proportional to the product of their circles’ sizes.

Figure 3 shows that the key structure of the lobbying network in this two-dimensional model revolves around specific issue areas and industries. As the shaded regions indicate, interest groups are clustered in the latent space according to their industry affiliation (such as telecommunications) or issue area of interest (such as veterans’ affairs). Importantly, these clusters emerge even though the interest groups involved usually do not lobby on the same bills (recall that most bills are lobbied by only a few groups). Instead, related groups are “pulled together” through their connections to the same politicians, who sponsor separate bills on which the groups separately lobby. As in the one-dimensional model, there is no clear partisan divide either across or within industry clusters. Rather, we find in each large cluster of interest groups a concentration of politicians from committees involved with the policy issues that affect those groups. For instance, Figure 4 shows that the legislators belonging to the “Finance & Insurance” cluster are likely to serve in the
Table 1: Regression Analysis of LSNM Covariates. Statistics of linear models of numerous covariates against the one- and two-dimensional LSNM latent dimensions for politicians. We compare full linear models with all available covariates against restricted models, omitting either committee-related covariates, geographic information, or ideology measures. In the bottom three rows, for each of the three latent dimensions, we give the $F$ statistics and $p$-values of the comparison $F$-test. The test illustrates that the committee membership covariates are the only covariates with significant explanatory power for the LSNM latent space organization.

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House Financial Services Committee (31 out of 62), and those belonging to the “Veterans’ Affairs” cluster are likely to serve in the House Committee on Veterans’ Affairs (10 out of 15).

To formally investigate the significance of industry and committee affiliations in determining an actor’s latent position in the lobbying network, we conduct a series of statistical tests. Specifically, we compare the performance of “Restricted” and “Full” linear models predicting latent positions by nesting the former in the latter. Table II presents the $F$-statistics and $p$-values summarizing the significance of including various sets of observable covariates. We find that committee memberships are the only significantly informative covariates of politicians’ positions in both the one- and two-dimensional models. In contrast, ideology and geography play insignificant roles. Similarly, we find that industry affiliation, determined by the leading digits of an interest group’s NAICS industry code, is always significant in predicting the latent space position of an interest group, while the interest group’s home state affects only one dimension of the two-dimensional LSNM significantly. Statistics for the latter findings are included in the Appendix in Table E.1.

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16We find evidence that one dimension of the two-dimensional LSNM can partly be explained by geography. We believe that this is a result of politicians in Congress tending to support the interests of groups belonging to...
Not all regions in our spatial representation are characterized by domain-specific political interests, however. We find that a region in the center of the latent space (marked by the dashed oval in Figure 3) is populated by politicians who are lobbied by groups with diverse interests. Furthermore, these politicians tend to sit not on committees that are directly related to the industries of the lobbying groups, but rather on so-called “power committees” that concern government oversight, federal spending, and taxation, most prominently the House Committee on Ways and Means, the House Committee on Appropriations, and the Senate Committee on Appropriations, as shown in Figure 4. Several of the interest groups that are the most active in lobbying, such as the Chamber of Commerce (which lobbied more bills in the 113th Congress than any other group), the Specialty Equipment Market Association (SEMA), and the American Civil Liberties Union (ACLU), lie in this region. The region also includes many senior politicians and party leaders, such as Patty Murray (D-WA), Barbara Boxer (D-CA), Kirsten Gillibrand (D-NY), Orrin Hatch (R-UT), Lindsey Graham (R-SC), and Paul Ryan (R-WI). We defer further interpretation of the characteristics they have in common to Section 4.2, where we will introduce mixed-membership community models in part to answer this question.

The analysis presented in this section demonstrates the presence of political communities in the lobbying network, which is organized according to industry interests and jurisdictional committee memberships. This description contrasts with networks of electoral politics, which scholars have found are organized around ideology. Although the identification of clusters and their visual demarcations might seem arbitrary, the spatial configuration of estimated latent positions exhibits substantively meaningful patterns. In fact, the geometry of the clusters is consistent with common intuitions about relationships among industries. For example, the “Technology” cluster is adjacent to both the “Telecommunications” and “Energy” clusters, while the “Universities” cluster is their own state [Bailey and Brady, 1998]. To examine such effects more formally, recent research has sought to incorporate observed covariates into network models directly (e.g., Minhas, Hoff, and Ward, 2016). We leave the challenging task of adapting such methods to our setting for future research.
adjacent to both the “Industrial Research & Defense” and “Healthcare” clusters. Having bolstered our belief that political communities exist in the lobbying network, we next seek to study them in a more explicit and statistically principled manner, adapting our modeling techniques to better suit the community structure.

4 Community Modeling of the Lobbying Network

In this section, we first introduce the canonical model of probabilistic community detection in bipartite networks—the Bipartite Stochastic Block Model (biSBM) of Larremore, Clauset, and Jacobs (2014)—to clarify the interpretation of the network structure that we identified in Section 3.

Unlike the LSNM, the biSBM explicitly models the interaction between a politician and an interest group as a function of their community memberships, allowing us to verify that politicians and interest groups with shared group memberships interact more frequently. Although this model confirms the existence of communities in the lobbying network, it also exposes the limitations of assigning a single community membership to each actor. To address this, we propose the Bipartite Link Community Model (biLCM) in Section 4.2. The proposed model allows for multiple community memberships as well as heterogeneous connection types, which are prevalent in most political networks. We derive an expectation-maximization (EM) algorithm to estimate the biLCM parameters, which characterize the mixed community memberships of interest groups and politicians in the 113th Congress. Our analysis illustrates a clear distinction between “specialist” and “generalist” interest groups and identifies the politicians with whom each kind is strongly connected.

4.1 Motivation: The Bipartite Stochastic Block Model

The Model Following the formulation of the LSNM in Section 3, we model $A_{i,j}$ as having a Poisson distribution, whose mean now depends exclusively on the community memberships of
interest group $i$ and politician $j$. Formally, we introduce a matrix parameter $B \in \mathbb{R}^{k \times \ell}$ with entries $B_{r,s}$, where $k$ is the number of interest group communities and $\ell$ the number of politician communities, viewed as hyperparameters. To fix the dimensions, we draw upon our exploratory analysis with the LSNM, which suggests roughly how many distinct communities occur in the lobbying network. Based on our analysis of Figure 3, we use eight politician and interest group communities\(^\text{17}\) The membership parameters are denoted by $x_i \in [k]$ and $y_j \in [\ell]$ for each $i \in [m]$ and $j \in [n]$. As before, we include popularity factors $\alpha_i$ and $\beta_j$ for each interest group and politician, respectively. The entries $A_{i,j}$ are then modeled as

\[
A_{i,j} \sim \text{Poisson}(\alpha_i \beta_j B_{x_i,y_j})
\]

independently, so that the joint probability is

\[
P(A \mid B, x, y, \alpha, \beta) = \prod_{i=1}^{m} \prod_{j=1}^{n} \text{Poisson}(A_{i,j} \mid \alpha_i \beta_j B_{x_i,y_j}).
\]

In the block modeling literature, the inclusion of popularity factors is known as \textit{degree correction}, because these variables account for the possibility of broad degree distribution in a network. The model described by equation (5) is therefore named the \textit{Degree-Corrected Bipartite Stochastic Block Model (dc-biSBM)}\(^\text{18}\).

Unlike our analysis of the LSNM, the dc-biSBM does not \textit{a priori} assume that the community structure underlying the lobbying network is \textit{assortative}. That is, we allow for the possibility that interest groups and politicians do not form shared communities within which they prefer to interact, but rather form “blocks” that characterize their interactions in other ways, so long as

\(^{17}\)Our substantive findings are robust to adjustments of the number of communities in the community models we work with. Rigorous model selection for this parameter is an active area of research in block modeling, so we leave such quantitative justifications for future research.

\(^{18}\)We discuss the advantages of dc-biSBM over the Bipartite Stochastic Block Model (biSBM) without degree correction for analyzing the lobbying network in Appendices C.1 and E.
the strength of the interaction between an interest group and a legislator depends only on their respective block memberships. Nonetheless, we find that the former situation indeed obtains, and interest groups and legislators interact strongly only when they share a community membership. We support this claim quantitatively in Appendix C.2.

**Computation** We perform maximum likelihood parameter estimation for the dc-biSBM using the EM algorithm implementation provided by [Larremore, Clauset, and Jacobs (2014)](#). Since this algorithm is not guaranteed to converge to the parameters maximizing the likelihood, we take the parameters attaining the highest likelihood value of 50 randomly initialized runs.

**Empirical Findings** In the LSNM, we interpreted the proximity of actors’ latent positions as an indication of their shared community membership. To validate this interpretation, we begin by comparing the LSNM latent position results from Section 3 with the community memberships found by the dc-biSBM.

Figure 5 presents strong evidence for the existence of community structure in the lobbying network. It overlays the finite cluster memberships (marked by different colors and shapes) identified by the dc-biSBM on the geometry obtained from the LSNM. To facilitate a direct comparison, we also include the grey boundaries previously used to classify interest groups and politicians. The figure shows that the LSNM and dc-biSBM community structures align remarkably well with each other. Specifically, the boundaries that we drew in the LSNM indeed correspond to distinct political communities or to mixtures of at most three political communities identified by the dc-biSBM. For example, we find that all of the interest groups and legislators in the LSNM’s “Veterans’ Affairs” community belong to a single dc-biSBM community (orange plus signs). Likewise, in the “Energy” cluster, 84% of the interest groups and 86% of the legislators in the LSNM community belong to a single dc-biSBM community (pink diamonds). Furthermore, two pairs of adjacent LSNM com-

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19 We combine the dc-biSBM groupings of interest groups and legislators into eight joint communities when we present our findings in this section, as justified by the previous observation of assortativity.
Figure 5: **Latent Space Position vs. Community Membership.** We plot estimated latent positions from the LSNM together with eight different markers indicating the estimated community memberships from the dc-biSBM. Each marker designates one community of politicians and one community of interest groups, paired because of their proximity in the latent space. The gray regions and the dashed oval are the ex post LSNM community boundaries from Figure 3.

Communities are merged into single dc-biSBM communities: “Universities” and “Industrial Research & Defense” (grey circles), and “Technology” and “Telecommunications” (green squares). Both instances support our earlier interpretation that adjacent LSNM communities represent closely related topical interests.

However, the dc-biSBM provides further insights about community structures that were not legible in the LSNM. First, the dc-biSBM uncovers two distinct communities near the center of the latent space. The first, which we name “Civil Society,” includes leading civil rights associations (**American Civil Liberties Union**) as well as lobby organizations for the rights of workers (**AFL-CIO**), senior citizens (**AARP**), women (**National Women’s Law Center**), the disabled (**Easter Seals**), gender and sexual minorities (**Human Rights Campaign**), and ethnic minorities (**American Jewish Committee**). The second, which we name “Retail &
1. Lafarge Corporation (LG)  
2. Applied Materials  
3. Texas Instruments  
4. McAfee, Inc.

1. American Motorcyclist Assn.  
2. CropLife America  
3. Assn. of American Railroads  
4. British Petroleum (BP)

Figure 6: **Overlapping Communities in LSNM and dc-biSBM.** We plot regions where pairs of communities discovered by the dc-biSBM overlap in their LSNM latent positions. These lie near the central region highlighted in Figure 3 and Figure 5 with a dashed oval, which we repeat in each plot for reference. For the sake of visual clarity, in each plot we include only the actors from the relevant pair of communities. We also highlight several illustrative examples of interest groups in each plot. (Markers and colors are the same as those used in Figure 5.)

Transportation,” includes agricultural groups (National Corn Growers), manufacturing associations (National Association of Manufacturers), automotive firms (Nissan), transportation associations (Association of American Railroads), retailers (Home Depot), fuel manufacturers (British Petroleum), and environmental conservation groups (American Rivers).

Second, the dc-biSBM clarifies a key structural feature of the lobbying network: many significant political actors, previously found in the center region of the LSNM, represent complex collections of interlocking political interests instead of a single industry. Figure 6 illustrates this point. Specifically, it zooms in on different parts of the center region of the LSNM, focusing on three pairs of adjacent communities. We find that these communities overlap, and that actors in their intersections tend to lobby on bills related to the subject matters of both communities. For instance, McAfee, Inc. (in the left panel), a security software firm (and subsidiary of Intel...
CORPORATION), lobbied on both 113th S. 1429 “Department of Defense Appropriations Act” (as did defense industry firms such as GENERAL DYNAMICS and NORTHROP GRUMMAN) and 113th H.R. 756 “Cybersecurity Enhancement Act” (as did technology firms including MICROSOFT, INC. and GOOGLE, INC). Accordingly, McAfee, Inc. in the LSNM lies near the intersection of the “Technology & Telecommunications” and “Universities & Research” communities. Similarly, BRITISH PETROLEUM (in the center panel), an oil energy firm, lobbied on both 113th H.R. 1191 “Keep American Natural Gas Here Act” (as did utility companies including NiSOURCE, INC. and DOMINION RESOURCES) and 113th H.R. 1462 “RFS Reform Act,” an amendment to the “Clean Air Act” concerning renewable fuels (as did retail organizations like the NATIONAL RETAIL FEDERATION and the SNACK FOOD ASSOCIATION). Accordingly, BRITISH PETROLEUM lies near the intersection of the “Energy” and “Retail & Transportation” communities. Finally, the CHAMBER OF COMMERCE (in the right panel) is classified into the “Civil Society” community; however, being among the most active interest groups in the lobbying network, the CHAMBER OF COMMERCE lobbied on a great variety of bills, ranging from 113th S. 601 “Water Resources Development Act of 2013,” to 113th S. 462 “United States-Israel Strategic Partnership Act of 2013,” to 113th S. 809 “Genetically Engineered Food Right-to-Know Act.”

Importantly, these findings suggest that the assignment of each actor to a single community by the dc-biSBM is severely limited, because many actors participate simultaneously in multiple communities in the lobbying network. Indeed, consistent with the analysis from the LSNM, we find that the actors in the center region are typically large groups with diverse interests and senior politicians with broad authority, both of whose legislative pursuits might plausibly be driven by varied interests. The dc-biSBM thus furnishes additional evidence that the regions of the latent space without clear community structure are characterized by varied interests. In the following section, therefore, we propose a new community model that represents the varied interests of political actors in an explicit probabilistic framework.
4.2 Proposed Methodology: The Bipartite Link Community Model

In this section, we propose the Bipartite Link Community Model (biLCM). Our model combines three features which, to the best of our knowledge, have not been previously combined in the community detection literature. First, it explicitly models mixed membership through link communities (“links” being a generic term from network theory for what we call “interactions”); second, it does so in the bipartite setting; and finally, it uses a statistically principled generative model. Our model is most similar to that of Ball, Karrer, and Newman (2011), but borrows the ideas for specialization to bipartite graphs from Larremore, Clauset, and Jacobs (2014). The most similar work we are aware of is that of Li, Zhang, and Zhang (2015), who treat the link community detection task in bipartite graphs as an optimization problem and solve it with an ad hoc genetic algorithm. Unlike that work, our model provides an underlying probabilistic model and therefore a natural statistical interpretation.

Mixed Memberships In political networks, actors often have memberships in multiple communities. As we saw in Section 4.1, many interest groups either have heterogenous interests across various policy domains or represent interests of diverse constituents. For instance, a cybersecurity firm might be interested in both national defense policy and internet regulation, and an oil energy firm might be interested in both restrictions on utility providers and pollution regulation. Large holding companies, meanwhile, will lobby in multiple arenas on behalf of their diverse subsidiaries. For instance, the firm Philips North America, which specializes in medical equipment, home appliances, and lighting solutions, lobbies both on bills related to healthcare and bills related to retail and transportation. Perhaps the broadest lobby group is the Chamber of Commerce, which represents business interests at large. Similarly, most politicians are interested in several policy domains (Lauderdale and Clark 2014 and sit on several committees that have distinct legislative jurisdictions.
Mixed Interactions  When it comes to interacting with legislators, the behaviors of interest
groups with broad interests fall into two categories. In some cases, an interest group lobbies
several different legislators who serve on different topical committees. For instance, continuing
an earlier example, British Petroleum lobbied on 113th H.R. 1191 “Keep American Natural
Gas Here Act” sponsored by Representative Edward Markey (D-MA), a member of the House
Committee on Energy and Commerce, as well as 113th H.R. 1462 “RFS Reform Act,” sponsored
by Representative Bob Goodlatte (R-KY), the vice chair of the House Committee on Agriculture
(a committee intimately related with renewable fuels, the subject of the legislation).

In other cases, an interest group interacts with a single politician on bills that do not belong to
the same political issue area. This occurs most frequently with aggregate interest groups or holding
companies and senior politicians who tend to have seats on power committees. For example,
as mentioned earlier, the Chamber of Commerce lobbied on the three diverse bills: 113th
Partnership Act of 2013,” and 113th S. 809 “Genetically Engineered Food Right-to-Know Act.”
In fact, all three of these bills were sponsored by the same legislator, Senator Barbara Boxer (D-
CA). Similarly, the Specialty Equipment Market Association lobbied on 113th S. 983
“Keep the IRS Off Your Health Care Act of 2013,” 113th S. 2635 “21st Century Endangered
Species Transparency Act,” and 113th S. 725 “Small Business Taxpayer Bill of Rights Act of
2013.” Again, all three were sponsored by a single legislator, Senator John Cornyn (R-TX).

Modeling Choices  These observations guide our choice of a model that allows political actors
to have simultaneous memberships in several legislative communities. One model that incorporates
this assumption is the mixed-membership stochastic block model (mmSBM), in which each actor has
a probability distribution over all communities (Airoldi et al., 2008). For example, Airoldi et al.
(2008) apply the mmSBM to the Sampson monastery dataset (Sampson, 1969), a popular example
Figure 7: Schematic Comparison of Community Models. We illustrate three generative network models: the single-membership dc-biSBM model, the mixed-membership mmSBM model, and the mixed-membership biLCM model. For the sake of simplicity, the first two models are illustrated in the assortative case, where each politician community interacts strongly with just one interest group community and vice-versa. Colors indicate community memberships and, in the mixed-membership cases, interaction types. (a) In the single-membership model, political actors within a community interact more frequently. (b) The mmSBM allows for mixed membership distributions while all interactions involving a given partner are constrained to be of a single type. (c) The biLCM allows for mixed membership as well as mixed interaction types. This captures the common lobbying interactions in which interest groups and politicians with diverse community memberships interact for different political reasons.

in the social networks literature. This dataset summarizes survey data on social relationships among 18 novice monks joining a monastery, and the model identifies a group of “waverer” monks who do not commit to one of the primary social factions in the monastery but rather maintain friendships with members of several different factions.

Although this model has some desirable features, the mmSBM has the property that an actor varies its community membership across its interactions with different partners but not with a specific single partner. We illustrate this important conceptual difference in Panels (b) and (c) of Figure 7. Panel (b) shows that a model in the mmSBM family would bizarrely insist that a single common community membership must account for each interaction (i.e., “link”) involving a given politician and interest group. However, as we discussed before, the mixed membership we

\[20\] We consider an extended version of the original mmSBM with assortative and weighted relationships that are closer to the data we observe in the lobbying network. Mathematically, the difference we are emphasizing is that an mmSBM adjusted to have Poisson-distributed weights on interactions would model each edge weight as a mixture of Poisson distributions, while the link community model we propose will model each edge weight as a sum of independent Poisson distributions, one for each link community in which the agents interact.
are interested in modeling is different: we are concerned with political actors whose community membership varies even across interactions with a single partner over the course of a session of Congress. Indeed, the bills that link a given politician and interest group often have different underlying motivations and subject matters, and thus should be modeled as such.

**The Model** We suppose that there are $k$ link communities, which we will always take as indexed by a variable $z \in [k]$. To emphasize the political interpretation, we will subsequently refer to these as *legislation communities*. Each interest group and politician has a vector of parameters $\alpha_{i,z}$ and $\beta_{j,z}$, respectively, which represents their involvement in legislation belonging to community $z$. The number of bills lobbied by interest group $i$, sponsored by politician $j$, and belonging to legislation community $z$ is modeled as Poisson with a mean proportional to $\alpha_{i,z}\beta_{j,z}$. To resolve the identification issue, we assume that for each fixed $z$, $\sum_{i=1}^{m} \alpha_{i,z} = \sum_{j=1}^{n} \beta_{j,z} = 1$, and we introduce another parameter $\kappa_z$ to capture the overall weight of group $z$, so that the number of bills between interest group $i$ and politician $j$ in legislation community $z$ has mean $\kappa_z \alpha_{i,z} \beta_{j,z}$. We assume that these Poisson variables are independent, so that the model for the interaction matrix entries is given by

$$A_{i,j} \sim \text{Poisson} \left( \sum_{z \in [k]} \kappa_z \alpha_{i,z} \beta_{j,z} \right),$$

and their joint distribution is

$$P(A \mid \alpha, \beta, \kappa) = \prod_{i=1}^{m} \prod_{j=1}^{n} \text{Poisson} \left( A_{i,j} \mid \sum_{z=1}^{k} \kappa_z \alpha_{i,z} \beta_{j,z} \right).$$

**Computation** We derive an expectation-maximization algorithm for this model in Appendix D.1, which involves alternating expectation and maximization update steps until the log-likelihood of the model converges. The update equations produced by our derivation are given below, including ancillary optimization parameters $q_{i,j}(1), \ldots, q_{i,j}(k)$. The first equation is the expectation step
for the ancillary parameters, and the last three equations are maximization steps for the model parameters.

\[ q_{i,j}(z) = \frac{\kappa_z \alpha_{i,z} \beta_{j,z}}{\sum_{k=1}^{K_z} \kappa_z \alpha_{i,z} \beta_{j,z}}, \]

(9)

\[ \kappa_z = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} A_{i,j} q_{i,j}(z)}{\sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_{i,z} \beta_{j,z}}, \]

(10)

\[ \alpha_{i,z} = \frac{\sum_{j=1}^{n} A_{i,j} q_{i,j}(z)}{\sum_{i=1}^{m} \sum_{j=1}^{n} A_{i,j} q_{i,j}(z)}, \]

(11)

\[ \beta_{j,z} = \frac{\sum_{i=1}^{m} A_{i,j} q_{i,j}(z)}{\sum_{i=1}^{m} \sum_{j=1}^{n} A_{i,j} q_{i,j}(z)}. \]

(12)

As with the dc-biSBM, this procedure is not guaranteed to converge to the parameters maximizing the likelihood, so we take the parameters obtained from 50 randomly initialized runs and choose those that attain the highest likelihood value.

**Empirical Findings**  We focus our analysis on how many different legislation communities an actor effectively belongs to. We first describe a simple way to quantify this notion. From the model definition, the mean total number of times interest group \( i \) lobbies in community \( z \) equals \( \kappa_z \alpha_{i,z} \), and likewise the mean total number of times politician \( j \) sponsors in community \( z \) equals \( \kappa_z \beta_{j,z} \). A natural choice of measurement then is to normalize these quantities to form probability distributions \( p_{i,z} = \frac{\kappa_z \alpha_{i,z}}{\sum_{z=1}^{K_z} \kappa_z \alpha_{i,z}} \) and \( q_{i,z} = \frac{\kappa_z \beta_{j,z}}{\sum_{z=1}^{K_z} \kappa_z \beta_{j,z}} \), and then consider the (Shannon) entropies \( H_i = H(p_{i,1}, \ldots, p_{i,k}) \) and \( H_j = H(q_{j,1}, \ldots, q_{j,k}) \), where \( H(c_1, \ldots, c_k) = - \sum c_z \log_2 c_z \).

In computing entropies, we take logarithms base 2 for the sake of interpretability: an entropy of \( H \) may then be interpreted as, roughly speaking, an agent typically participating in \( 2^H \) legislation communities. Simply put, agents with higher entropy values are members of more legislation communities than agents with lower entropy values.

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\[ ^{21} \text{We reuse the letter } H \text{ for these quantities since it will always be clear from context whether we are discussing the entropy of interest groups or of politicians. A more principled quantity to consider would be the mean of the entropy of empirical link distributions drawn from our model with a given set of parameters. We do this and find in practice that this mean agrees almost perfectly (Pearson’s } \rho = 0.99 \) \text{ with the quantity we compute.} \]
The left panel divides the LSNM latent space into hexagonal regions (keeping only those containing at least one actor). We shade each region by the average entropy of legislation community memberships of the agents in that region: the darker the color, the more communities the actors are active members of. The right panel plots example legislation community distributions at their corresponding latent positions. Three examples discussed earlier are highlighted and labeled for reference.

With this measure, the biLCM model can shed light on the properties of the region near the center of the LSNM latent space. Recall that we did not find legible community structure in this region based on the LSNM, while the dc-biSBM led us to suppose that this region might be characterized by varied interests. To evaluate this possibility, the left panel of Figure 8 repeats the latent space plot from Figure 3 but now divides the latent space into small hexagonal parcels and shades them according to the mean legislation community membership entropy of actors within them. We clearly observe that actors with more memberships (darker hexagons) are clustered near the center of the latent space.

We provide a schematic visual representation of the same phenomenon in the right panel, in which we represent each actor with a pie chart showing their estimated legislation community memberships. Again, we find that actors in the LSNM center region tend to have memberships in

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22The communities discovered by the biLCM are well aligned with those previously discovered by the dc-biSBM, so we reuse their labels, associated colors, and substantive interpretations. Quantitative results supporting this alignment are presented in Appendix E.
many different political communities (marked by pie charts divided among many colors). We highlight three examples discussed previously: the Chamber of Commerce, which as we suggested earlier has membership in many legislation communities; British Petroleum (BP), whose activities are mostly divided between the “Energy” (pink) and “Retail & Transportation” (purple) communities; and McAfee, Inc., whose activities are mostly divided between the “Universities & Research” (gray) and “Technology & Telecommunications” (green) communities. In addition, the biLCM consistently finds many of the same industry-specific communities at the edges of the latent space as we have seen before, such as the energy, technology, and finance industries, whose members typically participate in only one or two legislation communities (indicated by pie charts with only one or two dominant colors).

As we saw in Figure 4, the politicians in the central LSNM cluster tend to belong to power committees that have broad responsibilities of government oversight or budgetary allocation. The interest groups in this cluster also admit a simple and politically meaningful description: many of them are large associations that represent numerous small businesses, individuals, or causes, such as the Chamber of Commerce, the Specialty Equipment Market Association, Heritage Action for America, and the National Taxpayers Union. This analysis suggests that these actors participate in lobbying through aggregate action in the lobbying marketplace, either by lobbying politicians on power committees or by serving as a proxy for individuals or smaller firms to lobby many politicians who sit on committees with disparate jurisdictions. We believe that these findings indicate a unique aggregate mode of lobbying, wherein politicians with control over many policy areas are lobbied by numerous otherwise unrelated interest groups while groups representing the diverse interests of many constituents lobby numerous otherwise unrelated politicians.

In Table 2, we present the legislation community memberships of several interest groups, including some discussed previously. These distributions illustrate the full range of lobbying behaviors
<table>
<thead>
<tr>
<th>Interest Group</th>
<th>Distribution</th>
<th>Example Bills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microsoft</td>
<td>. . .</td>
<td><em>Startup Act 3.0</em> <em>Law Enforcement Access to Data Stored Abroad Act</em> <em>Satellite Television Access and Viewer Rights Act</em></td>
</tr>
<tr>
<td>Microsoft</td>
<td></td>
<td><em>Cybersecurity Enhancement Act</em></td>
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<tr>
<td>McAfee, Inc.</td>
<td>.</td>
<td><em>Department of Defense Appropriations Act</em></td>
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<tr>
<td>McAfee, Inc.</td>
<td></td>
<td><em>Energy Efficient Government Technology Act</em></td>
</tr>
<tr>
<td>Arch Coal</td>
<td>.</td>
<td><em>Climate Protection Act</em></td>
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<tr>
<td>British Petroleum (BP)</td>
<td>. . .</td>
<td><em>Keep American Natural Gas Here Act</em></td>
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<td>British Petroleum (BP)</td>
<td></td>
<td><em>RFS Reform Act</em></td>
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<tr>
<td>British Petroleum (BP)</td>
<td></td>
<td><em>Corn Ethanol Mandate Elimination Act</em></td>
</tr>
<tr>
<td>Philips North America</td>
<td>. . . .</td>
<td><em>Protect Medical Innovation Act</em></td>
</tr>
<tr>
<td>Philips North America</td>
<td></td>
<td><em>Medicare DMEPOS Market Pricing Program Act</em></td>
</tr>
<tr>
<td>Philips North America</td>
<td></td>
<td><em>Renewable Energy Parity Act</em></td>
</tr>
<tr>
<td>Chamber of Commerce</td>
<td>. . . . . .</td>
<td><em>Water Resources Development Act</em></td>
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<td>Chamber of Commerce</td>
<td></td>
<td><em>United States-Israel Strategic Partnership Act</em></td>
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<tr>
<td>Chamber of Commerce</td>
<td></td>
<td><em>Genetically Engineered Food Right-to-Know Act</em></td>
</tr>
</tbody>
</table>

Table 2: Legislation Community Distribution Examples: Interest Groups. We give examples of interest groups with small, intermediate, and large legislation community memberships, along with examples of bills illustrating their interests. Memberships are plotted as histograms, with eight bars corresponding to the eight legislation communities in the biLCM. We observe some overlap in the community memberships of these groups. For instance, Microsoft and McAfee, Inc. share an involvement in the “technology” community, colored green, while Arch Coal and British Petroleum share an involvement in the “energy” community, colored pink.

the biLCM identifies in the lobbying network. First, firms such as Microsoft and Arch Coal lobby on a single topic: both interact in one community (“Technology & Telecommunications,” colored green, and “Energy,” colored pink, respectively) with probability at least 85%. Second, firms with broader interests, such as McAfee, Inc. and British Petroleum, lobby primarily in two or three arenas. McAfee, Inc., for instance, interacts in the “Technology & Telecommunications” community with probability 31% and in the “Universities & Research” community, colored gray, with probability 52%. Finally, large holding companies such as Philips North America or lobbying organizations such as the Chamber of Commerce lobby most categories.
How do we know that the community structure the **biLCM** identifies arises from the structured interactions of interest groups and legislators rather than merely their individual propensities to lobby or sponsor? To provide an answer, we perform a permutation test on the lobbying network to obtain a null distribution of random networks with similar degree distribution to the lobbying network (see Appendix D.2 for technical details). We then compute entropies for the **biLCM** legislation community memberships in these random networks, and compare to the entropies from the model of the actual lobbying network. Since the **biLCM** can capture both single and multiple community memberships, this is a useful test of the statistical significance of the community structure we have been discussing: if the lobbying network has strong community structure compared to a “typical” similar network, then the **biLCM** should find fewer community memberships for actors in the lobbying network than for actors in a generic network. Indeed, we find that this is the case, observing that interest groups in the null model have much higher numbers of legislation community memberships than interest groups in model of the actual lobbying network.

On the whole, we find that the **biLCM** successfully describes the multiple community memberships of political actors and how interactions among actors occur in those communities. The proposed methodology allows researchers to quantitatively estimate the participation in various communities for each political actor. Furthermore, it allows researchers to relate interest groups with specialized and general interests to politicians with narrow and broad political connections. In sum, the proposed methods not only capture the community structure that we often cannot observe in political networks, but also provide a useful guide for gauging the varied interests and complex preference aggregation that drives lobbying in legislative politics.

---

23. We present analogous examples for politicians in Table E.2 finding that the legislators with the most legislation community memberships are often senior politicians or party leaders.

24. On the other hand, legislators in the null model have similar numbers of legislation community memberships to legislators in the model of the actual lobbying network. This agrees with our previous observation that varied interests are more typical for politicians than for interest groups: while many interest groups are focused on specific industries, most politicians are involved in a variety of policy domains.
5 Concluding Remarks

Lobbying is known to be an important channel through which special interest groups influence the U.S. legislative process. Even so, observable connections between interest groups and the politicians they target have proved elusive, since the individual politicians contacted in the course of lobbying need not be disclosed. Because interest groups must report the bills on which they lobby, however, and each bill has a unique sponsor, we are able to assemble a new *lobbying network* dataset that measures legislative interactions between interest groups and politicians for all bills introduced since the 106th Congress.

We show that this network can be usefully modeled with both latent space models and community models. Unlike previous applications of latent space models in political science, our models suggest that lobbying interactions depend not on an ideological spectrum, but rather on domain-specific community membership. Most of these domains are organized around industry interests and political issues, including energy, finance, and veterans’ affairs. Interest groups with concerns in these domains lobby politicians who sit on congressional committees with relevant jurisdiction, such as the House Committee on Energy and Commerce, the House Committee on Financial Services, and the House Committee on Veterans’ Affairs. Stochastic block models confirm and clarify this finding by explicitly modeling the community structure that we observe. Furthermore, our bipartite link community model illustrates that politicians who serve in power committees with broad responsibilities are active in multiple political communities and are linked to aggregate interest groups that represent heterogeneous political interests. The consistency between the latent space model and non-geometric community models in describing these patterns provides strong evidence that the interest group-politician network in U.S. legislative politics is not organized according to ideology. It is shared interests, not shared ideology, that drives lobbying.
By showing that it is helpful to view lobbying activity as occurring in a network of political actors, our findings suggest that other network analysis techniques may reveal further meaningful patterns in lobbying in the U.S. political system. Of course, the lobbying network contains more information than we have analyzed here, including bills and their texts, lobbyists and their lobbying firm affiliations, the money spent on lobbying, and the timing of lobbying and legislative events. Extending the models in this work to describe new types of agents and interactions should further enhance the understanding of lobbying and its political effects that we have obtained here.
References


Online Appendix

Mapping Political Communities: A Statistical Analysis of Lobbying Networks in Legislative Politics*†

In Song Kim‡ Dmitriy Kunisky§

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‡Associate Professor, Department of Political Science, Massachusetts Institute of Technology, Cambridge, MA, 02139. Email: insong@mit.edu URL: http://web.mit.edu/insong/www/
§Ph.D. Student, Department of Mathematics, Courant Institute of Mathematical Sciences, New York University, New York, NY, 10012. Email: kunisky@cims.nyu.edu URL: http://www.kunisky.com/
A Dataset Construction

A.1 Identifying Bills and Missing Congress Numbers

Identifying congressional bills in lobbying reports is difficult because bill numbers are repeated across Congresses, and often do not appear directly annotated with Congress numbers in lobbying reports. Using the report filing year to guess the Congress often leads to erroneous matches, because reports filed at the beginning of a new Congress tend to include disclosures of lobbying activities from the previous year (and therefore, if a new Congress has begun recently, from the previous Congress as well). For example, consider the following lobbying report filed by GOOGLE, INC. in 2013. It reads:

| Monitor legislation regarding online privacy including Safe Data Act (H.R. 2577, S. 1207) and Do not track proposals (H.R. 654). Monitor any Congressional or Administration efforts to impose privacy laws on search engines. Monitor Spectrum acts (S. 911, H.R. 2482). |

Figure A.1: First Quarter Report by GOOGLE, INC. in 2013

A naive guess would be that the bill H.R. 2577 refers to a bill from the 113th Congress, because the report was filed in 2013. However, it is clear from the report that this is a bill from the 112th Congress, the “SAFE Data Act.” We use the following strategies to mitigate this problem and correctly identify Congress session numbers under various circumstances.

1. **Bill Number Search:** We first identify bill numbers (e.g., H.R. 2577 above) using regular expression search in the report text. In the above example in Figure A.1, our algorithm would identify bill numbers H.R. 2577, S. 1207, H.R. 654, S. 911, and H.R. 2482. Note that all of these bills are from the 112th Congress rather than the 113th.

2. **Congress Identification:** Given a bill number found in a specific issue text (a section of the lobbying report), we attempt to identify the most likely Congress to which that bill would belong using other text around the bill number. We consider a range of candidate Congresses extending backwards from the Congress containing the year that the lobbying report was filed. By default, we consider the three preceding Congresses; in the above example, therefore, we would consider the 113th, 112th, and 111th Congresses. We then retrieve the bills having the same number as the given bill from each of these Congresses (omitting the Congresses that do not have a bill of that number), and compute a bag-of-words representation (after a tokenization and stopword filtering pipeline) of each of those bills, producing vectors \( \mathbf{v}_1, \ldots, \mathbf{v}_n \) representing the \( n \) candidate bills. We also compute
the same representation of the text around the mention of the bill number in the lobbying report, producing a vector $\mathbf{w}$ representing that text. We then choose the Congress number by maximizing the cosine similarity between the $\mathbf{v}_i$ and $\mathbf{w}$, choosing bill $i^*$ with index given by

$$i^* = \arg\max_{1 \leq i \leq n} \frac{\mathbf{v}_i^T \mathbf{w}}{\|\mathbf{v}_i\| \|\mathbf{w}\|}.$$  

If no bill having the same number exists in the entire range of Congresses we consider, we simply guess that the bill comes from the Congress of the year the lobbying report was filed.

3. **Congress Propagation:** If we successfully find a match for a Congress, it may be propagated to the other bills mentioned in the lobbying report, since, being scheduled on a quarterly basis, lobbying report will almost always only mention legislation from a single Congress. If different bills in a lobbying report disagree on the best-matching Congress, a majority vote may be taken, but this rarely occurs in practice.

4. **Bill Title Search:** Bills are sometimes only referred to by titles or alternate names. To account for this, we clean and tokenize the specific issue sections of the lobbying report, and perform a text matching operation against a table of bill titles. For instance, this operation would identify “Safe Data Act” in our previous example, even if the bill number H.R. 2577 were not mentioned.

5. **Bill Range Expansion:** It is also common for bills with nearby numbers to be related, and for lobbying reports to refer to ranges of bills when lobbying all of them at once. For instance, a lobbying report filed by MATTEL, INC. in 2002 contains the following text:

```
"H.R.3009, Trade Act of 2002. Certain miscellaneous tariff bills to suspend the rates of duty on certain toy-related articles (H.R.4182-4186; S.2099-2103). WTO market access negotiations for non-agricultural products Port and border security measures"
```

Figure A.2: Midyear Report by MATTEL, INC. in 2002

Therefore, if we find two bill numbers that are close (by default, we take this to mean that they share the same prefix and their numbers differ by at most 10), then we consider all other bills with numbers in between as also being lobbied in the same report. For instance, the pattern “H.R. 4182-4186” in the excerpt shown in Figure A.2 would be expanded into bills H.R. 4182, H.R. 4183, H.R. 4184, H.R. 4185, and H.R. 4186, all of which we would consider lobbied on by MATTEL, INC.
A.2 Filtering

Besides reducing the size of our data, the key task in filtering is to increase the minimum degree of the actors included for analysis in the lobbying network. This is especially important for the LSNM; it is intuitive that there is no principled way to position an actor in the lobbying marketplace if the actor does not lobby or sponsor very frequently. Computationally, this issue is reflected in poorer convergence properties for actors with fewer interactions. Thus, while it may appear appealing at first to filter the dataset independently for interest groups and politicians by considering summary statistics like total numbers of bills sponsored or lobbying reports submitted during a Congress, such filtering is not necessarily aligned with the goal of finding a large submatrix of the interaction matrix $A$ (or an induced subgraph of the bipartite lobbying network graph) with only agents of sufficiently high degree. In particular, we must avoid politician filtering causing some interest groups who survived interest group filtering to have their degree reduced again, or vice-versa.

We find that the simplest way to avoid such problems is to build the full interaction matrix $A$ without filtering first, and then filter based only on interactions in a way that explicitly ensures that only high-degree agents remain. To that end, we use a filtering procedure defined by two thresholds, denoted $T_I$ and $T_P$, which alternates removing all interest groups with degree lower than $T_I$ and removing all politicians degree lower than $T_P$, until no more interest groups or politicians are removed in a full iteration. This is a simple greedy algorithm for finding an induced subgraph of the full lobbying network with only high-degree nodes, which we find to suffice for our purposes. Typically only a few iterations are required for the algorithm to terminate, but never does just one iteration suffice on the datasets we consider, showing that the algorithm is indeed performing additional filtering beyond the first thresholding step.

We construct our main dataset by the above procedure with $T_I = 30$ and $T_P = 5$. This produces a dataset with 676 interest groups and 523 legislators. While, as mentioned in the main text, in the unfiltered dataset 97.96% of pairs of actors do not interact, in the filtered dataset this proportion is reduced to 90.23%.

B LSNM Additional Information

B.1 Parameter Identification

Several transformations of the parameters $\alpha, \beta, \theta_i, \psi_j$ leave the means of the $A_{i,j}$ unchanged, so we cannot expect the posterior distribution not concentrate on a single collection of parameter values without imposing further constraints. There are four important families of symmetries
under which our model is invariant, which we summarize below (for more detailed discussion, see e.g. Jackman (2001)). These are scaling, where for any invertible symmetric matrix $S \in \mathbb{R}^{d \times d}$, each $\theta_i$ may be multiplied by $S$ and each $\psi_j$ by $S^{-1}$; rotation, where for any orthogonal matrix $Q \in \mathbb{R}^{d \times d}$ each $\theta_i$ and $\psi_j$ may be multiplied by $Q$; popularity translation, where any constant may be added to each $\tilde{\alpha}_i$ and subtracted from each $\tilde{\beta}_j$; and mixed translation, where a vector $v \in \mathbb{R}^d$ may be added to each $\theta_i$ (resp. $\psi_j$) and $v^\top \psi_j$ (resp. $v^\top \theta_i$) subtracted from each $\tilde{\beta}_j$ (resp. $\tilde{\alpha}_i$).

We combine two strategies to accomplish identification, constraining the parameters to eliminate the above symmetries. Most of the task is accomplished by imposing hierarchical priors with constrained hyperparameters, as indicated by the form of the normal distributions appearing in equation (4). Specifically, the popularity translation invariance may be eliminated by assigning the $\tilde{\alpha}_i$ a normal prior with mean zero, and the mixed translation invariance by assigning the $\theta_i$ and $\psi_j$ a normal prior with mean zero. The scaling invariance is partly resolved by further setting the covariance matrices of the $\theta_i$ and $\psi_j$ to be equal; the only scaling transformations still admissible are those where $S$ is a symmetric orthogonal matrix. The rotation invariance is resolved by choosing this shared covariance matrix to furthermore be diagonal, which also constrains the above scaling invariances to those where $S$ is a permutation matrix, a diagonal matrix with diagonal entries $\pm 1$, or a product thereof.

The remaining task is to eliminate the discrete family of $2^d \cdot d!$ (e.g., two in the one-dimensional case) symmetries corresponding to applying arbitrary permutations and sign changes to vectors’ coordinates. Geometrically, these are compositions of reflections through certain hyperplanes in $\mathbb{R}^d$. We find that this task is most effectively accomplished not by assigning “symmetry-breaking” priors to the parameters of all agents, but rather by fixing the exact latent positions of a small number of distinguished agents. This generalizes the technique of Kubinec (2018) to multiple latent spatial dimensions. The main challenge in applying this technique is to choose the fixed agents carefully, so that it is not possible for one of the reflection symmetries to leave all of the fixed agents’ latent positions almost unchanged, in which case the constraints are said to “split the likelihood” (Bafumi et al. 2005). To address this, we first identify $2^d \cdot d!$ agents and fix their positions based on a fast variational estimation routine included in the Stan software package, and then run Monte Carlo sampling to perform the final inference. When $d = 2$, the agents to fix are chosen to maximize the number of samples drawn from the variational approximate posterior that lie in a single sector of the plane among the $2^2 \cdot 2! = 8$ sectors demarcated by the lines $y = 0$, $x = 0$, $y = x$, and $y = -x$ in the $(x, y)$ plane. Similarly, when $d = 1$, the agents to fix are chosen to maximize the number of samples lying in the positive or negative sides of the line. In both
cases, ties are broken by maximizing the norm of the agent’s mean latent position (following the heuristic of Kubinec (2018)).

This technique appears effective both on our dataset and on synthetic data drawn from the true data-generating process described by our Bayesian model. We consistently obtain Gelman-Rubin $\hat{R}$ statistics smaller than 1.01 for all parameters (as reported by Stan diagnostic logging) for both $d = 1$ and $d = 2$, and Pearson’s $\rho$ greater than 0.95 between the collections of posterior means estimated from different chains. Comparable correlations are also achieved with the “ground truth” values when data is drawn from the true data-generating process and when agent positions are fixed to their true values rather than the results of a variational approximation.

### B.2 Dimensionality Selection

We present some heuristics on choosing a dimensionality for the LSNM based on a simpler modeling technique, the spectral biclustering of Kluger et al. (2003). In this method, one computes the singular value decomposition (SVD) of the bipartite adjacency matrix $A$, and searches for piecewise constant structure among the leading singular vectors. The idea we will borrow from this method is simply that the leading singular vectors contain most of the relevant information for the analysis of $A$, proportionally to the magnitude of their singular values, essentially the same concept as in PCA analyses. We then make a plot of the singular values, analogous to a scree plot, and search for an “elbow” indicating the suitable number of singular vectors to use. That number gives an estimate of the dimensionality of the low-rank structure in $A$, and thus also a heuristic estimate of the dimensionality to take in the LSNM. In Figure E.2, we show that a dimensionality of two is a reasonable choice if this heuristic is admitted.

### B.3 Computation

The Stan code in Listing E.1 was used to fit our latent space models, run through the PyStan interface. In practice, by far the most important optimization for this model is the vectorization of the declaration of the Poisson distribution for edge weights. Factorizing the latent space position prior covariance matrix into Cholesky factors and using them to transform standard normal variables is a common optimization for multivariate normal priors, but our covariance matrix is so small that we find this not to affect (or even to degrade, in some cases) sampling performance.

For each latent space model we run four MCMC chains, each drawing 10,000 samples, the first 4,000 of which are discarded as a burn-in period, leaving us with 6,000 usable samples per chain, for a total of 24,000 samples. These are used to compute estimates of posterior means of all of
our models parameters as used in all visualizations in the main text.

**B.4 MCMC Diagnostics**

**Position Variances** We use a visualization that captures the variance structure of all of our latent space position estimates at once. For any agent, if we take $N$ samples of its $d$-dimensional position, we view these samples as the columns of a matrix $X \in \mathbb{R}^{d \times N}$. We let $\mu \in \mathbb{R}^d$ be the vector of means of the rows of $X$, and let $X_0$ be the centered matrix obtained by subtracting $\mu_i$ from each entry of the $i$th row of $X$. Then, $W = \frac{1}{N-1}X_0X_0^T \in \mathbb{R}^{d \times d}$ is the empirical covariance matrix of the position samples, which we diagonalize to obtain orthonormal eigenvectors $v_1, \ldots, v_d$ and associated non-negative eigenvalues $\lambda_1, \ldots, \lambda_d$. This is essentially a principal component analysis (PCA), except unlike the usual high-dimensional setting for PCA, we have many points in a low-dimensional space, $N \gg d$. Nonetheless, the computed quantities have the same interpretations: $\lambda_i$ is the amount of variance of the sampled positions in the direction $v_i$, and the shape of the set of sampled positions is approximately the ellipsoid with axes in the directions of the $v_i$ having lengths $\sqrt{\lambda_i}$.

For small $d$, in particular for $d = 2$ as we use for most of our models, these ellipses can be drawn for all embedded points at once. Small ellipses correspond to concentrated posterior distributions for latent positions, which justify the use of the mean position point estimate in our analysis in the main text. This visualization is given for the model inferred from our main dataset in Figure E.3.

**Popularity Factor Variances** For popularity factors $\alpha_i$ and $\beta_j$, it is more convenient to consider the variance of the estimates of the exponentials $\exp(\alpha_i)$ and $\exp(\beta_j)$: first, these are non-negative and so their statistics are easier to understand, and second they are the quantities we visualize and are ultimately more interested in, since the interaction mean scales linearly in each. We thus compute the standard error (standard deviation divided by the mean) of each of these quantities. As for the position variances, we visualize all of the standard errors at once, giving in Figure E.4 their histograms for our main dataset. We observe that the distributions of the standard errors are strongly concentrated on the low values; manual inspection reveals that the agents with the highest standard errors are typically those with the lowest degrees, which are less significant to our substantive analysis (these agents also tend to have smaller popularity factors, which can further inflate the standard error).

**Trace Plots** In Figure E.5, we show two representative sets of trace plots for latent space position and popularity factor parameters for one interest group and one politician. Sampling for
all parameters regardless of the group appears to mix rapidly following the warmup period.

C  biSBM Additional Information

C.1  Degree Correction

The uncorrected biSBM usually exhibits an undesirable property of grouping nodes by their degree, especially when the degree distribution of the network is not tightly concentrated, a feature that we observed previously in the lobbying network. In our case, the model is likely to cluster politicians who sponsor larger numbers of lobbied bills and interest groups who lobby many bills together, essentially grouping actors by their overall popularity rather than the specific legislation they tend to be involved with. To correct for this, we follow Larremore, Clauset, and Jacobs (2014) and introduce additional parameters $\alpha_i$ and $\beta_j$ per interest group and politician respectively, and adjust the model given by (5) to

$$A_{i,j} \sim \text{Poisson}(\alpha_i \beta_j B_{x_i,y_j}),$$

for $\alpha_i, \beta_j > 0$. We must now identify these new parameters, since among the sets of parameters $\alpha, \beta,$ and $B$, a positive constant can be multiplied to any one and divided from any other without changing the model. This is easily resolved by constraining $\sum_{x_i=a} \alpha_i = \sum_{y_j=b} \beta_j = 1$ for all $a \in [k]$ and $b \in [\ell]$. Letting $\text{deg}(\bullet)$ be the degree of an agent, the maximum likelihood values may be calculated as

$$\hat{\alpha}_i = \frac{\text{deg}(i)}{\sum_{x_i=x} \text{deg}(i')} , \quad \hat{\beta}_j = \frac{\text{deg}(j)}{\sum_{y_j=y} \text{deg}(j')} ,$$

the fractions of the edges leaving a node’s cluster that are leaving the node itself. Due to that interpretation of the new parameters, the resulting model is named the Degree-Corrected Bipartite Stochastic Block Model (dc-biSBM). As has been shown in Karrer and Newman (2011) and Larremore, Clauset, and Jacobs (2014), degree correction is an important modeling mechanism in networks with outlying high-degree nodes, in which setting the dc-biSBM typically obtains more meaningful clusterings than the biSBM. This distinction in our setting is discussed further in Appendix E.

C.2  Assortativity

In Figure E.6, we plot the mean interaction value between actors of each pair of communities discovered by the dc-biSBM. We observe that, after applying a suitable permutation to the community
labeling, that it is possible to match politician and interest group communities into strongly interacting pairs, such that most other pairs are weakly interacting. This suggests that the dc-biSBM output may be interpreted as placing politicians and interest groups into joint strongly interacting communities, a property known as assortativity.

**D biLCM Additional Information**

**D.1 EM Algorithm**

In this appendix, we present a more detailed derivation of the EM update equations for the link community model. After discarding constant terms, the log-likelihood of this model is given by

\[
\log P(A \mid \alpha, \beta, \kappa) = \sum_{i=1}^{m} \sum_{j=1}^{n} A_{i,j} \log \left( \sum_{z=1}^{k} \kappa_{z} \alpha_{i,z} \beta_{j,z} \right) - \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{z=1}^{k} \kappa_{z} \alpha_{i,z} \beta_{j,z} \tag{4}
\]

We then apply the standard technique of introducing parameters \( q_{i,j}(z) \) that form a probability distribution over \( z \) for fixed \( i \) and \( j \) and applying Jensen’s inequality to obtain the objective function of our optimization task, a lower bound on the log-likelihood:

\[
\mathcal{L}(A, \alpha, \beta, \kappa, q) \overset{\text{def}}{=} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{z=1}^{k} \left( A_{i,j} q_{i,j}(z) \log \left( \frac{\kappa_{z} \alpha_{i,z} \beta_{j,z}}{q_{i,j}(z)} \right) \right) - \kappa_{z} \alpha_{i,z} \beta_{j,z} \tag{5}
\]

\[
\leq \log P(A \mid \alpha, \beta, \kappa).
\]

We then seek to maximize \( \mathcal{L} \) via coordinate ascent. Maximizing with respect to the \( q_{i,j}(z) \) with all other parameters fixed simply sets these parameters to the values that make Jensen’s inequality sharp, which are

\[
q_{i,j}(z) = \frac{\kappa_{z} \alpha_{i,z} \beta_{j,z}}{\sum_{z=1}^{k} \kappa_{z} \alpha_{i,z} \beta_{j,z}}. \tag{6}
\]

It is also straightforward to differentiate with respect to \( \kappa_{z} \), obtaining the update

\[
\kappa_{z} = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} A_{i,j} q_{i,j}(z) \alpha_{i,z} \beta_{j,z}}{\sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_{i,z} \beta_{j,z}}. \tag{7}
\]

For the \( \alpha \) and \( \beta \) parameters, we must constrain our optimization to respect the normalization that for all \( z \), \( \sum_{i=1}^{m} \alpha_{i,z} = \sum_{j=1}^{n} \beta_{j,z} = 1 \). If add to \( \mathcal{L} \) Lagrange multiplier terms \( \lambda_{z} (1 - \sum_{i=1}^{m} \alpha_{i,z}) + \)
\[ \mu_z (1 - \sum_{j=1}^{n} \beta_{j,z}) \text{, then we obtain the updates} \]

\[
\begin{align*}
\alpha_{i,z} &= \frac{\sum_{j=1}^{n} A_{i,j} q_{i,j}(z)}{\lambda_z + \sum_{j=1}^{n} \kappa_z \beta_{j,z}}, \\
\beta_{j,z} &= \frac{\sum_{i=1}^{m} A_{i,j} q_{i,j}(z)}{\mu_z + \sum_{i=1}^{m} \kappa_z \alpha_{i,z}},
\end{align*}
\]

but now we see that to obtain the desired normalization we must in fact take \( \lambda_z \) and \( \mu_z \) such that they cancel out these denominators and leave us with the simpler

\[
\begin{align*}
\alpha_{i,z} &= \frac{\sum_{j=1}^{n} A_{i,j} q_{i,j}(z)}{\sum_{i=1}^{m} \sum_{j=1}^{n} A_{i,j} q_{i,j}(z)}, \\
\beta_{j,z} &= \frac{\sum_{i=1}^{m} A_{i,j} q_{i,j}(z)}{\sum_{i=1}^{m} \sum_{j=1}^{n} A_{i,j} q_{i,j}(z)},
\end{align*}
\]

which in particular do not involve any coupling between \( \alpha_{i,z} \) and \( \beta_{j,z} \), meaning that a single iteration of updates suffices for the M step of our EM algorithm, simplifying the calculation substantially compared to the nested coordinate ascents that are usually involved in mixed-membership stochastic block models.

### D.2 Null Model Analysis

The *configuration model*, a standard tool in the theory of random graphs, gives a natural way to build a null model for our analyses. In this model, the degree distribution of the graph is retained, but all connections are “rewired” randomly. More precisely, in our bipartite setting, if there are \( m \) interest groups indexed 1, \ldots, \( m \) and \( n \) politicians indexed 1, \ldots, \( n \), suppose that interest group \( i \) is connected to a total of \( d_i^I \) politicians, and politician \( j \) is connected to a total of \( d_j^P \) interest groups. Note that the total number of edges \( E \) is given by the sums of either of these quantities:

\[
E = \sum_{i=1}^{m} d_i^I = \sum_{j=1}^{n} d_j^P.
\]

Now, we perform the following random process to generate a draw from the configuration model:

1. From each interest group \( i \), produce \( d_i^I \) “stubs” or partial edges, which we may label \( e_{i,1}^I, \ldots, e_{i,d_i^I}^I \). Likewise, from each politician \( j \), produce stubs \( e_{j,1}^P, \ldots, e_{j,d_j^P}^P \).
2. Generate a uniformly random matching of the two sets \( \{ e_{i,k}^I \} \) and \( \{ e_{j,k}^P \} \) (note that by the preceding remark, the two have equal size.)
3. Generate a bipartite graph by putting an edge between interest group $i$ and politician $j$
whenever $e_{i,k}^I$ and $e_{j,\ell}^P$ were matched for some $k, \ell$ in the previous step (so that the total
number of edges is the total number of pairs $k, \ell$ for which these two stubs were matched).

It is simple to verify that the configuration model satisfies the following desirable properties. First,
every draw of the configuration model has the same degree distribution as the original graph; that
is, the collections of numbers $\{d_i^I\}$ and $\{d_j^P\}$ remain the same. And second, the configuration
model draws a uniform sample from the bipartite graphs having those degree distributions. In
this sense, the configuration model is the canonical null model that retains the degree distribution
of a graph, but “forgets” all other details.

To understand how exceptional our findings with the biLCM are, we analyze the link community
distribution entropy under the configuration model for our main dataset here. In particular, in
Figure E.7, we compare the entropy distributions obtained from the biLCM run on this dataset with
averaged distributions obtained from many draws of the configuration model applied to the same
dataset (as we would hope, those distributions concentrate fairly well around a single representative
null entropy distribution). We see that the entropies of the link community distributions for
interest groups in the actual dataset are much lower, meaning the link community distributions
are much more concentrated, than those for the null model. This gives further evidence that
the communities we find are statistically significant, and not, for instance, merely artifacts of the
graph’s degree distribution (not a farfetched possibility, as less sophisticated community detection
algorithms often suffer from detecting spurious communities consisting only of high-degree nodes).

### E Model Comparison

We give a few points of comparison among our latent space and community models, confirming
that the models all consistently reflect the same underlying properties of the lobbying network.

Figure E.8 shows the interaction matrix $A$, with its rows and columns ordered according to
the groupings given by the biSBM (top panel) and the dc-biSBM (bottom panel). Both models
show that there exist clear “checkerboard” community patterns in the lobbying network data,
and, as mentioned before, without degree correction the biSBM suffers from the flaw of grouping
high-degree nodes together. (Indeed, one of the interest group communities found without
degree correction consists of exactly the two interest groups with highest degree, the Chamber
of Commerce and Iraq and Afghanistan Veterans of America.) The dc-biSBM, in contrast, gives much more balanced community sizes and less concentration of the highest-degree
agents. Degree correction also yields community memberships which, especially for interest groups, correspond more closely to the clusters we found ex post in the LSNM.

Next, in the LSNM we assign latent positions to each agent, while in the community models we divide the agents into discrete communities. This prompts the question of whether the latent positions of the LSNM respect the clusterings suggested by the community models, i.e. of whether the communities identified the block models are localized in the latent space. We address this question in Figure E.9 and Figure E.10 for the biSBM and dc-biSBM models, respectively, observing that the latter both gives less weight to high-degree nodes, and is more aligned with the LSNM geometric representation.

Lastly, in the main text we claimed that the communities of the dc-biSBM and the biLCM were well-aligned, and could be viewed as having the same substantive interpretations. We illustrate this in Figure E.11, plotting the mean probability with which an actor from a given dc-biSBM community interacts in a given biLCM community and observing that, after a suitable permutation, for each dc-biSBM community there is indeed one best-aligned biLCM community.
Figure E.1: **Distribution of Political Actions.** We present the distributions of politician sponsorship and interest group lobbying counts over the 113th Congress. Note that these statistics are aggregated before any of the filtering performed to form the dataset we work with later. Bills sponsored only count those bills lobbied by at least one interest group. The activity counts and names of the most active politicians and interest groups are presented in the tables below.
Figure E.2: Bipartite Adjacency Matrix Singular Value Decay. We plot the singular values of the interaction matrix $A$, highlighting that the singular values decay rapidly and the first few appear to capture much of the total weight of this matrix. The first two singular values are highlighted, corresponding to our analysis primarily of a two-dimensional latent space model in the main text.

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<td>✓</td>
<td>✓</td>
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<tr>
<td>Dependent Variable</td>
<td>$F$-Statistic ($p$-Value)</td>
<td>$F$-Statistic ($p$-Value)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1D LSNM</td>
<td>13.72 ($&lt; 2.2e-16$)</td>
<td>0.86 (0.71)</td>
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<td></td>
</tr>
<tr>
<td>2D LSNM, Dimension 1</td>
<td>9.92 ($&lt; 2.2e-16$)</td>
<td>0.81 (0.78)</td>
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<td></td>
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<tr>
<td>2D LSNM, Dimension 2</td>
<td>13.25 ($&lt; 2.2e-16$)</td>
<td>1.65 (0.01)</td>
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</tbody>
</table>

Table E.1: Regression Analysis of LSNM Covariates (Interest Groups). Statistics of linear models of covariates against the one- and two-dimensional LSNM latent dimensions for interest groups. We compare full linear models with all available covariates against restricted models, omitting either industry classification covariates or geographic information. In the bottom three rows, for each of the three latent dimensions, we give the $F$ statistics and $p$-values of the comparison $F$-test. The test illustrates that the industry covariates are significantly explanatory of the LSNM latent space organization, and, with lesser confidence, geographic covariates are also explanatory of one dimension in the two-dimensional LSNM.
Figure E.3: **LSNM Latent Position Uncertainties.** We plot an estimate of the “typical set” of each actor’s latent position in the two-dimensional LSNM, in the form of an ellipse containing most of the latent position samples drawn by MCMC sampling. The computations giving the parameters of these ellipses are detailed in Section B.4.
Figure E.4: **LSNM Popularity Factor Uncertainties.** Distributions of the coefficients of variation (standard deviation as a fraction of the mean) of the sampled popularity factors for all interest groups and all politicians.
Figure E.5: **LSNM Trace Plots.** We show trace plots for the model parameters (two latent space dimensions and one popularity factor) drawn from two (out of four) MCMC chains following the warmup period, for one interest group, the **Chamber of Commerce**, and one politician, **Barbara Boxer (D-CA)**.
// Latent space network model.
//
// Implements the model
// $A_{ij} \sim \text{Poisson}(\mu_{ij})$
// where
// $\mu_{ij} = \exp(a_i + b_j - ||x_i - y_j||_2^2)$
// = $\exp(aa_i + ba_j + 2<x_i, y_j>)$
//
// The names of the variables in the model are:
// $a_i$ : row_factor_adj
// $b_j$ : col_factor_adj
// $x_i$ : row_embedding
// $y_j$ : col_embedding

data {
  int<lower=1> D;    // dimension of latent space
  int<lower=2> N_row;  // number of rows
  int<lower=2> N_col;  // number of columns
  int N_fixed_row;  // number of pinned rows
  int N_fixed_col;  // number of pinned columns

  int<lower=0> edges[N_row, N_col];  // connection strength data

  int fixed_row_index[N_fixed_row];    // indices of fixed row actors
  matrix[N_fixed_row, D] fixed_row_embedding; // positions of fixed row actors

  int fixed_col_index[N_fixed_col];    // indices of fixed column actors
  matrix[D, N_fixed_col] fixed_col_embedding; // positions of fixed column actors
}

transformed data {
  int flat_ix;
  int flat_edges[N_row * N_col];

  flat_ix = 1;
  for (j in 1:N_col) {
    for (i in 1:(N_row)) {
      flat_edges[flat_ix] = edges[i][j];
      flat_ix = flat_ix + 1;
    }
  }
}

parameters {
  vector<lower=0.01>[D] cov_embedding_diag;

  matrix[N_row, D] row_embedding;
  matrix[D, N_col] col_embedding;

  real mu_col_factor_adj;
  real<lower=0.01> var_row_factor_adj;
  real<lower=0.01> var_col_factor_adj;
}

vector[N_row] row_factor_adj;
row_vector[N_col] col_factor_adj;
}

model {
  vector[N_row * N_col] means;
  int fixed_row_flag;
  int fixed_col_flag;

  for (i in 1:N_row) {
    fixed_row_flag = 0;
    for (j in 1:N_fixed_row) {
      if (i == fixed_row_index[j]) {
        row_embedding[i,] ~ normal(fixed_row_embedding[j,], 1e-4);
        fixed_row_flag = 1;
      }
    }
    if (fixed_row_flag == 0) {
      row_embedding[i,] ~ normal(0.0, cov_embedding_diag);
    }
  }

  for (j in 1:N_col) {
    fixed_col_flag = 0;
    for (k in 1:N_fixed_col) {
      if (j == fixed_col_index[k]) {
        col_embedding[,j] ~ normal(fixed_col_embedding[,k], 1e-4);
        fixed_col_flag = 1;
      }
    }
    if (fixed_col_flag == 0) {
      col_embedding[,j] ~ normal(0.0, cov_embedding_diag);
    }
  }

  row_factor_adj ~ normal(0.0, var_row_factor_adj);
  col_factor_adj ~ normal(mu_col_factor_adj, var_col_factor_adj);

  means = to_vector(
    rep_matrix(row_factor_adj, N_col) +
    rep_matrix(col_factor_adj, N_row) +
    2.0 * row_embedding * col_embedding);

  flat_edges ~ poisson_log(means);
}

Listing E.1: Stan code defining a sampler for the posterior distribution of the LSNM. Note that for $d = 1$ the model is greatly simplified, and it is much more efficient to remove the extra dimension from the types related to the latent space position distribution.
Figure E.6: **Assortativity in dc-biSBM.** We plot the mean interactions between each pair of communities in the dc-biSBM. If the communities are suitably permuted, we observe that the diagonal entries of the resulting matrix are typically the largest, indicating that the model finds an assortative community structure.

Figure E.7: **Link Community Entropy Distribution vs. Null Model.** We plot the distribution of entropies of link community distributions obtained by the biLCM run on the full 113th Congress dataset, compared to an average (per histogram bin) of the same distribution for several draws of the null configuration model generated from the same dataset. We observe that for interest groups the actual link community distributions are much more concentrated than those obtained in the null model, while for politicians the link community distributions are comparable.
### Highest Entropy

<table>
<thead>
<tr>
<th>Senator</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harry Reid (D-NV)</td>
<td>. . . . . . .</td>
</tr>
<tr>
<td>Jerry Moran (R-KS)</td>
<td>. . . . . .</td>
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<tr>
<td>Patty Murray (D-WA)</td>
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<tr>
<td>Kay Hagan (D-NC)</td>
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<tr>
<td>Bill Cassidy (R-LA)</td>
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<tr>
<td>Chris Van Hollen (D-MD)</td>
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<tr>
<td>Amy Klobuchar (D-MN)</td>
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<tr>
<td>Charles Schumer (D-NY)</td>
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<tr>
<td>Debbie Stabenow (D-MI)</td>
<td>. . . . . .</td>
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<tr>
<td>Dianne Feinstein (D-CA)</td>
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<tr>
<td>Robert Portman (R-OH)</td>
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<tr>
<td>Kirsten Gillibrand (D-NY)</td>
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<tr>
<td>Robert Casey (D-PA)</td>
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<tr>
<td>Sherrod Brown (D-OH)</td>
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<td>Mark Warner (D-VA)</td>
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### Lowest Entropy

<table>
<thead>
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<tr>
<td>Cory Gardner (R-CO)</td>
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<td>Maria Cantwell (D-WA)</td>
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<tr>
<td>Jeff Merkley (D-OR)</td>
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<td>Shelley Capito (R-WV)</td>
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<td>Tom Cotton (R-AR)</td>
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<td>Tim Scott (R-SC)</td>
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<tr>
<td>John Isakson (R-GA)</td>
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<tr>
<td>Richard Burr (R-NC)</td>
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<tr>
<td>Bob Corker (R-TN)</td>
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<td>James Risch (R-ID)</td>
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<tr>
<td>Martin Heinrich (D-NM)</td>
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<td>Michael Crapo (R-ID)</td>
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<tr>
<td>Angus King (I-ME)</td>
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<td>John Hoeven (R-ND)</td>
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<td>Richard Shelby (R-AL)</td>
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</table>

Table E.2: **Legislation Community Distribution Examples: Politicians.** We show the legislation community memberships of senators having the most (left panel) and least (right panel) memberships, as quantified by the entropy of the legislation community distribution. As in Table 2, the distributions are represented as histograms, with eight bars corresponding to the eight legislation communities in the **biLCM**.
Figure E.8: **Effect of Degree Correction.** We show the results of the biSBM and the dc-biSBM by permuting the interaction matrix to group together the clusters that each model infers. Both models identify community structure, and, as expected, the dc-biSBM infers more balanced cluster sizes.
Figure E.9: **LSNM vs. biSBM.** We plot the latent positions of legislators and interest groups from the LSNM, split by their memberships in clusters from the biSBM. For the sake of visual clarity, we no longer vary the sizes of the plotted latent space positions based on popularity factors. Localization of groups in the latent space suggests that the LSNM is somewhat consistent with the biSBM, but much of the fine-grained interest group clustering we observe in Figure 3 is not reflected in the biSBM results.
Figure E.10: **LSNM vs. dc-biSBM.** We plot the latent positions of legislators and interest groups from the LSNM, split by their memberships in clusters from the dc-biSBM, with both run on the main dataset. Almost all clusters found by dc-biSBM are closely localized in the latent space, showing that the dc-biSBM captures much of the same information as the LSNM.
Figure E.11: **dc-biSBM vs. biLCM.** We plot communities of the dc-biSBM against those of the biLCM, measuring pairwise alignment by the mean probability of an actor in the given dc-biSBM community to interact in the given biLCM community. We observe that, when community labels are suitably permuted, it is possible to match communities from either model into well-aligned pairs.
References


