Integration of the Load Matching and Routing Problem with Equipment Balancing for Small Package Carriers…

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MichiganEngineering
Industrial and Operations Engineering

…or, Struggling to Apply OR in the Real World

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Industry, OR, and Academia

- My background
- What do trucking, passenger aviation, and automotive manufacturing all have in common?
- Challenging scheduling problems
  - Complex networks
  - Tightly constrained resources
  - Large numbers of interconnected decisions

Industry, OR, and Academia, cont.

- What do trucking, passenger aviation, and automotive manufacturing all have in common?
- Challenges in applying academic OR solutions
  - Emphasis on optimality
  - Simplifying for tractability
  - Stochastic and dynamic environments
  - Desire for solution regularity

Industry, OR, and Academia, cont.

- Academic goals:
  - Clean, elegant, provably optimal
- Industry desires:
  - High quality solutions
  - Timely results
  - Regularity
  - Robustness
  - Real-world detail

Large-Scale Network Problems in Transportation and Logistics

Challenges of Complex Systems

- Large numbers of decisions
- Tightly inter-coupled because of network structures
- Very constrained resources (time, money, raw materials, labor…)
- Timeliness

Math Programming Approach

- Pros:
  - Global viewpoint
  - General solution techniques
  - Established commercial solvers
- Cons:
  - Large numbers of constraints, variables
  - Non-linearities
  - Weak LP relaxations
  - No exploitation of problem-specific structures

Customized Algorithms

- Pros:
  - Exploit problem structure
  - More realistic
  - Fast run times, convergence
- Cons:
  - Feasibility?
  - Solution quality?
  - Adaptability?

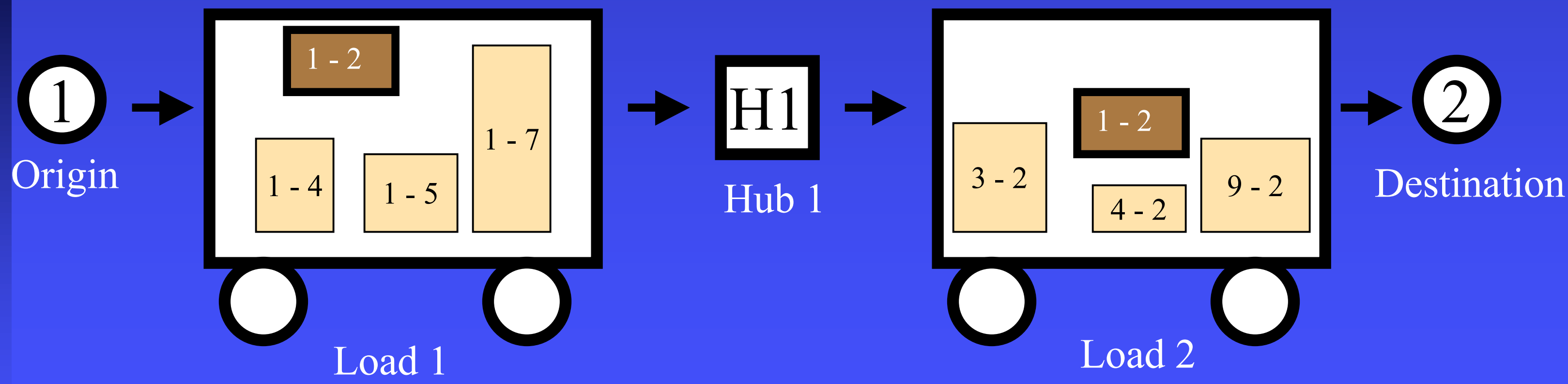
Our Hybrid Approach

- Seek to combine the strengths of both approaches
- MP-based *master problem*
- *Complex variable definitions*
  - Multiple decisions in a single variable
  - Capture problem complexity within variable definition
  - Eg. crew pairing
- Application-specific algorithms to generate columns
  - Wealth of resources for generating candidate columns
  - Leveraging user expertise
  - Trading solution time vs quality

Integrated Planning for Small Package Carriers

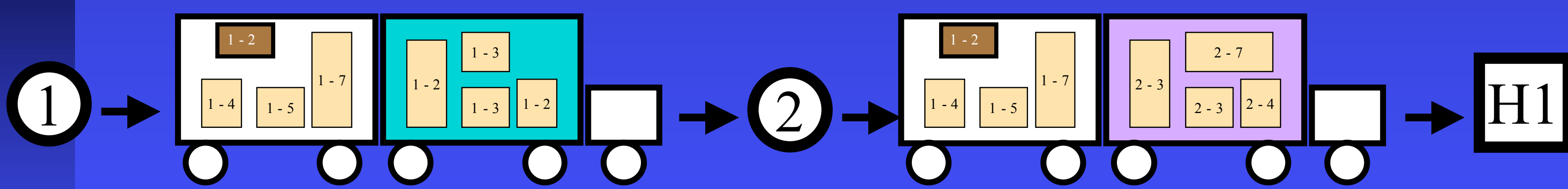
Express Package Networks

- Packages move from origin to destination via one or more hubs at which freight is unloaded, sorted, and reloaded
- Each trailer movement is called a *load*



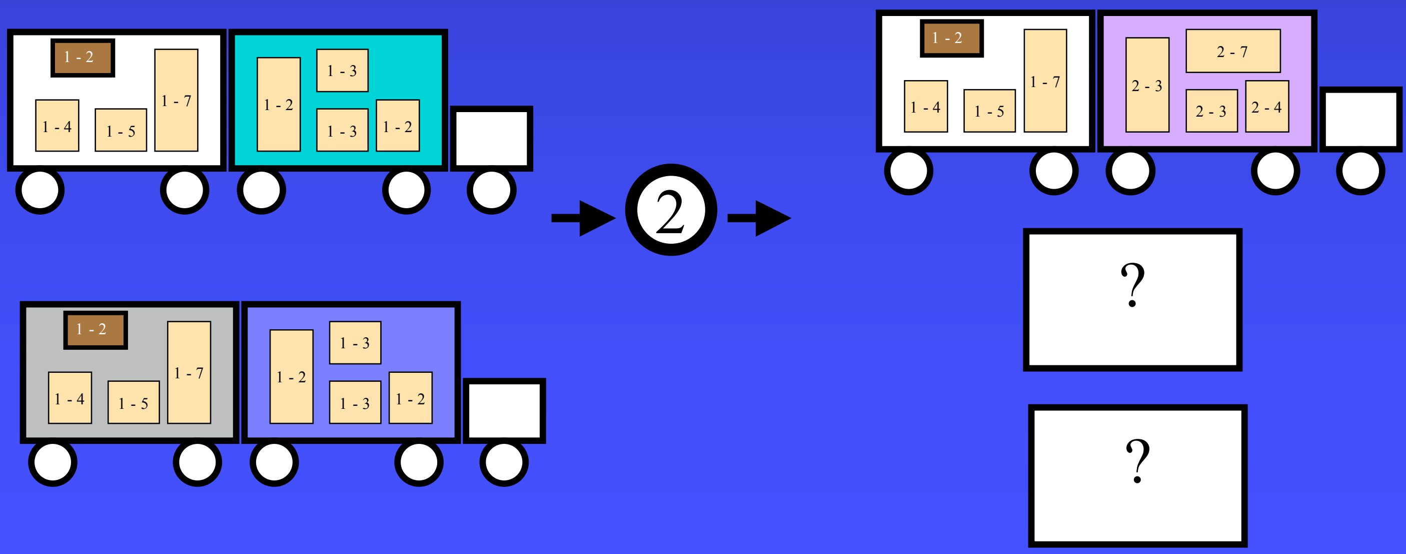
Load Matching

- Loads (trailers) are often coupled with other trailers for reduced transportation cost
  - The cost of pulling two trailers behind the same tractor is less than twice the cost of pulling a single trailer
- Loads may stop at intermediate points along the way to re-configure the tractor



Equipment Balancing

- Loaded trailer movements can result in system imbalances – some locations have more outbound loads than inbound, others more inbound than outbound
- We need to move empty trailers throughout the system to regain balance



Integrated Load Matching and Equipment Balancing Problem

- What loads should be matched?
  - When is it worth driving extra miles to combine two trailers?
- How should empty trailers be redistributed throughout the system?
  - At each location, total trailer flow out must equal total flow in
- Can we make these two decisions together for even greater savings?
  - Loaded and empty trailers can be paired together

Problem Description: Inputs

- Set of loads (each load has an origin, destination, trailer type, earliest available time, and latest delivery time)
- Permissible tractor configurations between each pair of locations
- Cost per mile of each configuration
- Mileage and travel time between each pair of locations

Problem Description: Constraints

- Loads must move contiguously in space and time from origin to destination
- Loads must satisfy time windows
- Each location must be balanced (i.e. total number of trailers in must equal total number out, including both loaded and empty trailers)

Problem Description: Objective

Minimize total transportation cost, incorporating cost function for different modes and configurations

MCF Model with Side Constraints

- This can be thought of as a multi-commodity flow problem with side constraints
- Each load is a commodity that must be moved from origin to destination
- Side constraints are needed to ensure timing restrictions, linearize the objective function, and balance the network

Multi-commodity flow based model

- Use time-space network to capture time constraints
- Decision variables
  - x_{ijk} – flow of commodity *k* on arc (i, j)
  - y_{ij} – flow of empty trailers on arc (i, j)
  - s_{ij} – flow of single trailer combinations on arc (i, j)
  - d_{ij} – flow of double trailer combinations on arc (i, j)

Multi-commodity flow based model

$$\min \sum_{(i,j)\in A} c_{ij}^{s}\, s_{ij} + \sum_{(i,j)\in A} c_{ij}^{d}\, d_{ij}$$

$$\text{s.t.} \quad \sum_{i:(j,i)\in A} x_{jik} - \sum_{i:(i,j)\in A} x_{ijk} = b_{jk} \qquad \forall\ j \text{ in } V,\ k \text{ in } K$$

$$s_{ij} + 2d_{ij} = \sum_{k\in K} x_{ijk} + y_{ij} \qquad \forall\ (i,j) \text{ in } A$$

$$\sum_{j\in V_f} \Big(\sum_{k\in K} b_{jk} + \sum_{i:(j,i)\in A} y_{ji} - \sum_{i:(i,j)\in A} y_{ji}\Big) = 0 \qquad \forall\ f \text{ in } F$$

$$x_{ijk},\ y_{ij},\ s_{ij},\ d_{ij} \text{ in } Z^{+}$$

Challenges of MCF

- Very large number of nodes, constraints
  - Time/space network
  - If non-facility meet points are permitted, grows further
- Weakness of LP relaxation – half a double is always better than a whole single

Alternative Model: Load Clusters

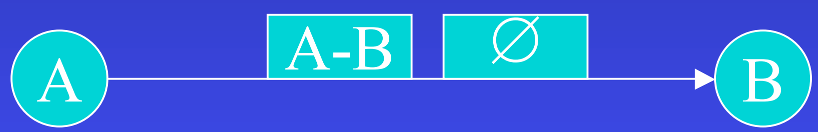
- As an alternative, we define the notion of a *load cluster* – a set of interconnected loads, the path each load takes, and the tractor configurations used to combine these loads
  - All loads must move fully from origin to destination
  - All loads must satisfy their time windows
  - No outside loads can be partially included in the configurations

Example Cluster

A load from A to B moves directly, as a single trailer:

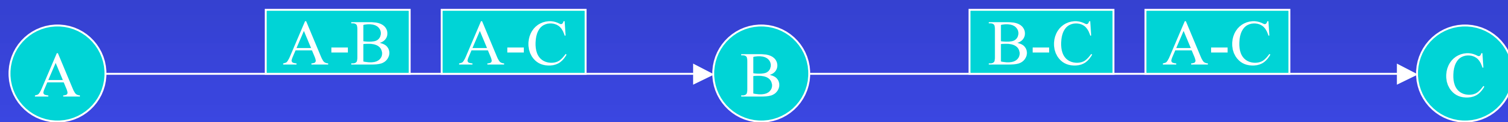
Example Cluster

A load from A to B moves directly, in combination with an empty trailer:



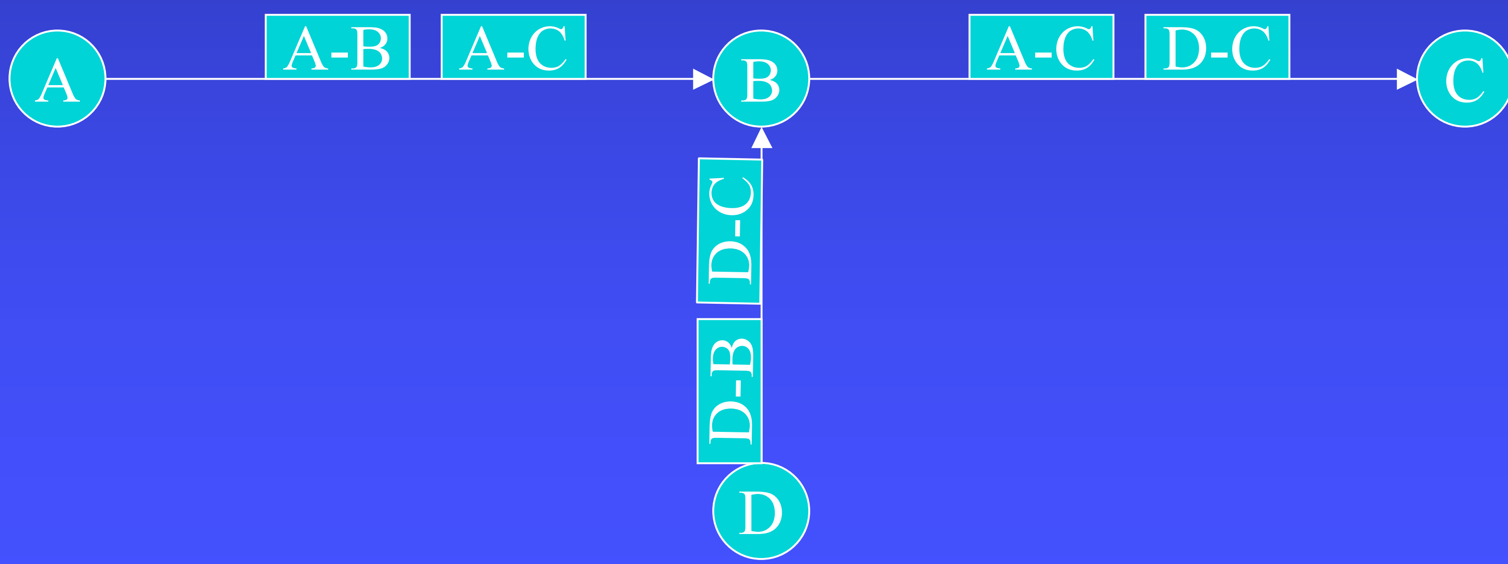
Example Cluster

Loads from A to B and A to C move via B, where the A to B load is dropped off and a load from B to C is picked up:



Example Cluster

Loads from A to B and A to C move via B, as do loads from D to B and D to C:



The Cluster-Based Model

- L is the set of loads
- C is the set of load clusters
- c_i is the cost of load cluster *i*
- F is the set of facilities
- δ_{il} is defined to be 1 if cluster *i* contains load *l* and 0 otherwise
- λ_{if} is the net surplus of trailers at facility *f* relative to cluster *i*
- x_i is a binary variable that takes value 1 if cluster *i* is chosen and 0 otherwise

The Cluster-Based Model

Min. $\quad \Sigma_{i \in C} c_i x_i$

St $\quad \Sigma_{i \in C} \delta_{il} x_i = 1 \qquad \forall\, l \in L$

$\Sigma_{i \in C} \lambda_{if} x_i = 0 \qquad \forall\, f \in F$

$x_i \in \{0, 1\} \qquad \forall\, i \in C$

Benefits

- The objective function is linear
- The LP relaxation is strengthened
- Timing issues are imbedded within the variable definition
- The number of constraints is greatly reduced
- Expanded solution space – the set of meet locations does not have to be limited to carrier facilities
- Multiple transportation modes, trailer configurations can easily be incorporated

Original Solution Plan

- Too many variables to explicitly enumerate in the model
- Use *branch-and-price*
  - Branch-and-bound to solve the IP
  - *Column generation* to solve the LP nodes
- What is the pricing problem???

Computational Experience

- Started off with "initial" set of cluster *templates* to find seed duals
- Found significant improvement in solution quality just from using the templates!
  - Approximately 10% on a small network, with expectations of better savings from larger networks
- Feasibility
- Regularity
  - Simplies cluster-based model
  - Makes MCF much more complex

Future Research

- Symmetry
- Time window analysis
- Assigning volume to trailer types

Conclusions, Challenges, and Future Research

Conclusions

- Many applied problems are hard!
- Simplification can lead to tractability but real-world infeasibility
- Optimality is not all it's cracked up to be…
- …but the quality of our solution really matters!
- Sometimes, a hybrid approach with a complex variable model can help

Challenges

- How to generate candidate columns
- Feasibility vs. optimality
- Lower bounds
- Degeneracy, convergence

Future Research

- Expanding problem scope
- Generalizing results
- Robustness