HEAT CONDUCTION THROUGH LAYERED REFRACTORY LININGS

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ABSTRACT: A predictive model is developed for transient heat transfer analysis of structures with layered heterogeneous media. The model is particularly suited for the analysis of refractory concrete lined coal gasification and other similar chemical processing vessels. To allow for the effects of high conductivity gases in voids and cracks on the overall conductivity of the media, an effective conductivity model is developed. The model incorporates the effects of temperature dependent nonlinear material properties. Validity of the model is shown by comparing the predictions with those from an analytical solution for a special case, another numerical solution, and experimental measurements from an actual test vessel. Sensitivity of the temperature distribution to the presence of hydrogen gas in the vessel environment is evaluated.

INTRODUCTION

In view of the current energy needs, a considerable increase in the use of coal gasification processes and, thus, a greater demand for large-scale commercial plants is anticipated. These plants must be capable of operating reliably under high temperature (1,500–3,000° F) (816–1,632° C) and pressure conditions (11,16). A coal gasification vessel is usually cylindrical in shape and is composed of an outer steel shell and inside layers of refractory concrete linings. At present, the design of such gasification vessels is based on simplified calculations using the knowledge from past experience of ammonia plants, steel mills, or glass plants. Development of improved design techniques for these vessels is therefore needed whereby the resistance of refractory materials to high temperature can be effectively utilized. Recent experimental (1) and analytical (6,25) studies have provided much insight into the thermomechanical behavior of monolithic refractory linings as a basis for developing improved design procedures.

A critical problem in the analysis and design of these complex structures is the determination of temperature distribution through the composite wall of the vessel during heat-up and cool-down cycles. Structural response of the refractory linings to high temperature effects is highly path dependent, and predictions based on steady-state temperature distributions cannot be expected to be accurate. The critical design condition of the vessel usually occurs in the transient state of the heat flow.

An excellent review is provided in Ref. 34 of previous studies on effective conductivity models for heterogeneous media and heat transfer through composite walls. In Refs. 9 and 28 development is reported of an effective conductivity model and a predictive capability for steady-state heat distribution through multilayered gasifier vessel walls. Nu-


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Numerical solutions of transient heat transfer through composite media are reported for slabs (36), box-girder bridge sections (30), and pressure vessels (29).

In this paper, a predictive model developed for transient heat flow through the layered refractory linings of dual component coal gasifier cylindrical vessels (Fig. 1) is reported. The model adopts the effective conductivity concept and accounts for heat conduction in porous media, gas conduction, radiation through pores, and the effect of cracks and gaps on thermal conductivity. A finite difference scheme, which incorporates temperature dependent material properties and boundary conditions, is used for the solution of a one-dimensional parabolic differential equation. The predictive model is then implemented in a heat transfer program. Verification of the predictive capability is made by comparing the results with the analytical solution for a particular case, another numerical solution, and available experimental measurements. Finally, a parametric sensitivity study is presented on the effect of hydrogen on temperature distributions.

**Overview of Thermal Conductivity Models**

The thermal conductivity of the materials under process conditions is generally different from those provided by laboratory tests. This is mainly because the gases in the process environment are different from those in the tests; also, the degradation of the materials (e.g., cracking) during the process cycles may produce different void volume, thus resulting in different heat transfer characteristics.

For a coal gasification process environment, the process gas usually consists of hydrogen, methane, carbon dioxide, carbon monoxide, steam, and air, among which hydrogen has the highest conductivity. In fact,
Crowley (10,35), who studied the effect of hydrogen on thermal conductivity of refractory concrete, reported that for most insulating refractory concrete when the air or vapor in the pores is replaced with a process gas containing 60–80% hydrogen, the thermal conductivity usually doubles. The increase in the thermal conductivity would result in higher shell temperatures and, as a result, additional cracks in the lining may form. Flow of hot gases directly to the shell may then occur. It is thus clear that for an accurate analysis and design of these process vessels, the development of a temperature-dependent thermal conductivity model is needed for the materials in the process environment. Such a model should be able to predict the thermal conductivity during the process from the provided laboratory test data.

Consider a porous material of which the pores are filled with gas. The solid phase has a conductivity, \( k_s(T) \), and the gas has a conductivity, \( k_a(T) \); both are functions of temperature, \( T \). For concrete type materials, the size and shape of pores is random. When the size and volume fraction of the pores is small, statistically, the overall effect can be approximated by randomly distributed small spheres with the same volume fraction. For this case, Maxwell (20) derived from potential theory an equation for effective conductivity:

\[
k = \frac{2k_s + k_a - 2V_a(k_s - k_a)}{2k_s + k_a + V_a(k_s - k_a)} k_s \tag{1}
\]

in which \( k \) = the effective conductivity of the equivalent homogeneous media; and \( V_a \) = the pore volume fraction. When the pore volume fraction is not small, the temperature fields surrounding the pores begin to “interact.” Mathematical modeling of this interaction is difficult. To deal with this problem one is required to fix the position of all pores; moreover, their position should be such that simple symmetry exists. Lord Rayleigh (26) treated the case for the cubical array of sphere pores where the field is perpendicular to a side of a cube. The result, with the correction by Runge (27), is given by

\[
k = \left(1 - \frac{3V_a}{\frac{2k_s + k_a}{k_s - k_a} + V_a - 0.525 \frac{k_s - k_a}{4} V_a^{10/3}}\right) k_s \tag{2}
\]

A variety of other models have been proposed in the past, e.g., an analytical solution for pores with other shapes or other arrangements (14,17,26), approximate solutions (4), empirical equations, and statistical solutions (3,32). Extensive surveys can be found in Refs. 7, 8, 13, 21, 24, and 34. Several useful results are noteworthy. Eucken (12) generalized Maxwell’s equation to multi-dispersed phases in a continuous media:

\[
k = \frac{1 - \sum_{j=1}^{l} \frac{2V_j}{2k_s + k_j} \frac{k_s - k_j}{k_s} k_s}{1 - \sum_{j=1}^{l} \frac{V_j}{2k_s + k_j} \frac{k_s - k_j}{k_s} k_s} \tag{3}
\]
in which \( l \) = the number of dispersed phases; and \( V_j, k_j \) = the volume fraction and conductivity of the \( j \)th dispersed phase. Fricke (14) first introduced a generalized shape factor, \( x \), derived for ellipsoid pores whereby Eq. 1 becomes

\[
k = \frac{xk_s + k_a - xV_a(k_s - k_a)}{xk_s + k_a + V_a(k_s - k_a)} k_s \quad \ldots \quad (4)
\]

and, accordingly, the numbers 2 in Eq. 3 can be substituted by \( x_j \), the generalized shape factor of each phase. The shape factor, \( x \), in general, depends on the shape of the ellipsoid and the ratio of \( k_s \) to \( k_a \).

In Ref. 9, using the results of Bruggeman (4), a mixture model was derived for the effective thermal conductivity. The effective thermal conductivity with \( l \) continuous phases is the solution of the \( l \)th degree polynomial

\[
\sum_{j=1}^{l} \frac{V_j(x_j + 1)}{k_j + x_j k} = 0 \quad \ldots \quad (5)
\]

This equation applies to cases where all phases are continuous. It was shown that when all the \( x_j \)s are equal to 2, a unique positive root exists.

Comparisons of the preceding summarized models, together with the upper and lower bound solutions, are shown in Fig. 2 for \( k_a/k_s = 0.1 \), and 5 for \( V_a \) up to 0.5. The differences between Eqs. 1, 2, and 5, when one considers the uncertainty in measuring material properties, are insignificant for all engineering purposes. Maxwell’s equation (Eq. 1) has been adopted in the following analysis. Equation 2 produces results close

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**FIG. 2.—Comparison of Effective Conductivity Models**
to those obtained by Maxwell’s equation. The mixture equation, suggested in Ref. 9, requires determination of shape factor \( x \) by judgment rather than rigorous derivation; the solution of the polynomial is tedious and its root in some cases may not be unique. The tests by Austin (2) support the choice of Maxwell’s equation.

**Proposed Model**

Heat flow through a heterogeneous media such as the refractory linings involves many parameters. Ideally, one has to consider heat conduction through porous media, gas convection and radiation through pores, heat transfer in cracks and gaps, influence of metal anchors, moisture migration effect, and influence of pressure, etc. An exhaustive treatment of all these effects is not intended in this paper. Instead, a simple, and effective conductivity model considering only the dominant factors is developed in the following.

Consider a porous material, having a laboratory measured thermal conductivity, \( k_c(T) \), the pores of which are filled with gas (usually air or vapor) having a thermal conductivity, \( k_g(T) \). The volume fraction of pores is \( V_a \). When the material is exposed to process gas with conductivity \( k_g(T) \), the effective conductivity can be found by applying Maxwell’s equation (Eq. 1):

\[
k(T) = \frac{1 + 2V_aks(T)}{1 - V_aks(T)} \tag{6a}
\]

in which

\[
s = \frac{k_g(T) - k_s(T)}{k_g(T) + 2k_s(T)}; \quad \dot{k}_s(T) = p(T) + \sqrt{p^2(T) + q(T)} \tag{6b}
\]

\[
p(T) = \frac{k_c(T) - \frac{k_a(T)}{2}}{2(1 - V_a)} \tag{6c}
\]

\[
q(T) = \frac{k_a(T)k_c(T)}{2} \tag{6d}
\]

in which \( \dot{k}_s(T) \) may be called the pseudo-solid phase conductivity. The process gas environment usually comprises more than one gas. The thermal conductivity of the gas mixture, \( k_g(t) \), has to be obtained first. For this, the method of Brokaw for binary mixture or the approximation of Burgoyne and Weinberg for multi-component mixtures, as described in Refs. 15 and 31, is suggested.

The foregoing procedure to estimate thermal conductivity in the process environment from test data automatically takes into account the important effect of gas radiation through pores. Gas convection and pressure have been shown to have little effect on effective conductivity (9,34); they are neglected in the present study.

To allow for the effect of cracks and gaps, it is recognized that cracks or gaps are usually either parallel or perpendicular to the heat flow direction. For example, in the coal gasifier, due to the axisymmetry of the vessel, the principal axes of stresses remain in the axial, radial, and hoop
directions. Consequently, radial gaps and cracks are parallel to the heat flow. Assume that one can estimate the volume fraction of gaps and radial cracks, \( V_r \), and of hoop and axial cracks, \( V_h \). By applying Eucken's equation (Eq. 3) and Fricke's shape factor (Eq. 4), Eq. 6 may be modified to

\[
k(T) = \frac{1 + 2V_a s + V_h \frac{k_h(T) - k_s(T)}{k_s(T)}}{1 - V_a s - V_r \frac{k_r(T) - k_s(T)}{k_r(T)}} \quad \text{.................. (7)}
\]

in which \( k_h(T) \) and \( k_r(T) \) = thermal conductivities of the gas in \( V_h \) and \( V_r \). They are obtained by adding radiation components to \( k_s(T) \). Loeb's equation in Ref. 19 as modified in Ref. 36 suffices for the purpose:

\[
k_h(T) \text{ [or } k_r(T)\text{]} = k_s(T) + k_l \quad \text{.......................... (8)}
\]

in which \( k_l \) = radiation contribution to conductivity = \( 4 \gamma d \varepsilon \sigma T_0^3 \); \( \gamma \) = geometrical factor (= 1 for present cases); \( d \) = maximum dimension of cracks in direction of heat flow; \( \varepsilon \) = emissivity of radiating surface; \( \sigma \) = Stefan-Boltzmann constant; and \( T_0 \) = average absolute temperature at the crack. Note that for Eq. 7, in applying shape factor \( x \), \( x = 0 \) is assumed for gaps and radial cracks (series model), and \( x = \infty \) is assumed for hoop and axial cracks (parallel model).

For the present study, the developed effective conductivity model has been incorporated in a one-dimensional finite difference transient heat transfer program. However, the model can be readily extended to two or three-dimensional formulations without difficulty.

**Numerical Solution**

The physical system considered is a long hollow cylinder with the composite wall heated from the center line uniformly along the axis. The temperature distribution can thus be considered to vary in the radial direction only. The governing equation is

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( k_r \frac{\partial T}{\partial r} \right) = \rho c \frac{\partial T}{\partial t} \quad \text{.......................... (9)}
\]

in which \( r \) = radial distance from center line; \( k = k(T, r) \) = effective thermal conductivity; \( \rho \) = density; \( c = c(T, r) \) = specific heat capacity; and \( T = T(r, t) \) = temperature.

Heat flow through actual coal gasification vessel walls is certainly more complex than the assumed one-dimensional flow. The gasification process is usually restricted to a certain height of the vessel so that the temperature distribution may not be uniform along the entire vessel height. However, there is evidence that the assumption of a unique temperature distribution along the vessel height may be used with no significant loss of accuracy in prediction (18). Heat flow through the linings is also complicated by the presence of anchors and other asymmetrical structural components requiring locally a three-dimensional modeling. But it has been shown (33) that the effect of anchors on temperature distribution is not significant.
FIG. 3.—Finite Difference Grid

The governing equation (Eq. 9) with prescribed temperature dependent radiation and convection boundary conditions is solved by the use of the finite difference method. The solution uses a central difference discretization in space and the Crank-Nicholson formula in time (Fig. 3). To account for nonlinearities in material properties, a two-cycle iterative procedure is adopted in the solution.

VERIFICATION

A computer program which implements the developed effective conductivity model and the described finite difference solution procedure has been written. The following sections describe the verification of the program first by the use of existing analytical and numerical solutions for special cases, and then by comparison with available experiments.

Analytical Solution.—An analytical solution for transient heat transfer through a typical coal gasification vessel wall with outer steel shell and two layers of refractory concrete lining was obtained from Refs. 22 and 23. The vessel geometry is shown in Fig. 4. As a special case, the material properties were kept constant with respect to temperature.

An example analysis with material properties and geometries similar to those of the test vessels (1) was performed by using both the analytical method and the developed transient finite difference program. The assumed material properties and geometry are summarized in Table 1. Initial temperatures in the vessel system and the environment were set to 70° F (21° C). The hot face was then heated up at a rate of 100° F/
TABLE 1.—Material Properties and Geometry of Vessel Used in Example

<table>
<thead>
<tr>
<th>i</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>1.1</td>
<td>0.25</td>
<td>31.42</td>
<td>—</td>
<td>British thermal units per hour feet F(^a)</td>
</tr>
<tr>
<td>(\rho c)</td>
<td>30</td>
<td>9.96</td>
<td>53.88</td>
<td>—</td>
<td>British thermal units per cubic feet F(^b)</td>
</tr>
<tr>
<td>(h)</td>
<td>(\infty)</td>
<td>(\infty)</td>
<td>(\infty)</td>
<td>1.5</td>
<td>British thermal units per hour square feet F(^c) feet per hour</td>
</tr>
<tr>
<td>(D)</td>
<td>0.191</td>
<td>0.158</td>
<td>0.764</td>
<td>—</td>
<td>feet per hour</td>
</tr>
<tr>
<td>(r)</td>
<td>1.5</td>
<td>1.875</td>
<td>2.5</td>
<td>2.594</td>
<td>feet</td>
</tr>
</tbody>
</table>

\(^a\)F = 1.731 \, \text{W/m} \cdot \text{k}.
\(^b\)F = 67,070 \, \text{J/m}^3 \cdot \text{k}.
\(^c\)F = 5.678 \, \text{W/m}^2 \cdot \text{k}.

Note: 1 ft/hr = 0.305 m/hr; 1 ft = 0.305 m.

hr (55\(^\circ\) C/hr). Ambient temperature was kept constant at 70\(^\circ\) F (21\(^\circ\) C); the convection and radiation coefficients for the outer face of the steel shell were assumed to be constant.

The first 18 eigenvalues for the homogeneous problem were calculated numerically. The results converged quickly as more eigenvalues were included. For the present example, incorporation of only the first 10 eigenvalues sufficed for an accurate solution. It is of interest to note that, although the closed form solution of the heat flow problem was available, evaluation of many integrals involving Bessel functions could be

![Graph](image)

FIG. 5.—Comparison of Transient Finite Difference Solution with Analytical Solution
FIG. 6.—Schematic of Vessel System used for Comparison with another Numerical Solution (9)

done only numerically, which involved more operations than those for the implemented numerical finite difference solution.

The finite difference solution utilized nine nodes across the wall thickness with a time step of 0.5 hr. The temperature profiles resulting from the two solutions at selected times are compared in Fig. 5. Results obtained from the finite difference solution, even with the coarse grid used, were almost identical to those from the analytical solution.

Comparison with another Numerical Solution.—To further demonstrate the applicability of the developed heat flow prediction capability, especially for cases with temperature dependent material properties and cracks or gaps, the results were compared with those from an existing numerical capability (9). While the predictive capability reported in Ref. 9 can be used to model nonlinear conductivities, cracks, and the effect of process gas, it is restricted to a steady-state solution. Example 1, as described in Ref. 9, was chosen for comparison. The system consisted of a 4.5-in. (0.114-m) thick dense refractory concrete, backed by a 7.5-in. (0.191-m) thick lightweight insulating refractory concrete, and a 3-in. (0.076-m) thick steel vessel liner. Vessel diameter was 10 ft (3.05 m) (Fig. 6). The cross section shown includes stainless steel anchors of 3/8 in. (0.0095 m) diameter which were assumed to be welded to the vessel liner at 9-in. (0.229-m) spacings. In the present analysis, however, the effect of anchors was neglected. It was later seen that the effect of anchors on the temperature distribution was very small.

A hot face temperature of 1,500°F (816°C) was used at the inside refractory face, and an ambient temperature of 90°F (32°C) was assumed external to the steel liner. A combined convective/radiative heat transfer coefficient of 3.5 Btu/hr·ft·°F (19.87 W/m²·k) between steel shell and external environment was assumed. The results of the two analyses, with the same number of nodes, material properties, voids, and cracks, are shown in Fig. 7. The hot face temperature in the transient analysis was heated up at 200°F/hr (111°C/hr) to 1,500°F (816°C) and then held constant for 42 hr. The results of the transient analysis were always close to the results of path 1, from Ref. 9, which was the temperature profile through refractory concrete. The path 2 temperature profile, which was the temperature profile through anchors, was also close to other curves. Both analyses gave exactly the same steel shell temperature and, thus, would give the same rate of heat loss.
Simulation.—The refractory lining simulated is lining No. 2 of the tests reported in Ref. 1. The general configuration of the lining is shown in Fig. 8.

The lining was heated up to 1,200° F (649° C) uniformly along the axial direction of the vessel by a centrally located heat source. Thermocouples were installed at various axial positions and various distances from the hot face at two opposite circumferential positions, which allowed for monitoring temperature profiles and histories during the heat-up. The developed transient heat transfer capability was used to simulate the temperature build-up in this test lining. The scheduled and measured hot face temperature histories, which were slightly different, are shown in Fig. 9. For convenience, the scheduled hot face temperature history was input in the analysis. It is believed that this difference between the scheduled and the measured temperatures would not alter the predicted trends.

Simulation runs were made first by assuming that the refractory lining was not cracked throughout the applied heating history. The predicted temperature histories at the center of each layer (insulating and dense) were compared with experimental measurements, as shown in Fig. 9. The temperature profiles through the lining thickness at two different times is shown in Fig. 10. The upper curves in Fig. 10 correspond to the temperature profiles at the peak hot face temperature. The lower curves represent the temperature profiles corresponding to the time immediately after the 400° F (204° C) hold period.

It is seen from these figures that, while the prediction for this uncracked case was accurate for both layers at the 400° F hold period, it somewhat deviated from experimental data at peak temperature for the insulating layer. The deviation was believed to be due to the cracking within the insulating layer which occurred during the 400° F (204° C)
hold period. Figure 9 shows that the measured temperature history at the center of the insulating layer started to decrease at $t = 25$ hr during the hold period. During the testing, cracking in the insulating layer was not directly observable. That it occurred in the insulating layer when the hot face reached $400^\circ$ F ($204^\circ$ C) was concluded from previous analyses (5,6,25). An acoustic emission (A/E) recording (1) made during the test
confirmed this point. The A/E data showed the initiation of extensive activity when the hot face temperature approached 400° F. The A/E activity at this stage was due to the cracking of the insulating layer; the dense layer, which was under direct observation, did not show any cracking.

To incorporate the effect of cracking into the transient heat transfer analysis, a crack volume ratio had to be introduced for the refractory lining layers. In a coupled heat transfer-stress analysis procedure (5,33), this ratio would be established from the prediction of the mechanical behavior and input into the heat transfer analysis as a time dependent parameter. For the purpose of the present study, the crack volume ratio and the radiation distance was judged from the experimental indications.

The assumed crack volume ratios and the associated radiation distances for the layers are shown in Table 2. The assumed cracked state

| TABLE 2.—Assumed Crack Volume Ratio and Radiation Distances for Simulation of Lining Test (1) |
|---------------------------------------------------------------|-----------------|-----------------|-----------------|
| Ratios/distances (1)                                        | Dense layer (2) | Insulating Layer |
| Layer number                                                | 1               | 2               | 3               |
| Thickness, in inches                                        | 4.5             | 3.75            | 3.75            |
| Hoop or axial crack volume ratio radiatio distance, in inches| 0.01            | 0.05            | 0.10            |
| Radial cracks or gaps volume ratio radiation distance, in inches | 0.1             | 0.1             | 3.75            |
|                                                            | 0.005           | 0.02            | 0               |
|                                                            | 0               | 0               | 0               |

Note: 1 in. = 0.0254 m.
simulated the conditions in the refractory lining corresponding to the peak hot face temperature. This crack volume ratio could not be expected to hold valid at other stages of heating; nevertheless, for the purpose of the present analysis, it was held constant throughout the analysis.

The predicted temperature histories at the center of each layer with assumed cracking are shown in Fig. 9. The temperature profiles through the lining thickness are shown in Fig. 10. It is seen that the predicted temperature history for the center of the insulating layer near the peak hot face temperature is in much better agreement with measured values than those for the uncracked case. Also, with the assumed cracking, the predicted temperature profiles for the insulating layer at peak temperature are much closer to the experimental ones. For the temperature profiles at the 400°F (204°C) hold period, the predictions for the uncracked case are in better agreement than those for the cracked case. This was expected because cracks in the insulating layer formed mostly after the 400°F (204°C) hold period.

**Parameter Study**

Sensitivity of the temperature distribution to various identified parameters was investigated (33). As an example, only the effect of high-conductivity process gas is reported herein. The physical model used for the parameter study was the lining previously considered in the simulation study. Heat-up to 1,200°F (649°C) was considered with assumed cracking volume ratio as indicated in Table 2.

As previously mentioned, the presence of high-conductivity process gas may significantly increase the overall thermal conductivity of the lining and, thus, the thermal stress and cracking in the lining. The effect of process gas on temperature distributions is shown in Fig. 11 where the process gas was replaced by pure hydrogen. The use of hydrogen
increased the shell temperature by 44° F (24° C). The hydrogen in this case was chosen solely for illustrative purposes; actual process gas type and its effect on thermal conductivity will depend on the specific gasification process adopted.

CONCLUSION

A simple, effective conductivity model, which takes into account the effect of temperature, cracks, and process gas, is developed. A numerical procedure for one-dimensional heat transfer through layered media, which adopts the developed conductivity model and an iterative solution scheme, is then formulated.

Validity of the proposed model and the verification of the developed numerical scheme is shown by comparing the predictions with the results from an analytical solution, predictions from another numerical capability, and by comparison with experimental measurements. It is shown that accurate predictions can be obtained by the developed capability which may be used as a basis for optimum designs of refractory lined gasification vessels.

The effect of cracks on the temperature distribution is important and should not be neglected in the analysis and design of the process vessels similar to those considered herein. High-conductivity process gas can substantially increase the temperatures in the lining and, therefore, must be considered in design.

ACKNOWLEDGMENT

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APPENDIX I.—REFERENCES


29. Tauchert, T. R., "Thermal Stresses in Coal Conversion Pressure Vessels Built


**APPENDIX II.—Notation**

The following symbols are used in this paper:

- \( c \) = specific heat capacity;
- \( D = k/pc \) = thermal diffusivity;
- \( d \) = maximum dimension of cracks in direction of heat flow;
- \( h \) = heat transfer coefficient;
- \( k \) = thermal conductivity;
- \( k_d \) = thermal conductivity of gases in test environment;
- \( k_c \) = measured thermal conductivity in test environment;
- \( k_g \) = thermal conductivity of process gas;
- \( k_h \) = thermal conductivity of gases in hoop or axial cracks;
- \( k_j \) = thermal conductivity of \( j \)th phase;
- \( k_r \) = radiation contribution to thermal conductivity;
- \( k_s \) = thermal conductivity of gases in radial cracks or gaps;
- \( k_a \) = thermal conductivity of solid phase;
- \( l \) = derived thermal conductivity of solid phase;
- \( n \) = number of phases;
- \( r \) = radial distance from central axis;
- \( t \) = time;
- \( V_v \) = volume ratio of voids;
- \( V_h \) = volume ratio of hoop and axial cracks;
- \( V_r \) = volume ratio of radial cracks and gaps;
- \( x \) = Fricke’s shape factor;
- \( x_j \) = shape factor for \( j \)th phase;
- \( \gamma \) = geometrical factor;
- \( \epsilon \) = emissivity of steel shell surface;
- \( \rho \) = density; and
- \( \sigma \) = Stefan-Boltzmann constant = \( 0.1714 \times 10^{-8} \text{ Btu}/(\text{hr ft}^2 \text{ F}^4) \) = 5.670 \( \times 10^{-8} \text{ W/(m}^2 \text{ K}^4) \).