CONCRETE IN BIAXIAL CYCLIC COMPRESSION

By Oral Buyukozturk, M. ASCE and Tsi-Ming Tseng, A. M. ASCE

ABSTRACT: An experimental program was conducted to study the behavior of concrete under low-cycle high amplitude biaxial cyclic compression. Biaxial loading was achieved by subjecting square concrete plates to in-plane loading where compressive stress was applied in one direction while confining the deformation of the specimen in the orthogonal direction. Three main types of tests were performed: Monotonic loading to failure; cycling of compressive stresses to a limiting envelope curve; and cycling of compressive stresses to prescribed values. In each category, tests were performed on specimens under different levels of strain confinement, and for comparison, on unconfined specimens. Complete stress-strain histories were recorded and analyzed to assess the effect of confinement on concrete behavior under different nonproportional load conditions. A simple predictive model for the constitutive behavior of concrete in biaxial cyclic compression is proposed. Predicted behavior from the model which does not require any a priori information from experiments is found to be in good agreement with the measured response.

INTRODUCTION

In recent years, considerable interest has developed in the strength of concrete members under repeated loadings. Reliable information on strength, failure mode, ductility and energy absorption capacity is required for the design of such systems as offshore gravity, nuclear containment, bridges, and other reinforced concrete structures, particularly those which must withstand seismic loading conditions.

Many reinforced and prestressed concrete structures are subject to cyclic loadings such as those generated by traffic, wind, waves and earthquakes. Stress cycling at relatively low levels results in an approximately linear structural response. These cyclic stresses may cause fatigue failure of the structures or structural members at load levels below their ultimate static strength. Repeated loads, causing cycling at high stress levels which result in nonlinear concrete response, may also occur. There are many examples of severe damage and failure of reinforced and prestressed concrete structures under a relatively small number of cycles of high amplitude. These are generally characterized as low-cycle high-amplitude failures.

In many structural applications, complex behavior of concrete under biaxial or triaxial cyclically varying stresses must be considered. At present, extremely limited experimental and prototype experience on such cyclic behavior is available. Somewhat more, but again limited, information is available on monotonically loaded concrete under biaxial and triaxial conditions.

1Assoc. Prof. of Civ. Engrg., Massachusetts Inst. of Technology, Cambridge, Mass. 02139.
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Most of the early research on plain concrete subjected to variable load histories was directed toward obtaining a fatigue limit for the material. These early works have been reviewed in detail in Ref. 15. Experiments on plain concrete subjected to uniaxial cyclic loadings have been reported in Refs. 21, 8, 17 and 10. Test results obtained from these experiments indicate that the stress-strain relationship of concrete under cyclic compressive load histories describes an envelope, which may be considered unique and identical to the stress-strain curve obtained from monotonically increasing strain; there are indications, however, that the envelope may vary for different strain rates.

From experiments there is evidence that the behavior of concrete in cyclic loading is greatly influenced by microcracking (18–20). It appears that cyclic loading enhances crack propagation at stress levels in excess of approximately 70% of the ultimate strength (17). Cracking occurs at aggregate-mortar interfaces and in the mortar matrix itself. In a recent investigation (12) evidence was found indicating that the behavior of concrete under uniaxial cyclic loading is highly controlled by the nonlinear behavior of the mortar constituent.

Reported investigations on plain concrete subjected to multiaxial cyclic loading conditions provide very little information. Recent experimental investigations on pressure-confined plain concrete under cyclic compression were reported in Refs. 3 and 13. Biaxial cyclic compression tests have been reported in Ref. 1 for concrete specimens subjected to constant and alternating stress ratios.

The main objective of the investigation reported here was to study experimentally the behavior of concrete in low-cycle high amplitude biaxial cyclic compression. For this purpose, cyclic compressive stresses were generated by the application of repeated loadings on square, flat concrete specimens in one direction, while the specimen was subjected to predetermined levels of strain confinement in the second direction. The specimens were subjected to two loading phases: (1) Initial confining phase, where the specimen was loaded horizontally to a predetermined strain value; and (2) application of cyclic compressive stress in a vertical direction, during which the horizontal strain imposed in phase one was kept constant. To establish a comparative basis for the biaxial cyclic data, separate tests were performed to investigate the behavior of the concrete specimens under uniaxial cyclic, and monotonically increasing biaxial stresses.

In what follows, first, the experimental program is described. The biaxial compression test results and their evaluation are presented. Then, a simple analytical model to predict the response of concrete under multiaxial cyclic loadings is developed, and the results are compared with experimental findings. Finally, conclusions from this investigation are given in the last section.

**Experimental Program**

In this section, test specimens, materials, and test procedures are briefly described. Selected tests are described here; and a description of the entire test program is given in Ref. 24.
Test Specimens.—The specimens tested were 5-in. × 5-in. (12.7-cm × 12.7-cm) flat concrete plates with 1-in. (2.5-cm) thickness. In all tests only one type of concrete, having a water-cement ratio of 0.60 by weight, sand-cement ratio of 2.5, and gravel-cement ratio of 3.5, was used. Walbro pea gravel with a maximum size of 0.263 in. (0.7 cm) and Portland Type III (high early strength) cement was used in the mix. Specimens were cured in water until about 2 days prior to testing at 7 days. Average uniaxial strength of the specimens was approximately 4,200 psi (300 kg/cm²).

Loading Arrangement.—The specimens were loaded in two orthogonal directions. A 50 metric ton capacity Automatic Materials Testing System (MTS) with accompanying PDP-11 computer was used for applying vertical loads. An independent loading frame was constructed for applying loads in the horizontal direction. Horizontal loads were applied with the use of a manually operated 15 ton capacity screw jack by which predetermined levels of strain confinements were maintained. The specimens were loaded using brush-like bearing platens developed for a previous investigation (2). In this arrangement, the thin steel plates were flexible enough to offer little restraint against flexural deformation in the direction normal to the applied load, but were such as to transmit the load to the concrete without buckling. (No reduction in restraint was provided in the third direction.) Swivel mechanisms, by means of steel ball bearings, were implemented in the loading arrangement to provide proper alinement of the applied loads.

Strain Measurements.—Resistance strain gages, type FAE-100N-12SX with a length of 1.2 in. were used to measure strains in the vertical and horizontal directions (Fig. 1). Two strain gages (one for each direction) were mounted on each face of the specimens and average strains for each direction were recorded. Monitoring and plotting equipment included digital strain indicators, two X-Y recorders and the MTS x-y recorder.

Test Operations.—Each biaxial test consisted of two loading phases (Fig. 1): (1) Initial confining phase, in which by manual operation of the jack, the specimen was loaded horizontally to a predetermined strain value; and (2) application of cyclic (or monotonic) stress in the vertical direction through the MTS loading ram during which the initially applied horizontal strain remained constant. In all tests, the MTS was operated under stroke (displacement) control. The stroke rate imposed in the monotonic uniaxial and biaxial tests was such that failure of the spec-

![Diagram](image-url)

**FIG. 1.—Loading of Specimen: (a) Loading Phase 1: Confinement in Horizontal Direction; (b) Loading Phase 2: Monotonic or Cyclic Loading in Vertical Direction**
imens occurred in 15–20 min. In the cyclic tests, duration of a typical load-unload cycle was about 2-1/2-5-1/2 min depending on the test series.

**Test Series**

Tests performed in three different series will be reported in this section. These test series are classified according to the type of loading history imposed on the specimen (Table 1). Specific tests in each series were repeated to confirm the consistency of the results.

1. **Monotonic Loading to Failure.**—Unconfined and confined specimens were tested to failure by monotonically increasing the vertical strain. For the confined specimens, under described boundary conditions, vertical and horizontal stresses increased monotonically, but in a nonproportional fashion. Note that in biaxial tests of concrete previously reported (9,11,14,22), proportional loading schemes were adopted.

2. **Cycles to Envelope Curve.**—Unconfined and confined specimens were loaded vertically up to a given value of vertical strain, unloaded to zero vertical stress, and then reloaded until the stress-strain curve followed the trend of the previous loading portion of the curve. The specimen was, again, unloaded to zero vertical stress, and the procedure was repeated in the same fashion until failure occurred.

3. **Cycles to Prescribed Values of Vertical Stress.**—Confined specimens were subjected to a given vertical cyclic stress history. Thereafter, the specimens were loaded monotonically to failure.

The previously described test series allowed the evaluation of the effects of different confinements on stress-strain behavior of concrete. In all confined tests the horizontal confinement levels were characterized by the values of predetermined horizontal strain, \( \epsilon_{hc} \), which remained

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<tr>
<th>Specimen group (1)</th>
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<td>2.4</td>
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<td>3.4</td>
<td>( C/C_{y} \sigma_{h} = 0.18 f'_{c} )</td>
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<td>2.5</td>
<td>U/C( y )</td>
<td>3.5</td>
<td>( C/C_{y} \sigma_{h} = 0.37 f'_{c} )</td>
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<td>1.6</td>
<td>C/M ( \sigma_{h} = 0.60 f'_{c} )</td>
<td>2.6</td>
<td>C/C( y ) ( \sigma_{h} = 0.20 f'_{c} )</td>
<td>3.6</td>
<td>( C/C_{y} \sigma_{h} = 0.55 f'_{c} )</td>
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Note: U = unconfined; C = confined; M = monotonic; and C\( y \) = cyclic.
constant during each test, and the initial value of the stress in the horizontal direction, \( \sigma_{h,i} \), introduced by the application of \( \epsilon_{hc} \) during the first phase of loading. Horizontal stresses fluctuated as a function of the applied vertical load during the second phase of loading. In order to establish a basis for comparison, tests on unconfined specimens were also performed. For each test series, concrete batch properties were determined through the performance of uniaxial, monotonic load tests (\( f'_{c} = \) peak stress, \( \epsilon_{0} = \) strain corresponding to peak stress).

Complete stress-strain histories in the plane of the specimen (\( h \) and \( v \) directions in Fig. 1) were recorded. No measurements were made in the direction perpendicular to the plane of the specimen. For unconfined specimens, vertical stress, \( \sigma_{v} \), versus vertical strain, \( \epsilon_{v} \), and vertical strain versus horizontal strain, \( \epsilon_{h} \), relationships were established. For the confined specimens, vertical stress versus vertical strain and horizontal stress increment, \( \sigma_{h} \), versus vertical strain relationships were recorded.

**TEST RESULTS AND EVALUATION**

In this section results obtained from the previously described tests will be summarized. Some selected, representative data will be given, and basic behavioral trends highlighted. Complete data analysis and details may be found in Ref. 24.

1. **Monotonic Loading to Failure**

   The confined specimens initially exhibited a linear behavior with an effective modulus (slope of the \( \sigma_{v} \) versus \( \epsilon_{v} \) curve) slightly higher than that corresponding to the unconfined specimen. At a value of vertical stress approximately equal to the uniaxial elastic limit, the confined concrete started to exhibit inelastic behavior with a stiffer response compared to that of the uniaxial case. Vertical stresses and strains at failure were higher than those for the uniaxial compression test. Both stiffness and strength increased with the level of confinement.

   For confined specimens the horizontal stress increment (i.e., stress magnitude added to the initially introduced stress due to strain confinement), \( \sigma_{h} \), versus applied vertical strain, \( \epsilon_{v} \), curves were fairly linear (Fig. 2). The initial slope of the curve was nearly the same for specimens confined by stress levels below 0.50 \( f'_{c} \) and decreased at higher values

![Image of Figure 2](image-url)

**FIG. 2.—Horizontal Stress Variation Under Confined Monotonic Loading**

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of $\sigma _h$. The initial ratio of horizontal to vertical stress increments was found to be considerably lower than that predicted by linear elastic isotropic theory, according to which $\sigma _h/\sigma _v$ is equal to the initial Poisson's ratio, $\nu$, of the uniaxial compression test, which is approximately equal to 0.20. For example, in test 1.5, the initial ratio $\sigma _h/\sigma _v$ was equal to 0.08.

The observed behavior which is related to microcracking in concrete suggests that the concrete under confinement behaves as a stress-induced orthotropic material with properties in the vertical and horizontal directions dependent on the initial confining conditions.

In the present tests, where stresses varied nonproportionally, strains at failure were up to 25% higher than the uniaxial peak strain $\varepsilon _p$, while stresses at failure were up to 40% higher than $f'_c$. Stress and strain states at failure somewhat deviated from those observed in previous biaxial tests (e.g., see Ref. 9), where stresses varied proportionally. In such tests the maximum increase in strength due to confinement was less than 30% of the uniaxial strength (see Fig. 11 for comparison). These differences can be attributed to the different boundary conditions and loading paths adopted in the investigation reported here.

2. Cycles to Envelope Curve

Uniaxial Cyclic Test.—Figs. 3 and 4 show the $\sigma _v$ versus $\varepsilon _v$ and $\varepsilon _h$ versus $\varepsilon _h$ curves, respectively, of uniaxially cycled specimen 2.4. General behavior exhibited in this test resembles that reported in other uniaxial cyclic tests carried out by different investigators. The concrete exhibited a typical hysteretic behavior (8,21), where the area within the hysteresis loops representing the energy dissipated during a cycle became larger.

![FIG. 3.—Uniaxial Cyclic Loading to Envelope Curve](image1)

![FIG. 4.—Relationship between Vertical and Horizontal Strains Under Uniaxial Loading, Test 2.4](image2)
as vertical strain increased. It was observed (Fig. 3) that the reloading curves were nearly linear along their length up to the intersection with the unloading curve of the cycle, after which they decreased in slope. The unloading curves were slightly nonlinear along most of their length, and exhibited a marked increase in curvature as they approached zero vertical stress. A continuous degradation of the concrete was observed, as evidenced by the decrease of the slopes of the reloading curves. The envelope to the cyclic curve was seen to coincide with the uniaxial monotonic curve.

For uniaxial cyclic tests, $\varepsilon_v$ versus $\varepsilon_h$ relations are shown in Fig. 4. The points where the reloading curves intersect the unloading curves in a cycle may be termed "common strain points." These points do not coincide with the common points of the $\sigma_v$ versus $\varepsilon_v$ curves.

An upper envelope to the $\varepsilon_v$ versus $\varepsilon_h$ curve may be formed by joining the upper peaks of the curve, while a lower envelope may be formed by joining the lower peaks. The lower peaks correspond to nonrecoverable (or residual) strain states attained upon completion of unloading to zero vertical stress. The lower envelope, thus defined, may be designated as a lower bound to uniaxial cyclic strain histories. The upper envelope may possibly coincide with the monotonic $\varepsilon_v$ versus $\varepsilon_h$ curve. More tests are required, however, to examine the uniqueness of these upper and lower envelopes.

**Biaxial Cyclic Tests.**—Figs. 5 and 6 show the $\sigma_v$ versus $\varepsilon_v$ curves for confined cyclic tests with $\sigma_h = 0.20 f'_c$ and $0.60 f'_c$, respectively. In general, $\sigma_v$ versus $\varepsilon_v$ behavior of confined specimens subjected to cyclic stress was found to be similar to that of specimens under uniaxial cyclic loading. The initial linear portion of the envelope curves had, at all confinement levels, nearly the same slope which was slightly higher than the initial modulus in uniaxial loading. In the inelastic ranges, however, lateral confinement induced much stiffer behavior than that obtained under uniaxial conditions, as evidenced by the rise of the biaxial cyclic envelopes above the uniaxial cyclic envelope curve (Fig. 7).

In all cases, reloading was practically linear (except at very low stress

![FIG. 5.—Confined Cyclic Loading to Envelope Curve, Test 2.6 ($\sigma_h = 0.20 f'_c$, $\varepsilon_{hc} = 0.08 \varepsilon_v$)](image1)

![FIG. 6.—Confined Cyclic Loading to Envelope Curve, Test 2.8 ($\sigma_h = 0.60 f'_c$, $\varepsilon_{hc} = 0.27 \varepsilon_v$)](image2)
levels) up to the common points (Figs. 5–6). In all confined tests the slope of the first reloading curve was slightly higher than that of the first loading branch, indicating a hardening effect on the concrete produced by the first load-unload cycle. This behavior is the same as that observed in the uniaxial cyclic tests. The reloading slopes progressively decreased as the number of cycles increased.

Nonrecoverable (or plastic) strains are those corresponding to a zero stress level on the unloading stress strain curve. Fig. 8 shows the relationship between the residual strain, $\epsilon_r$, in the vertical direction and the corresponding envelope strains in the same direction at unloading, $\epsilon_u$, for the uniaxial and biaxial cyclic tests. Quantities are normalized with respect to $\epsilon_o$. From the figure, the plastic strains appear to be dependent mainly on the vertical strain at unloading, and do not seem to be significantly affected by the stresses and strains in the horizontal direction. At values of $\epsilon_u/\epsilon_o$ higher than 0.60, the plastic strains in the confined tests appear to be slightly reduced with respect to those in the unconfined tests. However, the difference is not significant and for practical purposes unique values may be considered. For comparison, the curve obtained in Ref. 8 based on their uniaxial cyclic tests is shown in Fig. 8. It coincides initially with the data obtained in this investigation for low values of $\epsilon_u/\epsilon_o$. At higher values, there is some deviation which may be attributed to the differences in experimental techniques, boundary conditions, form, and size of the specimens.

Generally, the confined specimens exhibited a degradation process similar to that of the uniaxially cycled specimens. Such a process can be
characterized by the progressive decline of the initial slopes of the re-loading curves, and has been generally attributed in the uniaxial cyclic tests to a steady increase in microcracking as cycling proceeds. However, in the biaxial tests, microcracking was inhibited due to confinement; thus, it appears that the degradation process may not be solely attributed to the microcracking at the mortar-aggregate interface. The nonlinear behavior of mortar itself and its effect on the degradation process should also be considered. This has also been implied in a recent investigation (12). However, in the present investigation, the confinement applied was only in the plane of biaxial loading. One should not exclude the possibility that the degradation process may, to a certain extent, be attributed to microcracking or straining in the unconfined direction out of the plane. Such data were not obtained in the tests reported here.

In Figs. 9 and 10, $\sigma_h$ versus $\varepsilon_v$ relations are shown for confined cyclic tests 2.6 and 2.8, respectively. A peculiar characteristic of these relations is that reloading curves initially coincide with the unloading curves in the unload-reload cycles. This might be related to the effect that the horizontal stress does not drop to zero at the end of each cycle, and thus reloading occurs at a relatively high value of stress. From uniaxial cyclic tests it was observed that if reloading in a given cycle occurs at a relatively high value of stress, the reloading curve would then tend to follow more closely the unloading curve, thus exhibiting a reduced hysteretic behavior. Thus, in the biaxial tests the observed deformation behavior in the horizontal direction may be considered analogous to that of the uniaxial cyclic behavior at high stresses.

The upper and lower peaks of the $\sigma_h$ versus $\varepsilon_v$ curves correspond to the peaks of the $\sigma_v$ versus $\varepsilon_v$ curves. The envelopes of these peak points are shown in Fig. 11. The initial linear portion of the upper envelope
FIG. 9.—Horizontal Stress Variation Under Confined Cyclic Loading to Envelope Curve, Test 2.6 ($\sigma_{hi} = 0.20 f'_c$, $\epsilon_{hc} = 0.08 \epsilon_o$)

FIG. 10.—Horizontal Stress Variation Under Confined Cyclic Loading to Envelope Curve, Test 2.8 ($\sigma_{hi} = 0.60 f'_c$, $\epsilon_{hc} = 0.27 \epsilon_o$)

coincides with the corresponding biaxial monotonic curve. At higher values of strain, the upper envelope falls below the monotonic curve. This deviation is greater at higher confinement levels. The lower envelope falls below the initial horizontal stress level. Its deviation from this level is greater at higher confinement levels. The initial confining conditions, therefore, influence the variations of horizontal stress during confined cyclic compression.

In general, stresses at failure in the biaxial cyclic tests were found to be close to the failure stresses corresponding to monotonic loadings. Average strength envelope for the specimens from the biaxial monotonic and cyclic tests are shown in Fig. 12. It thus appears that within the load histories followed in this investigation concrete failure occurs upon attainment of a given stress state, i.e., a certain combination of $\sigma_v$ and $\sigma_h$ represented by the curve shown in Fig. 12, independent of whether the loading is monotonic or cyclic. Furthermore, it also appears that, within the load conditions reported here, load histories may lead to strengths greater than those under proportional loads to failure. This has already been implied in a recent study reported in Ref. 1 (see Fig. 12 for comparisons).

FIG. 11.—Upper and Lower Envelopes of $\sigma_v$ versus $\epsilon_v$. Curves Corresponding to Confined Cyclic Tests to Envelope

FIG. 12.—Biaxial Strength of Concrete of $\sigma_v$ versus $\epsilon_v$. Curves Corresponding to Confined Cyclic Tests to Envelope
Cycles to Prescribed Values of Vertical Stress

In this test series an unconfined specimen was cycled to the envelope curve until failure. Specimens under different confinement levels were then cycled to the same vertical stress levels as in the unconfined specimens. Then they were loaded monotonically to failure. Fig. 13 shows the \( \sigma_v \) versus \( \varepsilon_v \) curve of a specimen confined to \( \sigma_{hi} = 0.55 f'_c \) while Fig. 14 shows the \( \sigma_h \) versus \( \varepsilon_v \) curve. Vertical stress-strain behavior was significantly affected by the confinement. Total and plastic strain accumulations in the confined tests at the end of the cycles were lower than those in the uniaxial test. It was observed that the ultimate strength of the confined specimen was not influenced by previous cycling.

Simple Analytical Model

A simple analytical model which describes the cyclic stress-strain behavior of concrete under a general loading conditions is presented in this section. The model is based on an incrementally orthotropic formulation and uses an equivalent one-dimensional approach to represent the multiaxial behavior (4–7). The model has recently been used in three-dimensional analysis of refractory concrete linings under cyclic temperature loadings (23).

Based on the orthotropic stress-strain relation developed in Ref. 23, the problem of forming the material incremental stiffness matrix is reduced essentially to the evaluation of the tangent modulus in each direction \((E_1, E_2, \text{ and } E_3)\). To evaluate the modulus in each direction, it is convenient to adopt an equivalent uniaxial stress-strain relationship

![FIG. 13.—Confined Cyclic Loading to Prescribed Stress Levels, Test 3.6 (\( \sigma_{hi} = 0.55 f'_c, \varepsilon_{hi} = 0.27 \varepsilon_c \))](image)

![FIG. 14.—Horizontal Stress Variation Under Confined Cyclic Loading to Same Vertical Stress Levels as in Uniaxial Cycling, Test 3.6 (\( \sigma_{hi} = 0.55 f'_c, \varepsilon_{hi} = 0.27 \varepsilon_c \))](image)
from which the modulus can be obtained based on the strain or stress level. The directions of these equivalent uniaxial strains are assumed, in this development, to coincide with the principal stress directions. To account for the multiaxial effect, Poisson’s effect and microcracking confinement effect are usually separated. In such an approach the Poisson’s effect may be taken out, e.g., by defining the equivalent strain in each direction in the following manner:

$$\varepsilon_{ei} = \frac{\varepsilon_i}{1 - \frac{\sigma_j + \sigma_k}{\sigma_i}} \quad (1)$$

in which $\varepsilon_{ei} = \text{equivalent strain in the } i \text{ direction}; \varepsilon_i = \text{principal strain in the } i \text{ direction}; \sigma_i, \sigma_j, \sigma_k = \text{principal stresses in the } i, j, \text{ and } k \text{ directions}; \text{ and } i, j, k = \text{permutation indices of } 1, 2, 3.$

However, in applying Eq. 1 to practical analysis, difficulty arises due to singularities for certain combination of stresses. For the purpose of the present development, the equivalent strain will be obtained, instead, by updating at each increment with

$$\Delta \varepsilon_{ei} = \frac{\Delta \sigma_i}{E_i} \quad (2)$$

The effect of microcracking confinement due to stresses in the lateral directions on stiffness is accounted for by adjusting the stress and strain at the peak of the equivalent stress-strain curve. The peak stress is obtained by passing a line from the origin through the current principal stress point in the principal stress space until it penetrates the failure surface. The equivalent strain at peak is assumed to be directly proportional to the peak stress.

The equivalent stress-strain relation used is the one proposed in Ref. 16:

$$\frac{\sigma}{\sigma_p} = \frac{n}{n - 1} \left( \frac{\varepsilon_e}{\varepsilon_p} \right)^n \quad (3)$$

in which $\sigma_p = \text{principal stress at peak}; \varepsilon_p = \text{equivalent strain at peak}; n = \text{shape factor}; \sigma = \text{principal stress}; \text{ and } \varepsilon_e = \text{equivalent strain}. \text{ Eq. 3 is assumed to be valid independently for each direction. The curve is completely defined by any three of the four parameters, } \sigma_p, \varepsilon_p, n, \text{ and } E_o, \text{ which is the initial slope of the } \sigma-\varepsilon_e \text{ curve. Knowing } \sigma_p, \varepsilon_p, \text{ and } E_o, n \text{ is calculated by}

$$n = \frac{1}{1 - \frac{\sigma_p}{E_o \varepsilon_p}} \quad (4)$$

Having obtained $n$, the slope at a point with equivalent strain $\varepsilon_e$ can be determined by differentiating Eq. 3 with respect to $\varepsilon_e$: 472
\[
E = \frac{\left[ 1 - \left( \frac{\epsilon_r}{\epsilon_p} \right)^n \right] n(n - 1) \frac{\sigma_p}{\epsilon_p}}{\left[ n - 1 + \left( \frac{\epsilon_r}{\epsilon_p} \right)^n \right]^2}
\]
\[
\text{For the present purpose, in applying Eq. 3, the softening part of the curve is neglected, i.e., concrete loses its strength beyond the peak.}
\]

The preceding uniaxial stress-strain relation is extended to include cyclic loading conditions. The unloading portion of the stress-strain curve is assumed to be a parabola, which is assumed to have an infinite slope at the beginning of unloading and pass through a point with a specific magnitude of residual strain, \( \epsilon_r \), at zero stress. Relation between residual strain and strain at unloading plotted in Fig. 8 is fitted by
\[
\frac{\epsilon_r}{\epsilon_p} = 0.162 \left( \frac{\epsilon_u}{\epsilon_p} \right) + 0.334 \left( \frac{\epsilon_u}{\epsilon_p} \right)^2
\]
in which \( \epsilon_r = \) residual strain at zero stress; and \( \epsilon_u = \) strain at unloading. For reloading, Eq. 3 is used again. Definition of \( \sigma_p \) and \( \epsilon_p \) remains the same. However, the initial slope is redefined by the straight line pointing toward the last point of unloading.

The analytical model is used to simulate the biaxial compression tests reported in this paper. Comparison with uniaxial test is shown in Fig. 2. For confined biaxial compression tests, comparison of the predicted and test results are shown in Figs. 5 and 9 for Test 2.6 with \( \sigma_{h} = 0.20 f' \). The prediction of the stress-strain response in the vertical direction (\( \sigma_v \) versus \( \epsilon_v \)) for this specimen is satisfactory as seen in Fig. 5. The prediction of the \( \sigma_h \) versus \( \epsilon_v \) response as shown in Fig. 9 shows adequately the variation of lateral stress due to confinement; hysteresis loops, however, are predicted with the model while very little hysteretic behavior was observed in the tests. This is due to the simplified formulation adopted in this development for unloading and reloading curves.

**Conclusion**

The results obtained from this investigation on the behavior of plain concrete in biaxial cyclic compression can be summarized as follows:

1. Concrete specimens confined in one direction behaved orthotropically when loaded in a direction normal to the confinement. Concrete stress and strain characteristics in the vertical and horizontal directions were dependent upon the initial confining conditions.
2. Envelopes to the confined and unconfined cyclic stress-strain curves corresponding to the direction of loading practically coincided during the initial stage of loadings. At higher values of strain the envelope curves were higher at greater confinement levels.
3. In both uniaxial and biaxial cyclic tests to the envelope, the slope of the first reloading curve was greater than that of the loading branch, indicating a hardening effect on the concrete produced by the first load-unload cycle. As cycling proceeded, there was a continuous degradation
of the elastic moduli, indicated by the progressive decrease of the slopes
of the reloading curves.

4. Plastic strain magnitudes due to unloading from the envelope curve
were not significantly affected by the level of confinement, and may be
defined as a function of the envelope strain at unloading.

5. The degradation process of concrete under cyclic loading before peak
stress appeared to be affected by the inelastic behavior of mortar. How-
ever, since the confinement applied in the present investigation was only
in the plane of biaxial loading, this degradation process may also be
attributed to microcracking or straining in the unconfined direction out
of the plane. This effect needs to be studied further.

6. Within the load histories employed in this investigation, strength
envelope for the specimens appeared to be independent of whether the
loadings were cyclic or monotonic. Furthermore, load histories seemed
to lead to strengths greater than those under proportional loads to failure.

7. The proposed orthotropic model appears to represent the concrete
behavior adequately under cyclic compressions. The formulation is sim-
ple so that the model can be easily implemented in practical finite ele-
ment analysis. However, no generality is claimed for this predictive ca-
pability; there is a need for the development of a constitutive model
which would capture the fundamental behavioral characteristics of the
specimens observed in this investigation, and which would have well-
defined parameters for convenient implementation in practical analysis.

The investigation reported in this paper represents an initial effort to
study the behavior of concrete under biaxial cyclic loading. The number
of tests performed was limited. Further research to verify the obtained
results and to include the effect of confinement levels other than those
reported in this study is needed. Furthermore, additional tests are re-
quired to study the effects of other variables such as concrete strength,
and rate of loading.

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APPENDIX I.—REFERENCES

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**APPENDIX II.—NOTATION**

*The following symbols are used in this paper:*

- $\varepsilon_v =$ vertical strain;
- $\varepsilon_h =$ horizontal strain;
- $\varepsilon_{hc} =$ initial confinement horizontal strain;
- $\varepsilon_o =$ strain at peak stress in uniaxial test;
- $\varepsilon_e =$ equivalent strain;
- $\varepsilon_p =$ equivalent strain at peak stress on equivalent stress-strain curve;
- $\varepsilon_r =$ residual plastic strain at zero stress;
- $\varepsilon_u =$ strain at unloading;
- $\sigma_v =$ vertical stress;
- $\sigma_h =$ horizontal stress increment;
- $\sigma_{hi} =$ initial horizontal stress for confinement (correspond to $\varepsilon_{hc}$);
- $\sigma_H =$ $\sigma_{hi} + \sigma_h$; and
- $\sigma_p =$ principal stress at peak.