Alternative Approaches for Real-Time Estimation and Prediction of Time-Dependent Origin-Destination Flows

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Abstract

This paper examines two different approaches for real-time estimation/prediction of time-dependent Origin-Destination(O-D) flows. Both approaches lend themselves to formulation as state-space models. The first approach is an extension of previous work by the authors (Ashok and Ben-Akiva(1993)). The key idea in this approach is to define the state-vector in terms of *deviations* in O-D flows instead of the O-D flows themselves. We demonstrate that approximations to this model make the real-time estimation process computationally more tractable with little deterioration in quality of estimates. In the second approach, the state vector is defined in terms of deviations of departure rates from each origin and the shares headed to each destination. This approach attempts to capture the differential variation of departure rates and shares over time.

Performance of the proposed models is evaluated using actual traffic data from different sources. Preliminary results indicate that the filtering procedure is robust and that compared to the original model, a formulation based on departure rates and shares yields better predictions with some loss of accuracy in filtered estimates.

Dynamic Traffic Management Systems have been paid increasing attention in recent years. These systems include dynamic driver information and adaptive traffic control. An important component of these systems is a dynamic traffic assignment (DTA) model (Ben-Akiva et al.(1994)). The DTA requires as input a time-dependent Origin-Destination matrix for the network.

Research on estimating such time-dependent O-D matrices can be broadly divided into two groups – pertaining to "closed" networks and pertaining to general networks. A closed network implies that complete information is available on the entry and exit counts of the network at all points in time. Several estimators have been proposed for the former (e.g. Bell(1991), Chang and Tao(1996), Chang and Wu(1994), Cremer and Keller(1987), Nihan and Davis(1987), Van der Zijpp(1996)). These approaches suffer from the limitation that all entry and exit counts should be known – an unrealistic scenario in a general urban traffic network. For general networks, Cascetta et al.(1993) provide an extension of the static O-D estimation framework. Though their model cannot be used online in the real-time context because it does not have any predictive or updating element, it can be used as an off-line tool for creating a set of "historical estimates" of O-D flows for the network. An enhanced model with predictive ability was developed by Inaudi et al.(1994). In this model, the "sequential" model of Cascetta et al. is used for estimation. Values thus estimated are then used to generate predictions by a separate "filtering" approach that combines historical and estimated O-D information using the concept of "deviations" proposed by Ashok and Ben-Akiva(1993) (see below). The main disadvantage of this approach is that the prediction component is completely exogenous to the estimation resulting in a statistically inefficient estimator. An alternative approach based on state-space modeling was suggested by Okutani(1987). The state vector in this formulation is the vector of unknown O-D flows. However, the auto-regressive formulation of the transition equation is simplistic and only captures temporal interdependencies between O-D flows. No structural information on trip making is incorporated into the formulation. Finally, another approach based on state-space modeling with the state-vector defined in terms of O-D flow *deviations* was presented by Ashok and Ben-Akiva(1993). This removes the problem with the auto-regressive specification in the earlier work. However, the problem remains very computation intensive since this procedure not only estimates O-D matrices for the current interval but also updates estimates for prior intervals. Kachroo et al.(1995) extended this approach to account for serial correlation of errors in the autoregressive formulation.

This paper contributes to the state of the art in dynamic O-D estimation and prediction in two ways. First, we present an approximation technique that involves estimating each O-D flow exactly once and holding it constant for future departure intervals. Second, we employ this approximation to two different approaches to real-time estimation and prediction. The first approach is based on earlier work by the authors (Ashok and Ben-Akiva(1993)) while the second is a new formulation that is based on departure rates from origins and destination shares instead of O-D flows. We provide results from several case studies where the performance of each model was evaluated using actual traffic data.

1 Model Formulation

1.1 The Idea of Deviations

A key idea in our approach is the use of *deviations* of O-D flows (or departure rates from each origin and shares to each destination) as the state vector instead of the O-D flows themselves. Previous formulation of this problem as a state-space model by Okutani(1987) uses for the transition equation a simple autoregressive process on the O-D flows. Such a process only captures temporal interdependencies among O-D flows and ignores structural information on trip making patterns. The pattern of O-D trips in a transportation network is a complicated function of the spatial and temporal distribution of activities and cannot be modeled by a simplistic auto-regressive procedure.

Suppose however that O-D flows have been estimated for several previous days or months. Since these flows subsume in them all the information about trip making patterns and their spatial and temporal variations, incorporating these in the formulation is highly desirable. A simple way then of capturing structural information is to use *deviations* of O-D flows (or departure rates and destination shares) from best historical estimates instead of the O-D flows themselves as state-vector in a state-space model. Using deviations has other benefits. A normal distribution for traffic variables (link volumes, O-D flows, etc) is a useful property for available statistical tools such as the Kalman Filter. Deviations are more amenable to approximation by a normal distribution than the original traffic variables since they can take on both positive and negative values.

1.2 The Basic Formulation

1.2.1 The Transition and Measurement Equations

Consider a network with n_{LK} links and n_{OD} O-D pairs. It is assumed that n_l of these links are equipped with counting stations. Denote by x_{rh} the number of vehicles between the rth O-D pair that left their origin in interval h and by x^H_{rh} the corresponding best historical estimate. Further, let the corresponding $(n_{OD} * 1)$ vectors of all O-D flows be given by $\mathbf{x}_h$ and $\mathbf{x}^H_h$ respectively. To set up the *transition* equation, it is hypothesized that the deviations in O-D flows from a historical base value at period h can be related to the deviations in O-D flows of previous time periods by the autoregressive form:

$$x_{rh+1} - x^H_{rh+1} = \sum_{p=h-q'}^{h} \sum_{r'=1}^{n_{OD}} f^{r'p}_{rh} (x_{r'p} - x^H_{r'p}) + w_{rh} \tag{1}$$

where the coefficients $f^{r'p}_{rh}$ describe the effect of the deviation $(x_{r'p} - x^H_{r'p})$ on the deviation $(x_{rh+1} - x^H_{rh+1})$ and w_{rh} is a random error. q' is the number of lagged O-D flow deviations (excluding the deviation in the interval h) assumed to affect the O-D flow deviation in interval $h+1$; q'+1 therefore is the degree of the autoregressive process.

Thus equation (1) captures correlation over time among deviations which arise from unobserved factors that are correlated over time. Such factors include weather conditions, special events, temporary changes in the transportation network, etc. Also, the model in the above form is highly general and assumes dependence of deviations corresponding to one O-D pair on deviations corresponding to other O-D pairs in prior periods. In practical applications (for example in the freeway case studies in this paper) this may be relaxed and relationships between deviations across different O-D pairs may be ignored.

In matrix form, the equation can be expressed as:

$$\mathbf{x}_{h+1} - \mathbf{x}^H_{h+1} = \Sigma^h_{p=h-q'} \mathbf{f}^p_h (\mathbf{x}_p - \mathbf{x}^H_p) + \mathbf{w}_h \tag{2}$$

where $\mathbf{f}^p_h$ is an $(n_{OD} * n_{OD})$ matrix of effects of ($\mathbf{x}_p$ - $\mathbf{x}^H_p$) on $(\mathbf{x}_{h+1} - \mathbf{x}^H_{h+1})$ and $\mathbf{w}_h$ an $(n_{OD} * 1)$ vector of random errors. The following assumptions are made about the error vectors:

1. $E[\mathbf{w}_h] = 0$

2. $E[\mathbf{w}_h \mathbf{w}'_m] = \mathbf{Q}_h \delta_{hm}$ where $\delta_{hm} = 1$ if h=m and 0 o.w. $\forall h, m$ and $\mathbf{Q}_h$ is an

$(n_{OD} * n_{OD})$ variance-covariance matrix.

In situations where the assumption of no serial correlation may break down (for example, due to a poor historical matrix), a variant of the estimation algorithm proposed in this paper can be employed (see for example Kachroo et al.(1995)).

Computation of the matrix $\mathbf{f}_h^p$ involves estimating linear regression models for each O-D pair. The error covariance matrix $\mathbf{Q}_h$ could be approximated from the residuals of these regressions. As stated earlier, the regressions would be much simplified by assumptions about the structure of the auto-regressive process, for instance, by assuming that $\mathbf{f}_h^p$ is diagonal.

To express the *measurement* equation, denote by y_{lh} the observed traffic counts at detector station l during interval h and by $\mathbf{y}_h$ the corresponding $(n_l * 1)$ vector. Then the measurement equation for y_{lh} can be written as:

$$y_{lh} = \sum_{p=h-p'}^{h} \sum_{r=1}^{n_{OD}} a_{lh}^{rp} x_{rp} + v_{lh} \qquad (3)$$

where a_{lh}^{rp} is the fraction of the rth O-D flow that departed its origin during interval p and crossing the counting point on link l during interval h. v_{lh} is the measurement error while p' is a measure of the maximum number of time intervals taken to travel between any O-D pair for the network. In matrix form the above equation reduces to:

$$\mathbf{y}_h = \Sigma_{p=h-p'}^{h} \mathbf{a}_h^p \mathbf{x}_p + \mathbf{v}_h \qquad (4)$$

where the matrix $\mathbf{a}_h^p$ is an $(n_l * n_{OD})$ *assignment* matrix of contributions of $\mathbf{x}_p$ to $\mathbf{y}_h$ and $\mathbf{v}_h$ is the vector of measurement errors. Assume that

1. $E[\mathbf{v}_h] = 0$
2. $E[\mathbf{v}_h \mathbf{v}'_m] = \mathbf{R}_h \delta_{hm}$ where $\delta_{hm} = 1$ if h=m and 0 o.w. $\forall h, m$ and $\mathbf{R}_h$ is an $(n_l * n_l)$ variance-covariance matrix.
3. $E[\mathbf{w}_h \mathbf{v}'_m] = 0 \; \forall h, m$; i.e., transition and measurement errors are uncorrelated.

The interpretation of Equation (4) is straightforward. The flow across any detector station during interval h is comprised of contributions from O-D flow vectors corresponding to departures during h,h-1,...,h-p'. The assignment matrix consists of the proportions of these O-D flows that constitute the link flow. The error term reflects the possibility of imperfect measurements.

Again, there could be situations in which the assumption of no serial correlation might break down. An example would be a specific detector that – perhaps because of incorrect calibration – consistently over-estimates or under-estimates a link volume. Again in such cases, this assumption might be relaxed.

In deviation form, the above equation can be rewritten as:

$$\mathbf{y}_h - \mathbf{y}_h^H = \Sigma_{p=h-p'}^{h} \mathbf{a}_h^p (\mathbf{x}_p - \mathbf{x}_p^H) + \mathbf{v}_h \tag{5}$$

where $\mathbf{y}_h^H = \Sigma_{p=h-p'}^{h} \mathbf{a}_h^p \mathbf{x}_p^H$.

It should be noted again that the vector $\mathbf{x}_h^H$ is known $\forall h$ and is obtained from a "database" created off-line. We describe in Section 3 how this database might be constructed and updated.

While the error covariance matrix $\mathbf{R}_h$ can be easily computed from historical data, computation of the matrices $\mathbf{a}_h^p$ is a complicated exercise. The fractions contained in these matrices depend on path choice probabilities as well as the stochastic mapping of time-dependent path flows to link flows. One way of determining the former could be by means of discrete choice models. To estimate the latter, one would need in addition, knowledge about time-dependent travel times. Travel times could be obtained from a traffic surveillance system (e.g. sensors or probe vehicles) or from a simulation model (See for example Ben-Akiva et al.(1994)). We provide some guidelines for computing the assignment fractions in Section 2. A detailed treatment can be found in Ashok and Ben-Akiva(1996).

An examination of Equation (4) indicates that a link count corresponding to time-interval h provides information not only about $\mathbf{x}_h$ but also about $\mathbf{x}_{h-1}$, $\mathbf{x}_{h-2}$, ..., $\mathbf{x}_{h-p'}$. Similarly, Equation (2) implies a serial correlation in O-D deviations that extends over multiple time periods. This suggests that in order to fully exploit all available information, each O-D flow be estimated multiple times. More specifically, it suggests that each O-D flow be estimated $max(p'+1, q'+1)$ times. The standard technique of achieving this is through State Augmentation (as in Ashok and Ben-Akiva(1993) and Okutani(1987)). In this approach, the state-vector is augmented to include deviations on O-Ds for prior departure intervals up to $s = \max(p', q')$. This yields after some algebra,

$$\mathcal{Y}_h = \mathbf{A}_h \mathcal{X}_h + \mathcal{B}_h + \mathbf{v}_h \tag{6}$$

where
$\mathcal{Y}_h = \mathbf{y}_h - \mathbf{y}_h^H$
$\mathcal{X}_h = \mathbf{X}_h - \mathbf{X}_h^H$
$\mathcal{B}_h = \mathbf{A}_h \mathbf{X}_h^H - \mathbf{y}_h^H$
and

$$\mathcal{X}_{h+1} = \mathbf{\Phi}_h \mathcal{X}_h + \mathbf{W}_h \tag{7}$$

In the above equations, $\mathbf{X}_h$ and $\mathbf{X}_h^H$ are $(n_{OD}(s+1)*1)$ augmented vectors; $\mathbf{A}_h$ is an augmented $(n_l * n_{OD}(s+1))$ matrix; $\mathbf{\Phi}_h$ is an augmented and appropriately modified $(n_{OD}(s+1) * n_{OD}(s+1))$ matrix and the error $\mathbf{W}_h$ is an $(n_{OD}(s+1)*1)$ vector with zeros in the bottom $(n_{OD}s)$ cells and an $(n_{OD}(s+1)*n_{OD}(s+1))$ variance matrix $\mathcal{Q}_h$.

With Equations (6) and (7), we complete the specification of our state space model. A standard approach to estimating the state vector in such a system is by way of a *Kalman Filter*. The equations comprising the Kalman Filter solution to the problem are fairly standard and are omitted here (see for example Gelb(1974)).

1.2.2 An Approximation

From the nature of the augmentation described in Equations (6) and (7), it can be seen that during each interval, $n_{OD}(s+1)$ flows are estimated – alternatively each O-D flow is estimated $s+1$ times. This imposes an enormous computational strain especially for large and congested networks potentially rendering a real-time application of the model infeasible. For example, the size of the variance covariance matrix of the state (which is in general a full matrix) after augmentation is $(n_{OD}(s+1) * n_{OD}(s+1))$, manipulation of this matrix in the estimation equations therefore becomes cumbersome. As the congestion level in the network increases, the problem becomes worse because the number of lagged states s could increase with increase in travel times. The approximation we propose is based on the conjecture that much of the information about an O-D flow is likely to be provided the first time it is counted. If this were true, O-D flows corresponding to prior departure intervals could be held constant at their prior estimated values and only the flows for the current departure interval need to be estimated. The measurement and transition equations in the state-space model would then be expressed as follows:

$$\mathbf{y}_h - \mathbf{y}_h^H = \mathbf{a}_h^h(\mathbf{x}_h - \mathbf{x}_h^H) + \mathbf{b}_h + \mathbf{v}_h \tag{8}$$

and

$$\mathbf{x}_{h+1} - \mathbf{x}_{h+1}^H = \mathbf{f}_h^h(\mathbf{x}_h - \mathbf{x}_h^H) + \mathbf{c}_h + \mathbf{w}_h \tag{9}$$

where

$\mathbf{b}_h = \Sigma_{p=h-p'}^{h-1} \mathbf{a}_h^p(\hat{\mathbf{x}}_p - \mathbf{x}_p^H)$,

$\mathbf{c}_h = \Sigma_{p=h-q'}^{h-1} \mathbf{f}_h^p(\hat{\mathbf{x}}_p - \mathbf{x}_p^H)$ and $\hat{\mathbf{x}}_p$ is an estimate of $\mathbf{x}_p$.

The extent to which the conjecture might be true in a given situation depends on a number of factors. Obviously, it is more likely to hold with low measurement errors – a second measurement might not contribute much over the first. Less obvious is that it might also hold in the presence of very *high* measurement errors – in that case, measurements become so bad that each additional set offers hardly any improvement. Another factor is the error in the transition equation. If the errors $\mathbf{w}_h$ have a high variance, the importance of the counts as extra sources of information increases and the conjecture is less likely to hold.

Estimation of the O-D deviations from the above system is similar in spirit to that presented earlier except for the presence of constants in the transition and measurement equations and will again be omitted.

1.3 Alternate Formulation

To see the motivation behind the alternate formulation, we realize that any O-D flow can be expressed as the product of two components – the number of trips emanating from the origin of the O-D pair and the proportion of these trips headed towards the destination of that O-D pair. It was observed during empirical studies (see for example Ashok(1996)) that these two components exhibited different variability with time. While the number of trips was highly variable over a typical A.M. peak period, the shares (at least the larger ones) were relatively stable. Allowing for this differential variability in the estimation process could arguably increase the predictive power of the model.

To formalize the approach, define t_{ih} as the number of trips originating from origin i during interval h and the corresponding $(n_O * 1)$ vector by $\mathbf{t}_h$ where n_O is the number of origins in the network. Denote the rth share for departures from the origin during interval h by ψ_{rh} and the corresponding $(n_{OD} * 1)$ vector by $\mathbf{\Psi}_h$. The new formulation would then involve estimation of $\mathbf{t}_h$ and $\mathbf{\Psi}_h$ during each interval h.

1.3.1 Transition and Measurement Equations

For similar reasons as previously stated, we wish to consider *deviations* of trips and shares as the state variables. Accordingly, define $\mathbf{t}_h^H$ and $\mathbf{\Psi}_h^H$ as the corresponding best historical estimates. To incorporate differential variability for trips and shares, we specify two sets of transition equations as follows:

$$\mathbf{t}_{h+1} - \mathbf{t}_{h+1}^H = \Sigma_{p=h-q_1'}^h \mathbf{\Phi}_h^p (\mathbf{t}_p - \mathbf{t}_p^H) + \mathbf{w}_h^1$$

$$\boldsymbol{\Psi}_{h+1} - \boldsymbol{\Psi}^H_{h+1} \quad = \quad \Sigma^h_{p=h-q'_2} \boldsymbol{\Upsilon}^p_h (\boldsymbol{\Psi}_p - \boldsymbol{\Psi}^H_p) + \mathbf{w}^2_h \tag{10}$$

where $\boldsymbol{\Phi}^p_h$ and $\boldsymbol{\Upsilon}^p_h$ are *transition* matrices for trips and shares analogous to the $\mathbf{f}^p_h$ defined earlier. $\mathbf{w}^1_h$ and $\mathbf{w}^2_h$ are error vectors of dimensions $(n_O * 1)$ and $(n_{OD} * 1)$ respectively while $q'_1 + 1$ and $q'_2 + 1$ are the orders of the autoregressive processes. The errors satisfy assumptions similar to those stated earlier. Estimation of the transition matrices and the covariance matrices would be based on historical data as before.

The measurement equation is more complicated. Since it involves a mapping between O-D flows and link counts and the former involves a product of trips and shares, it is non-linear in the state variables. In matrix form, it can be stated as follows:

$$\mathbf{y}_h = \Sigma^h_{p=h-p'} \mathbf{a}^p_h \boldsymbol{\Xi}_p \mathbf{t}_p + \mathbf{v}_h \tag{11}$$

In the above equation, $\boldsymbol{\Xi}_p$ is a $(n_{OD} * n_O)$ matrix in which each row has exactly *one* non-zero element corresponding to one O-D pair r. We note that there is a unique mapping between the vector $\boldsymbol{\Psi}$ and the matrix $\boldsymbol{\Xi}$ – knowledge of either implies that the other can be constructed. All other terms in (11) have the same meaning as before. We note that we could rewrite the above equation in deviation terms on the same lines as before.

Again, to exploit the possibility that each trip could be measured in more than one time interval, one might wish to augment the state vector with state variables corresponding to prior intervals. Alternatively, one could employ the approximation suggested in Section 1.2.2 to avoid the computational burden.

One of the most popular ways to tackle the problem of non-linear estimation in dynamic systems has been to use the *Extended* Kalman Filter (EKF) algorithm (see for example, Gelb (1974)). In our problem, this involves a first-order Taylor linearization of the measurement equation about the best available estimate of the state vector. The resulting update equations for the filter closely resemble those of the conventional Kalman Filter. The estimates obtained from the EKF could be improved by performing successive iterations of linearization and re-estimation leading to an *Iterated* EKF.

2 The Assignment Matrix

An important input into the above framework is the assignment matrix that maps the O-D flows on to the link counts. The elements of this matrix can be expressed as analytical functions of the (time-dependent) link travel times and route-choice proportions (see Cascetta et al.(1993) or Ashok and Ben-Akiva(1996)). Since estimation of an O-D flow for departure interval h is done *at the end of* interval h, if travel times for interval h can be obtained from the surveillance system (from speed measurements at sensors, probe vehicles, etc.), these fractions can be computed using the analytical equations. Note that the route-choice fractions are usually not directly observable, so a route-choice model is required. An example of such a model can be found in Cascetta et al.(1996).

Consider next a situation where travel times in the network are entirely unobservable. In such a case, the assignment matrix is *endogenous* to the model. For static O-D estimation, researchers (Florian and Chen(1993), Yang(1995), etc.) have proposed the use of bilevel programming techniques where an *upper level* problem represents the O-D estimation process while the *lower level* problem represents a network equilibrium assignment. However, such techniques are difficult to apply to the problem at hand because (a) no analytical dynamic traffic assignment model that is widely accepted exists and (b) even if such a model were to exist, to apply it in real-time would entail solving a large nonconvex programming problem during each time interval which is impractical.

An alternative in such a situation is to use an iterative O-D estimation – computation of assignment matrices – O-D estimation technique. For a given O-D matrix, a simulation model (such as Ben-Akiva et al.(1994)) would be used to compute the assignment matrix. This process could be initiated using the one-step predicted O-D flows generated at the end of the previous interval. There are two disadvantages to this technique. First, convergence is not guaranteed. Second, the O-D flow estimates could be biased since the estimation during each step of the iteration does not allow for errors in the assignment matrix. Regarding the first, empirical study (Section 5) indicates that the filtering procedure is fairly robust with respect to the quality of assignment matrices and hence one would expect the quality of the O-D flow estimates to get better with each iteration. The second point is addressed in detail in Ashok and Ben-Akiva(1996) where generalized models that explicitly capture uncertainty in the assignment matrix as part of the O-D estimation/prediction framework are developed. Further discussion of these is beyond the scope of this paper.

It is again emphasized that an iterative process would only apply if the travel times are not measured directly or are liable to suffer from substantial imprecision. Otherwise, the assignment matrices would be computed directly from the observed (and perhaps some historical) travel times.

3 The Historical Database

Thus far, we have assumed the existence of a historical database of O-D matrices by departure time. This database would be constructed from results of estimations conducted in previous days, using either the two models presented earlier or the more general models in Ashok and Ben-Akiva(1997) (See below). The database would be stratified by day-of-week, type of weather, special events, etc. Furthermore, results from the estimation of each day would be used to update the database. There could be different ways of carrying out this updating. The simplest technique is to use the latest available estimate (the estimate obtained during the last day) since this encapsulates all prior history. Another alternative could be to use a moving average of the estimates for the last few days. A third alternative is to use a smoothing formula of the following form:

$$x_{rh}^{H,n} = x_{rh}^{H,n-1} + \alpha(\hat{x}_{rh}^{n} - x_{rh}^{H,n-1}) \tag{12}$$

where $x_{rh}^{H,n}$ represents the historical value corresponding to O-D pair r and departure interval h after n days, $\hat{x}_{rh}^{n}$ denotes the estimate on day n and α is a scalar between zero and one.

The final question that remains to be answered pertains to starting the process. For the first few days, the various inputs to the models such as the error-covariance matrices, the autocorrelation matrix, etc. are likely to be unknown or only approximately known. In such a situation, simpler models such as those proposed by Cascetta et al.(1993) can be used. Cascetta et al. propose two estimators. In the first, the O-D flow matrices $\mathbf{x}_h^*$ are obtained sequentially from solving constrained optimization problems of the form:

$$\hat{\mathbf{x}}_h = argmin[g_1(\mathbf{x}_h, \mathbf{x}_h^a) + g_2(\mathbf{y}_h, \hat{\mathbf{y}}_h)] \tag{13}$$

over $\mathbf{x}_h > 0$. $\mathbf{x}_h$ is the current value of the demand vector (the variable over which the expression is optimized), $\mathbf{x}_h^a$ is an apriori or starting guess of $\mathbf{x}_h$ (can be obtained by setting $\mathbf{x}_h^a = \hat{\mathbf{x}}_{h-1}$), $\mathbf{y}_h$ is the measured link counts while $\hat{\mathbf{y}}_h$ is the vector of counts obtained by assigning the decision variable $\mathbf{x}_h$. g_1 and g_2 are functions that depend upon the estimation framework.

An alternative procedure suggested by them solves for the unknown O-D flows of *several* periods simultaneously. This is computationally expensive since it involves solution of a large optimization problem. The equivalent of equation (13) for obtaining the O-D flow vectors for N periods is given by:

$$(\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, ..., \hat{\mathbf{x}}_N) = argmin[g_1(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N; \mathbf{x}_1^a, \mathbf{x}_2^a, ..., \mathbf{x}_N^a) + g_2(\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_N; \hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2, ..., \hat{\mathbf{y}}_N)] \quad (14)$$

where the minimization is over $\mathbf{x}_i \geq 0 \quad \forall i = 1, 2, ..., N$. All terms in the above expression have the usual meaning. Again g_1 and g_2 depend upon the estimation framework.

In closing, we might mention here that Ashok and Ben-Akiva(1997) present an enhanced version of Cascetta et al.'s model that is particularly well suited for construction and updating of the historical database. This model allows for incorporation of the autoregressive process into equation (14). Details of the formulation along with an efficient computational strategy for estimating the model may be found in the cited reference.

4 System Observability

A discussion of "observability" which is a desirable property of such dynamic systems is in order here (This section is based on earlier work by the authors (Ashok and Ben-Akiva (1993)). Essentially, observability defines our ability to determine the initial state vector uniquely from a set of measurements. Under conditions of non-observability, the effects of the initial estimates do not disappear with time and therefore it is critical to obtain accurate initial values. An analogous situation may be found in conventional static matrix estimation where one typically starts with an apriori matrix and uses traffic counts to modify the apriori estimates. The apriori matrix is necessitated by the fact that the number of observations (traffic counts) is much less than the number of O-D pairs. The apriori matrix in effect increases the number of observations. However, the estimates obtained would always depend on the apriori information provided. In the model proposed here, the initial state vector and the transition equation provide information similar to that provided by the apriori matrix in conventional estimation by exploiting temporal interdependencies among the O-D flows. The nice feature, however, of dynamic or real-time updating is that under conditions of observability, the influence of the initial value of the state vector would disappear with time.

The most obvious and critical factor affecting observability in our problem is the ratio (n_l/n_{OD}). For a given number of O-D flows, measuring more (independent) counts increases the chances of observability being satisfied. The degree of linkage between O-D flows and counts is another factor. An extreme example of this arises when an entire column of the assignment matrix is zero implying that a particular O-D flow never gets measured – this might however be overcome if that O-D flow is related to other measurable O-D flows by means of a non-diagonal transition matrix. A third consideration is the degree of linkage between O-D flows over time. Again, an extreme example arises when the transition matrix is fully populated by zeros.

In the context of observability, results from empirical studies (Section 5) conducted on the first formulation are encouraging. It was observed that for different values of initial starting conditions, the model produced almost identical filtered estimates. Another positive indication while testing the model was that it invariably succeeded in finding a gain matrix that moved the predicted estimates closer to the true values.

5 Case Study

We used three data sources in this research. We first describe the main characteristics of each.

5.1 Data Description

5.1.1 The Massachusetts Turnpike

The first dataset consisted of traffic data for 3 days for I-90 (the Massachusetts turnpike), a toll freeway stretching from the New York State Border to Weston (Route 128) – a distance of about 120 miles. There are 15 entry/exit ramps; hence, the network has 15*14=210 possible entry/exit or O-D pairs. However, in the analysis, only East Bound traffic was considered; hence the number of O-D pairs was reduced by half. Information was available on entry and exit ramps of each vehicle and entry and exit times. Entry times were not considered accurate enough, hence they had to be back-calculated from exit times assuming an average speed (see below).

Since the data was in disaggregate form, it had to be aggregated to obtain time-dependent traffic counts for each of the 14 links in the network. This aggregation was carried out by assuming an average vehicle speed of 55 mph and using this speed to calculate the entry time of each vehicle on each link. This assumption of a uniform average speed for all vehicles at all times is unrealistic. However, for the purposes of implementing and evaluating the proposed models, all that is needed is a set of "reasonable" O-D matrices, counts and assignment matrices consistent with each other. The issue of whether the speeds assumed for generating counts are realistic is not directly relevant.

Hypothetical counting stations were assumed to be located at the entrance to each link on the network. The values of the assignment matrices and O-D flows were computed such that the relationship

$$y_{lh} = \sum_{p=h-p'}^{h} \sum_{r=1}^{n_{OD}} a_{lh}^{rp} x_{rp} \qquad (15)$$

held exactly. Since the network was linear, no path choice model was required for this case study and the movement of each vehicle could be tracked using the constant speed assumption.

The data for the third day (of the three days for which data was available) was chosen for implementing the different models. The true O-D flows for the first day were used as historical flows. The autoregressive coefficients were computed by simple Ordinary Least Squares regressions using the deviation of O-D flow values (or trips and shares) of the second day from those of the first. Two assumptions were made in the process of obtaining these matrices. Firstly, it was assumed that the autoregressive structure remained constant over the whole day so that one could have enough observations in one day to estimate all the parameters. Secondly, it was assumed that a deviation in flow between O-D pair r (or trip/share deviation for the second approach) for a period was related *only* to the rth O-D flow deviation (or trip/share deviation) of prior intervals. The variances of the transition equation errors was approximated from the residuals of these regressions. An autoregressive process of order 4 provided the best fit. Because of the exactness of the estimated quantities in equation (15), there was no measurement error in the problem. The analysis was carried out for the A.M. Peak from 6.00 A.M. to 9.45 A.M. for 15 minute intervals. Since it took a maximum of 120 minutes to traverse the network, there were ((120/15) + 1)*105=945 O-D flows to be estimated in each interval. If the approximation of Section 1.2.2 is employed, only 105 O-D flows needed to be estimated in each interval.

5.1.2 I-880 near Hayward, California

The second dataset covered a 5.2 mile (NorthBound) stretch of I-880 near Hayward, California. This section had 4 on-ramps and 5 off ramps with 20 O-D pairs. Ten minute detector data on traffic volumes and average speeds was available at 10 detector locations for a 2.5 hour morning peak period. Data over seven days was available. The advantage of using this dataset was that unlike the Turnpike data, congestion level was heavy during certain time intervals with speeds reaching 15-20 mph at some locations.

Historical O-D matrices were created for the first six days using Equation (13) within a GLS estimation framework (i.e., g_1 and g_2 represented quadratic error functions). Data from the last day was used for testing the models. The transition matrices and the error variances were computed exactly as in the earlier dataset, except that data over six days was used. Again, an autoregressive process of order four was used. Since the maximum travel time between any O-D pair was about 9 minutes, the value of $s = \max(p', q')$ was max(1,3) = 3. Thus there were (3+1)*20=80 O-D flows to be estimated in each 10-minute estimation interval in the augmented model. For the approximate procedure, only 20 O-D flows needed to be estimated. The measurement error covariance matrix was computed from the residuals obtained for each interval from the first six days.

5.1.3 The Amsterdam Beltway

This is a 32 km freeway encircling the city of Amsterdam with 20 entrance and exit ramps. Both the datasets mentioned earlier, suffer from the limitation that they do not capture route choice. This dataset was intended to address this limitation by allowing for two routes between each O-D pair – clockwise and counterclockwise. Moreover, in this dataset, we used a combination of actual and synthetic data to gain additional insight into model performance. After removing sensors with large errors from the dataset, information on observed average speeds and link counts was available at 65 locations over one minute intervals for one day. This information was aggregated over 15 minute intervals for the morning peak. Synthetic data on O-D flows (and speeds) was generated for an additional day by repeatedly applying Equation (1) using the first day estimates as the historical database. Details on the data generation process and the calibration of the transition error covariances may be found in Ashok(1996). There were 291 O-D flows to be estimated in each interval.

To obtain the historical database from the first day, Equation (13) (with a GLS estimator) was used on the observed speeds and link counts over 15-minute estimation intervals. For route-choice, a simple logit model with path travel time as the only attribute for each route was used. Details on calibration of this model may be found in Ashok(1996). Because the data for the second day was generated using Equation (1), this dataset did not provide a realistic setting for evaluating the trip/share formulation presented in this paper. However, it served two useful purposes. First, it provided an example that involved route-choice. Second, it provided an opportunity to further investigate the sensitivity of the original formulation to errors in the assignment matrix and link counts. The general testing procedure is shows in Figure 1. Error

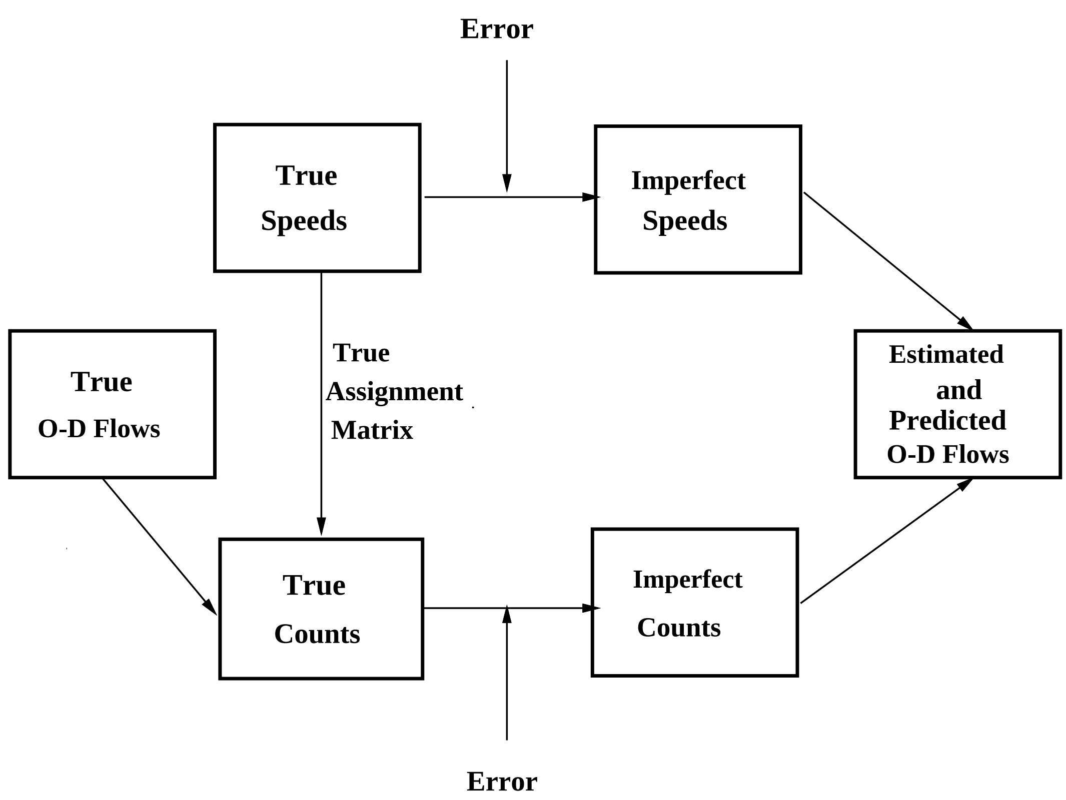


Figure 1: Testing procedure with synthetic data

was added to the counts and speeds using the equations

$$Measured\_counts = True\_counts * (1 - \delta_{cts} + 2 * \delta_{cts} * U) \quad (16)$$

$$Measured\_speeds = True\_speeds * (1 - \delta_{spd} + 2 * \delta_{spd} * U) \quad (17)$$

where U is a random number drawn from a uniform distribution between zero and one. By assigning different values for δ_{cts} and δ_{spd}, the sensitivity of the model to errors in counts and speeds could be investigated. Measurement error variances could be exactly computed as a function of δ_{cts}. For each scenario, multiple runs were conducted.

5.2 Results

Different models were tested in this study. The first model (Model *Base*) is the basic augmented model described in Section 1.2.1. The second model (Model *Base-Appx*) uses the same formulation along with the approximation described in Section 1.2.2. Models *T/s-Appx* are based on the alternate formulation that uses originating trips and destination shares (Section 1.3.1), again using the approximation. For this latter formulation, we require that the estimated shares lie between zero and unity and that the shares corresponding to each origin add to unity. Since the EKF procedure does not guarantee this, we truncate each negatively estimated share to zero (our empirical results indicated that this was a relatively infrequent occurrence and and happened only for O-D pairs with extremely low flow) and normalize the shares during each interval. Models *T/s-Appx A* and *T/s-Appx B* describe the results with and without these constraints. Also, for Models *T/s-Appx A* and *T/s-Appx B*, tests were carried out on sensitivity of the results to number of iterations in the implementation of the Iterated EKF.

For evaluation of the results of various models from the first and third datasets, the following error measures were used.

1. Root Mean Square (RMS) Error = $\sqrt{\frac{\sum_i (x_i - \hat{x}_i)^2}{N}}$
2. Root Mean Square Error Normalized (RMSN) = $\frac{\sqrt{N \sum_i (x_i - \hat{x}_i)^2}}{\sum_i x_i}$

   where the x and $\hat{x}$ represent the true and estimated O-D flows respectively and the summation is over all O-D pairs and all intervals for which analysis was carried out.

For the second dataset, the true O-D flows were unknown. Hence we compared the link counts obtained from re-assigning the estimated O-D flows with the measured

		Base	*Base A*	*Base B*	*Base-Appx*	*Historical*
RMS Error	Filtered	5.3449	*	*	5.7798	8.7015
	1-Step Predicted	9.1061	13.1631	10.9520	9.1916	8.9501
	2-Step Predicted	8.7558	18.2373	9.4515	8.7290	9.1146
	3-Step Predicted	9.3090	25.3705	11.6322	9.2937	9.2490
RMSN Error	Filtered	0.2905	*	*	0.3141	0.4729
	1-Step Predicted	0.4783	0.6913	0.5752	0.4878	0.4701
	2-Step Predicted	0.4502	0.9378	0.4860	0.4525	0.4687
	3-Step Predicted	0.4721	1.2866	0.5899	0.4701	0.4690

Table 1: RMS and Normalized RMS Error Values (I-90)

link counts. This error measure should be interpreted with caution since (a) the measured counts are themselves erroneous and (b) it is possible that even though the estimated link counts closely match the measured link counts, the estimated O-D flows differ considerably from the true O-D flows.

We discuss first the results corresponding to the I-90 dataset. The first column in Table 1 shows estimation and prediction results for Model *Base*. Shown alongside are two models (*Base A* and *Base B*) that use alternate schemes for predictions. Model *Base A* corresponds to a "no prediction" case – the values estimated by Model *Base* during a given interval are the "predictions" for all future intervals. Model *Base B* corresponds to a prediction method that uses constant deviations i.e., the deviation in O-D flow estimated by Model *Base* during a given interval is assumed to be identical for all future intervals. Next in the table are errors corresponding to Model *Base-Appx* (i.e., Model *Base* along with the approximation in Section 1.2.2). The final column displays the errors when the historical O-D flows for each interval were used in lieu of estimates/predictions from the models.

We make the following observations. First, the RMS errors are relatively low and the filtered estimates from both Models *Base* and *Base-Appx* are much more accurate than the historical values. We next see that there is some loss in accuracy in moving from *Base* to *Base-Appx* especially in the filtered estimates; however these have to be traded off against the vast computational gains – instead of estimating 945 parameters in *Base*, only 105 parameters need to be estimated in *Base-Appx*. While the predictions from *Base* are significantly better than those from Models *Base A* and *Base B*, they are not much different from the historical values. This could be because there was not much variability in O-D flows during the three day period and the historical flows provided good approximations to the O-D flow on the day of interest.

Following the observation that most of the O-D flows in the case study were

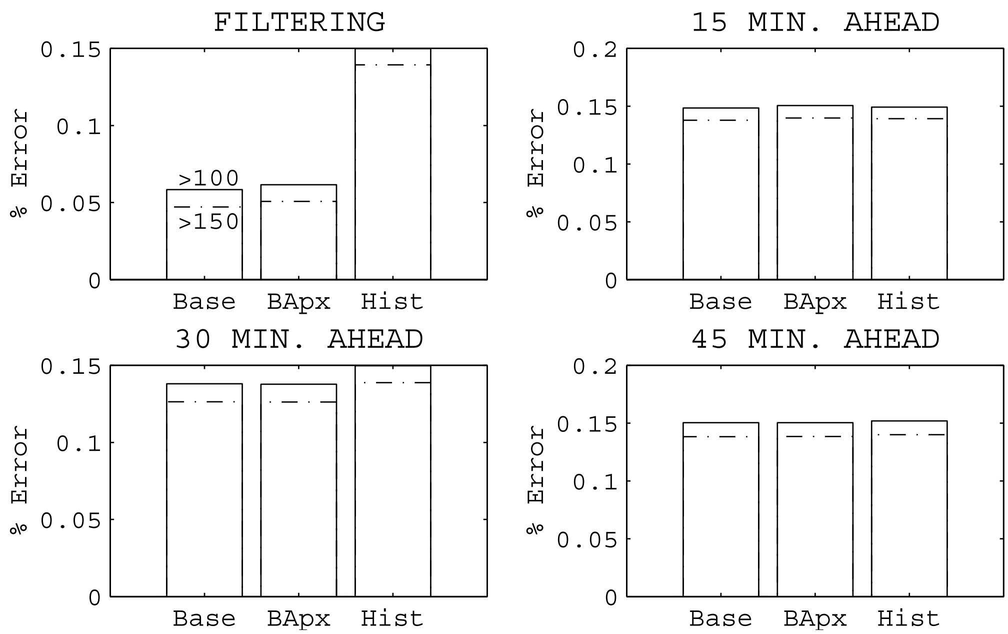


Figure 2: RMSN errors for high flows: I-90

extremely small, we next computed errors for Models *Base* and *Base-Appx* only for the high flows. Figure 2 shows that RMSN errors are drastically reduced showing that both models perform significantly better while estimating and predicting high O-D flows.

Of course one reason for the small gap in performance between Models *Base* and *Base-Appx* could be the fact that there was no measurement error in the problem. This could explain why most of the information about an O-D flow could be obtained from just one measurement. To perform a fair comparison between the two models, we first perturbed the assignment matrix using the following formula:

$$a_{new} = a_{correct}[(1-\delta) + U * 2\delta]$$

for different values of δ. U is a random number drawn from a uniform distribution between zero and one. The performance of the filtered estimates for a value of $\delta = 0.2$ (which can be interpreted as a within $\pm 20\%$ error in the assignment matrix) is shown in Figure 3. Expectedly, the errors are higher for both models with the gap between Models *Base* and *Base-Appx* widening.

Models *Base* and *Base-Appx* were next tested using an extremely poor historical database of O-D flows (actually corresponding to a P.M. peak period) to investigate their sensitivity to historical information. Figure 4 (only for high O-D flows) shows

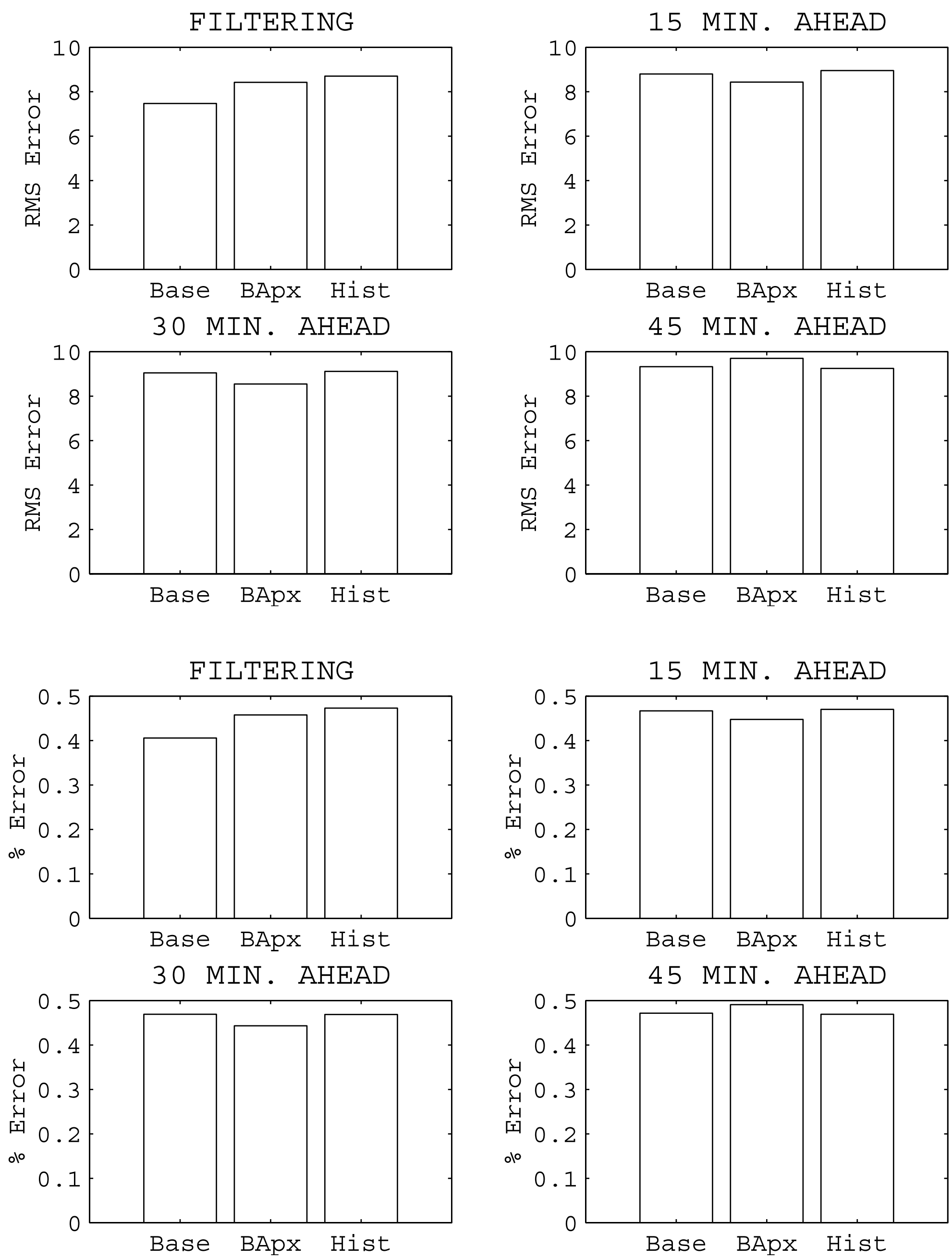


Figure 3: Errors with incorrect assignment matrix: I-90

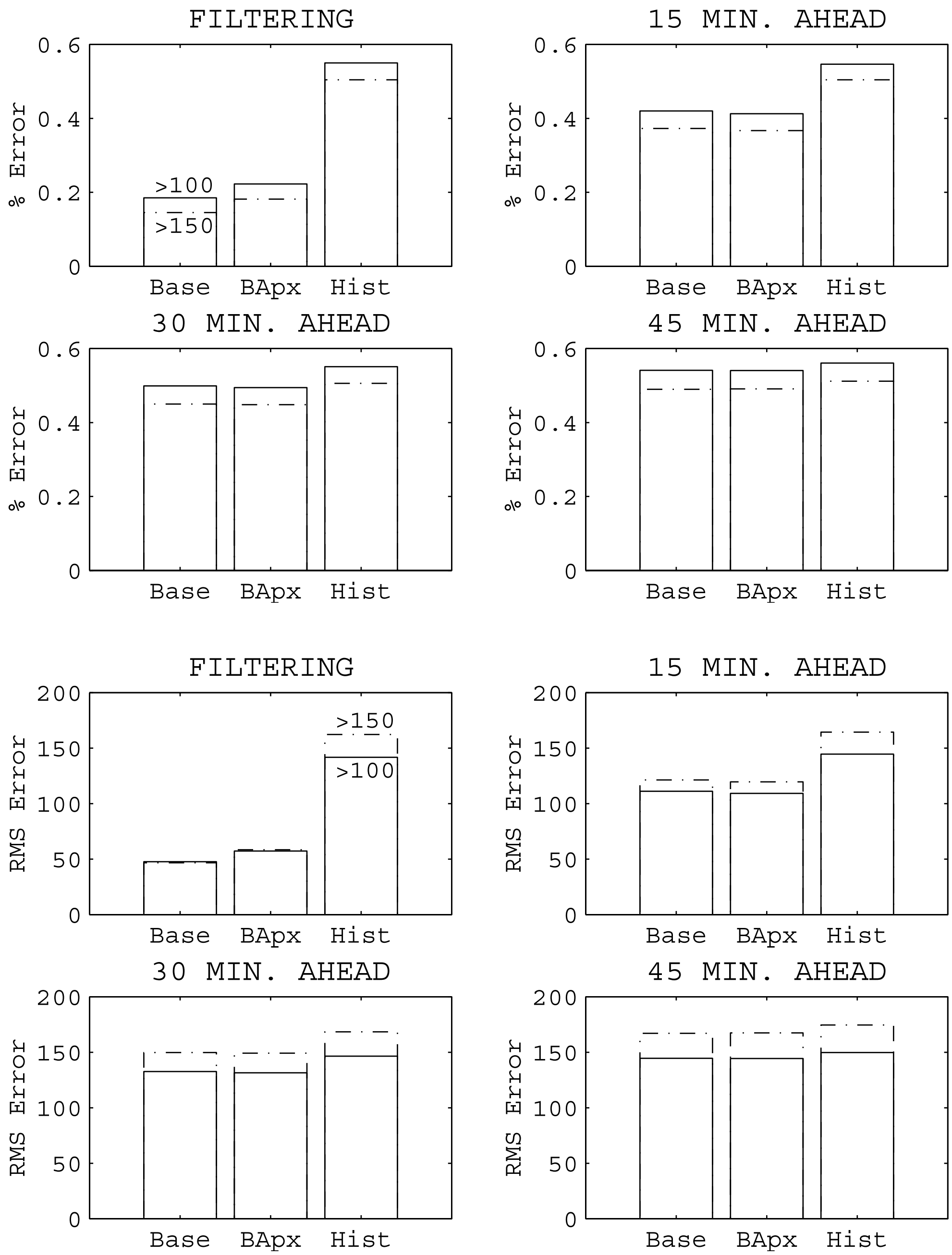


Figure 4: Errors with poor historical information: I-90

		T/s-Appx A	*T/s-Appx B*	*Historical*
RMS Error	Filtered	6.8346	7.0359	8.7015
	1-Step Predicted	8.9679	8.9654	8.9501
	2-Step Predicted	8.3042	8.3049	9.1146
	3-Step Predicted	9.2028	9.1839	9.2490
RMSN Error	Filtered	0.3714	0.3824	0.4729
	1-Step Predicted	0.4710	0.4709	0.4701
	2-Step Predicted	0.4270	0.4271	0.4687
	3-Step Predicted	0.4667	0.4657	0.4690

Table 2: RMS and RMSN Errors for alternate formulation (I-90)

that both models continue to be significantly superior to historical values.

Overall, results seem to indicate that the augmentation of the state-vector with variables corresponding to prior departure intervals does not offer significant improvement to warrant the extra computations that are required.

Table 2 and Figure 5 show RMS and RMSN errors for models based on the departure-rate/share based formulation. We observe that both Models *T/s-Appx A* and *T/s-Appx B* out-perform the linear models in predictive power. At an intuitive level, this could be explained by the fact that the transition equation now allows for differential variability of departing trips and shares. On the other hand, they exhibit worse performance in filtering which could be because of the non-linearity in the measurement equation and the resulting approximation. Also, Model *T/s-Appx B* is marginally inferior to Model *T/s-Appx A* in its filtered estimates.

The number of iterations in implementation of the Iterated EKF algorithm depends upon both the sensitivity of the results to number of iterations and the computational effort associated with each additional iteration. Figure 6 shows the filtering errors associated with different number of iterations using models *T/s-Appx A* and *T/s-Appx B*.

Results from the I-880 dataset showed the same overall trends. Figure 7 compares the various models with the corresponding historical values. Note again that the errors in this figure are with respect to link counts and not the true O-D flows. Again, *Base-Appx* displays higher errors relative to Model *Base*. Another interesting observation relates to the variances of the estimated O-D flows in *Base*. Since each O-D flow is estimated multiple (in this case four) times, one would expect the variance of the estimates to decrease with each successive estimate. This was borne out in the results. Figure 8 shows the relationship between the variance of the filtered estimates and the number of estimates for typical O-D pairs. It can also be seen that most of

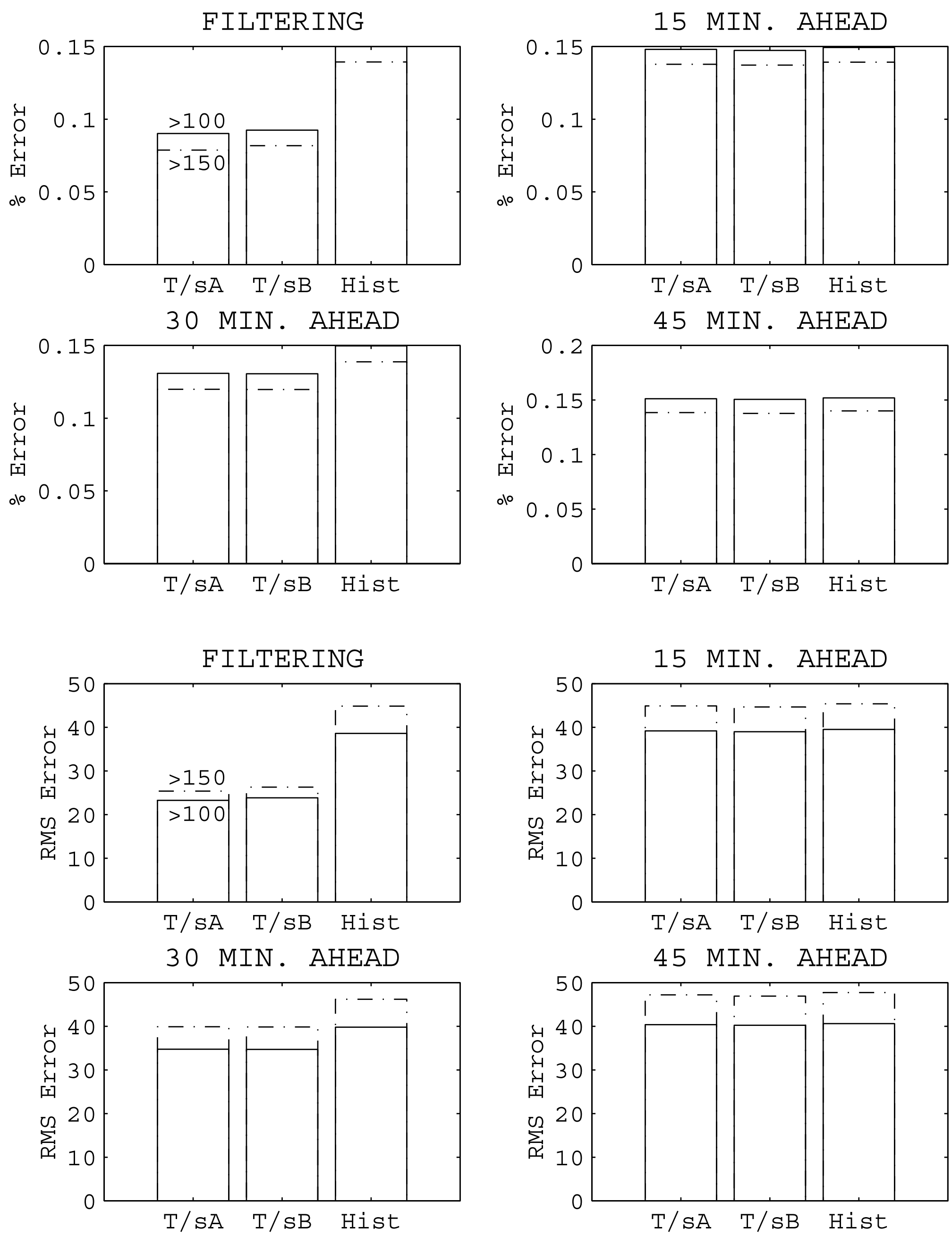


Figure 5: Errors for high flows using alternate formulation: I-90

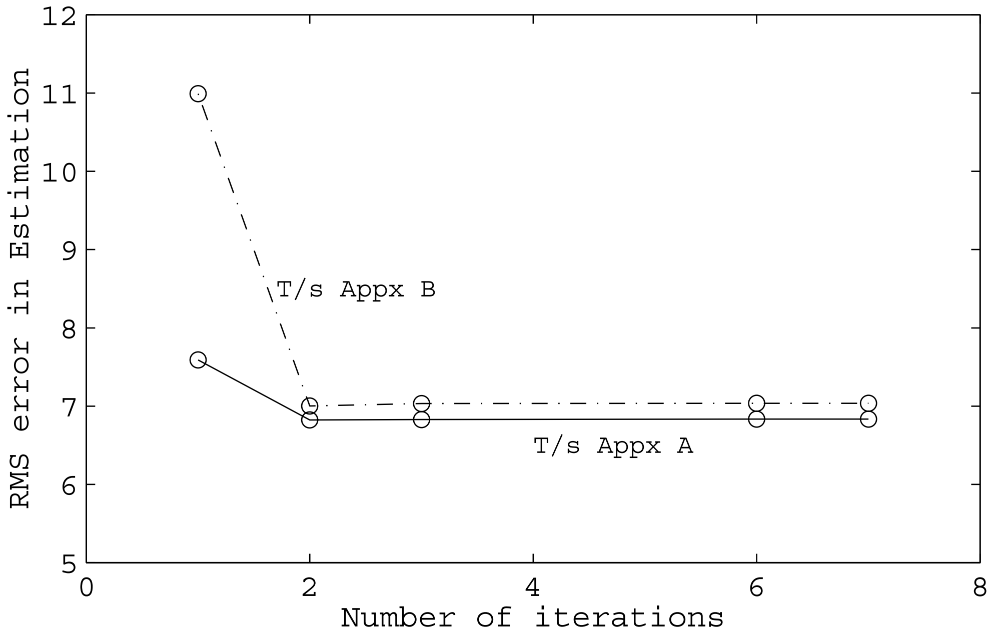


Figure 6: Error reduction with number of iterations: I-90

the reduction in variance takes place within two estimations. This could be because in this case-study, vehicles can remain on the network for at most two successive time-intervals. Finally, Models *T/s-Appx A* and *T/s-Appx B* out-perform their linear counterpart (*Base-Appx*) in predictive power just as in the I-90 dataset. Figure 9 shows, for a typical O-D pair, that the estimated O-D flows from the various models are quite different though they follow the same general trend. Figure 10 shows the total number of departing trips estimated for the corresponding origin.

Finally for the Amsterdam Beltway, Figure 11 shows the degradation in performance of *Base-Appx* as the measurement error parameter δ_{cts} (equation (16)) is varied. This graph was generated for *exact* speeds, i.e., δ_{spd}=0. Errors are stratified by size of O-D flow. Each bar shows the RMS/RMSN errors for a specific value of δ_{cts}. For comparison, errors in employing historical O-D flows are shown in the last bar. It can be seen that the model is fairly robust with respect to quality of link counts. A similar conclusion is reached in Figure 12 which investigates the extent of bias as the error parameter in speeds (equation (17)) is varied. This graph was generated for δ_{cts}=0.1. The bias becomes significant, however, for very large values of δ_{spd}.

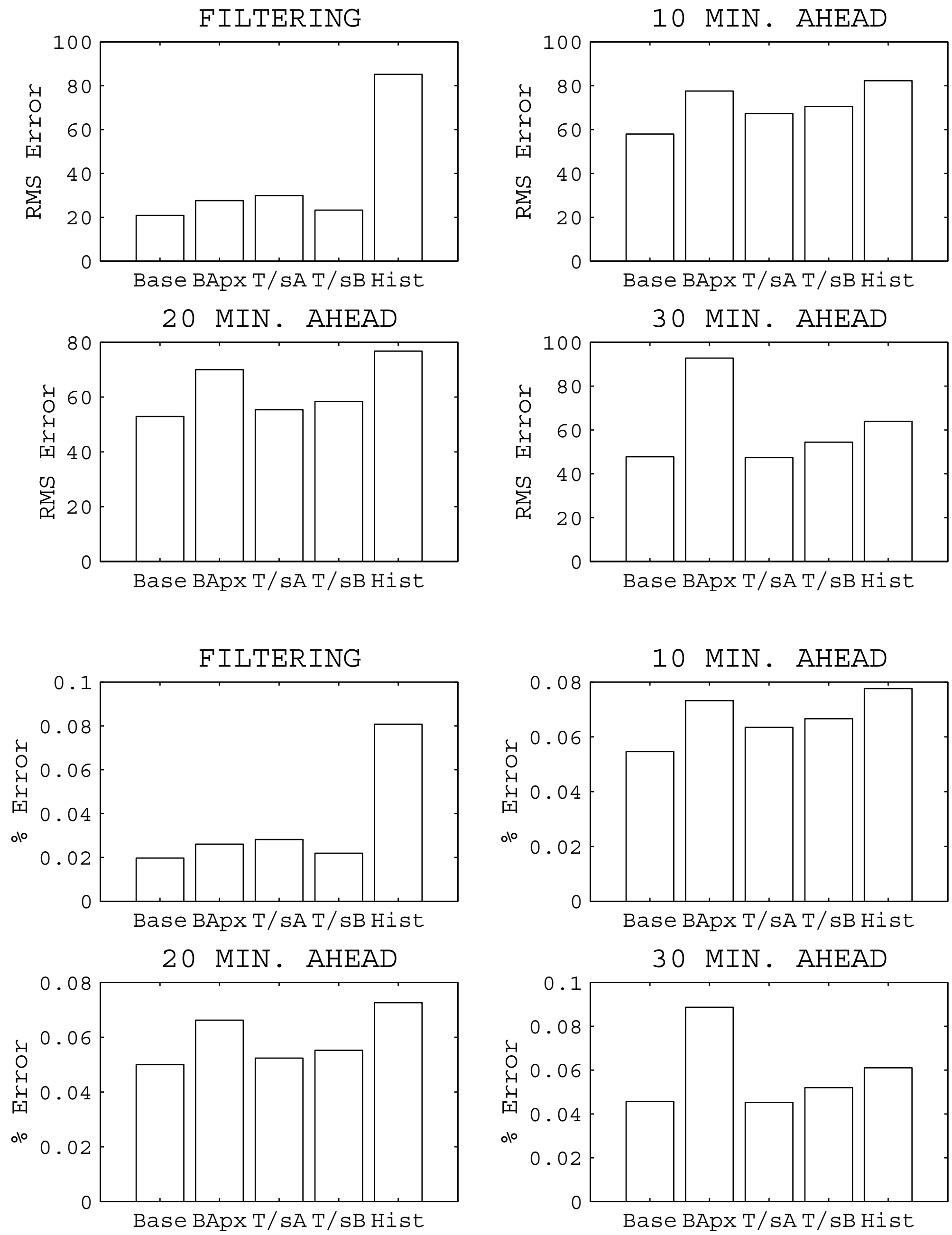


Figure 7: Errors for I-880

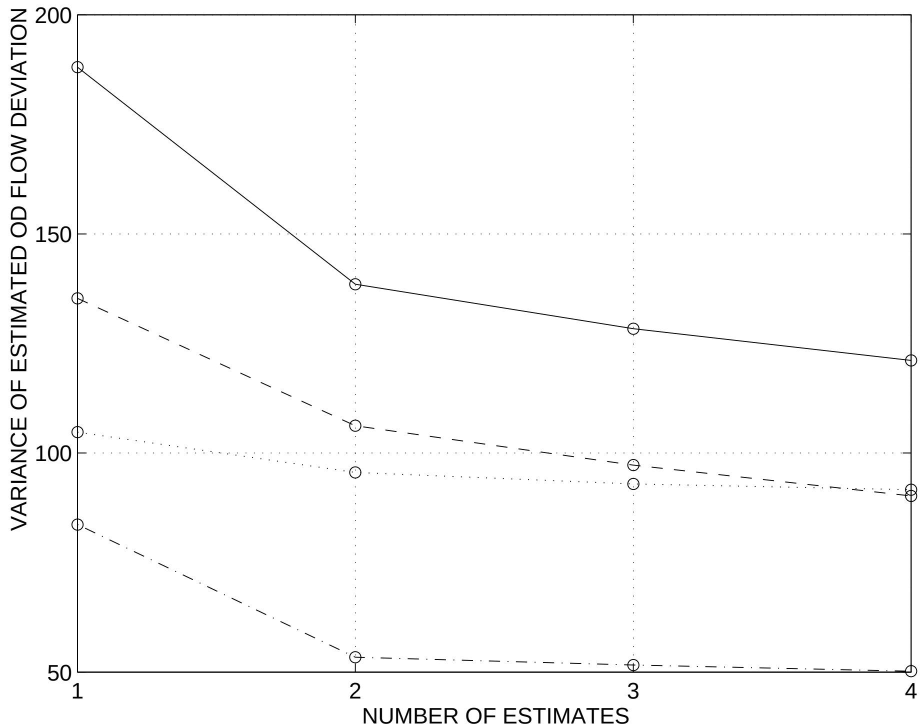


Figure 8: Variance of Estimated OD flow deviations: I-880

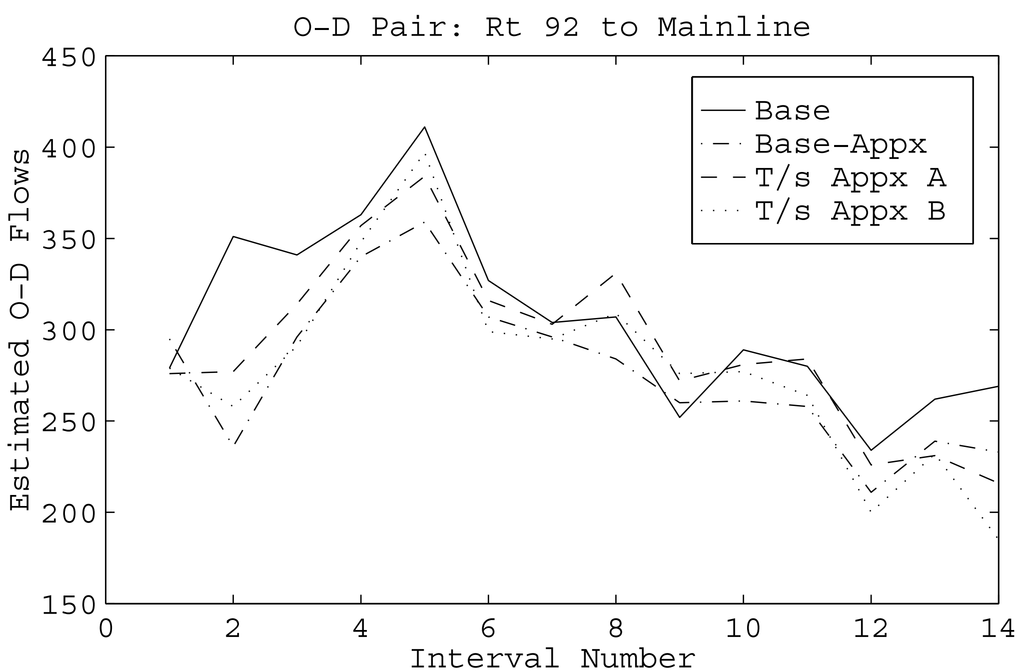


Figure 9: Estimated O-D flows: I-880

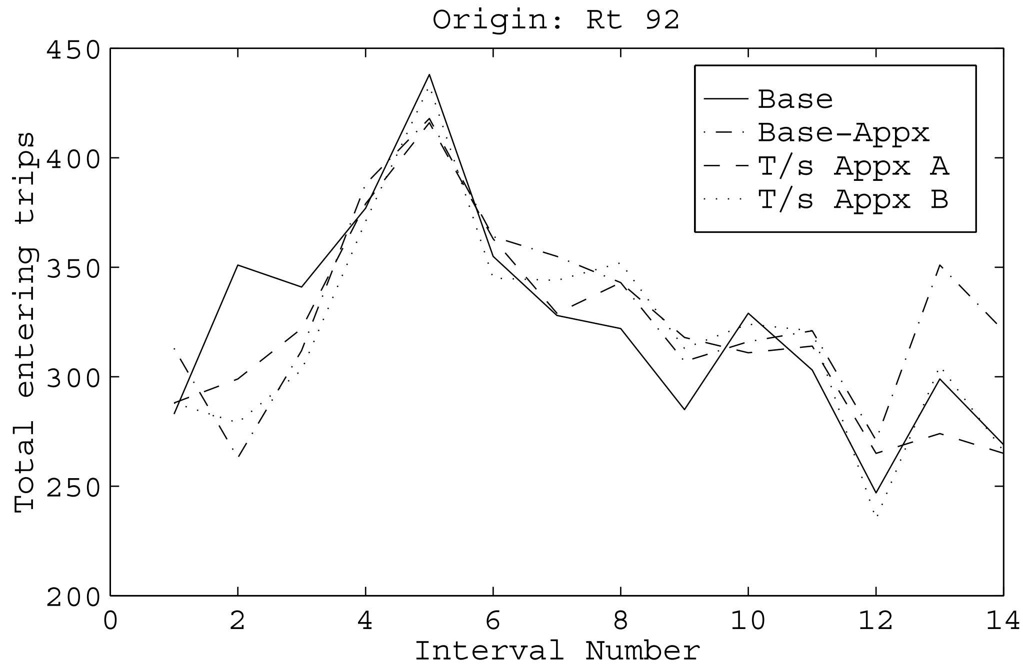


Figure 10: Estimated departing trips: I-880

5.3 Major Findings

The following are the major findings of the case studies.

1. The approximation introduced in the measurement equation by Model *Base-Appx* has only a slight impact on quality of estimated O-D flows relative to *Base*. The model is fairly robust with respect to quality of historical estimates, link counts and speeds. Errors in filtered and predicted flows are significantly lower for O-D pairs with higher flows.

2. The approximate models offer significant computational savings over the basic augmented model. An idea of the computational savings can be seen from the fact that because of the high dimensionality of the state vector, Model *Base* (for the I-90 data) had to be implemented on a Cray XMP EA/464 supercomputer while all the approximate models could be implemented on standard workstations. The biggest problem in this paper was the Amsterdam Beltway with 291 O-D pairs. A "brute-force" implementation of *Base-Appx* for this dataset on a SGI Indy 200MHz R4400 workstation using the Matlab matrix manipulation package took about five seconds for each estimation interval. It is expected that this time can be significantly reduced by using efficient data structures

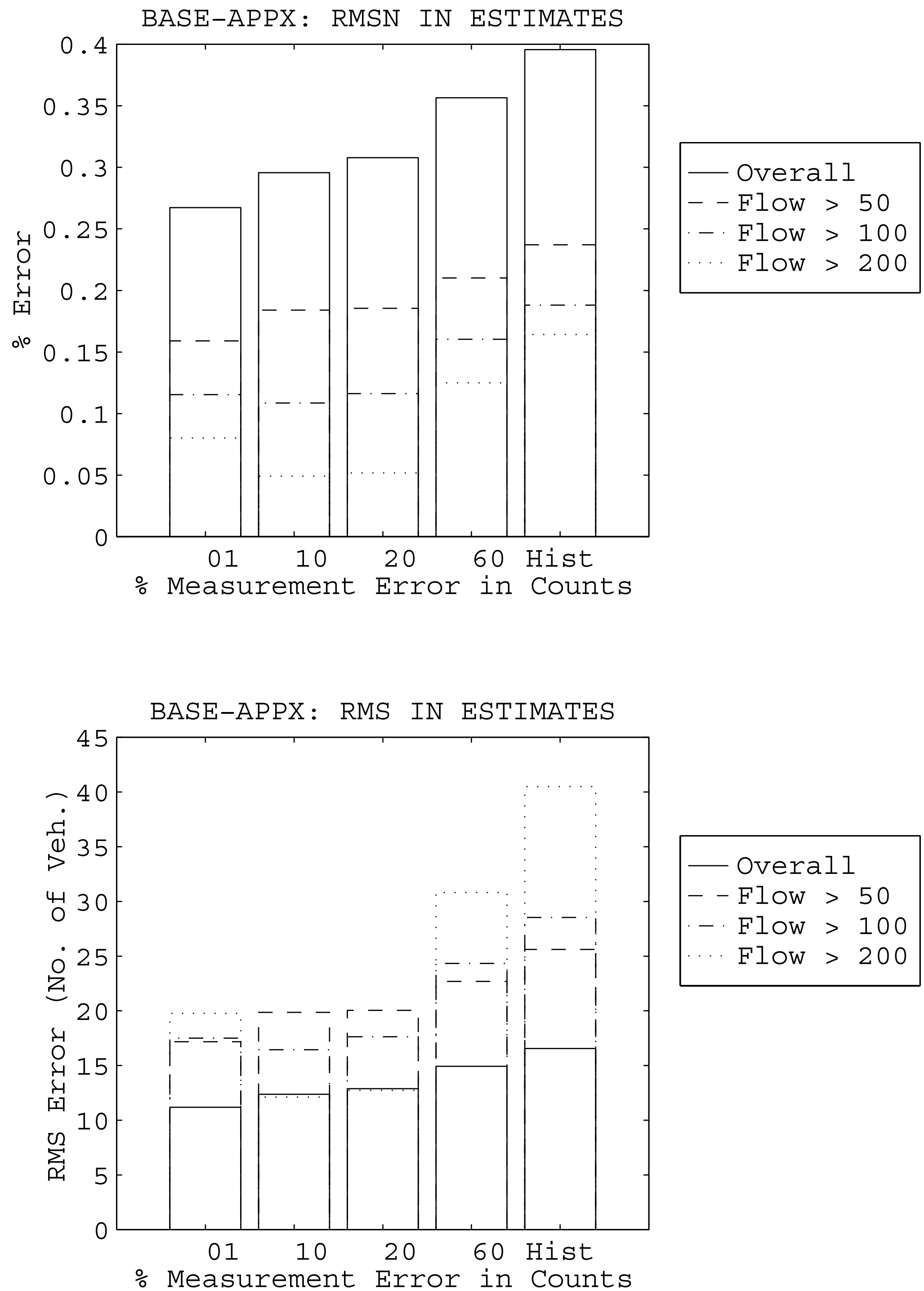
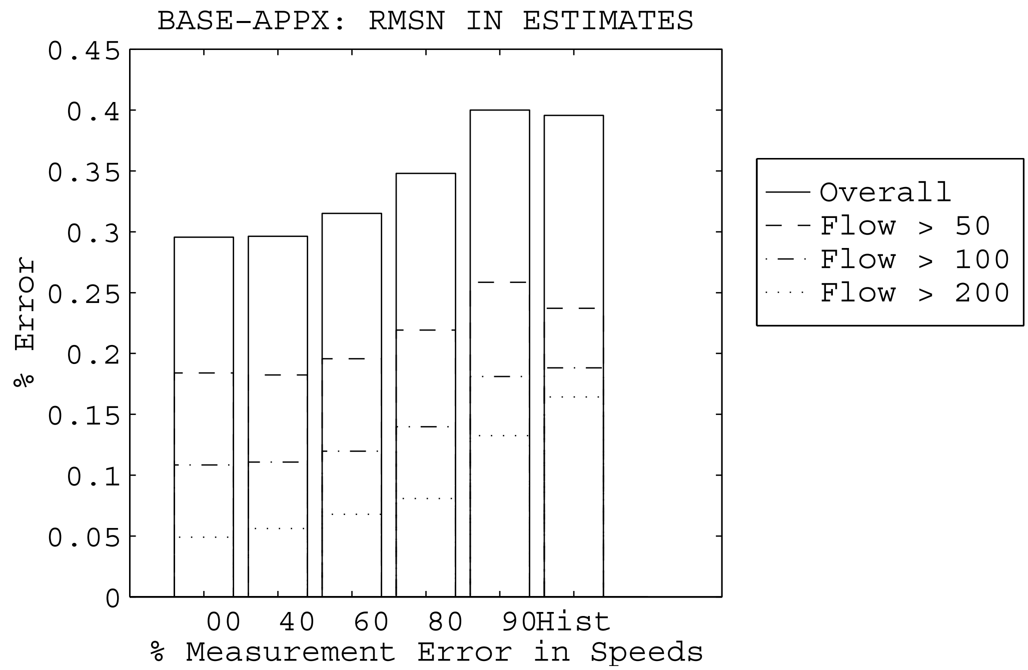


Figure 11: Errors in *Base-Appx* as a function of accuracy of counts: Beltway



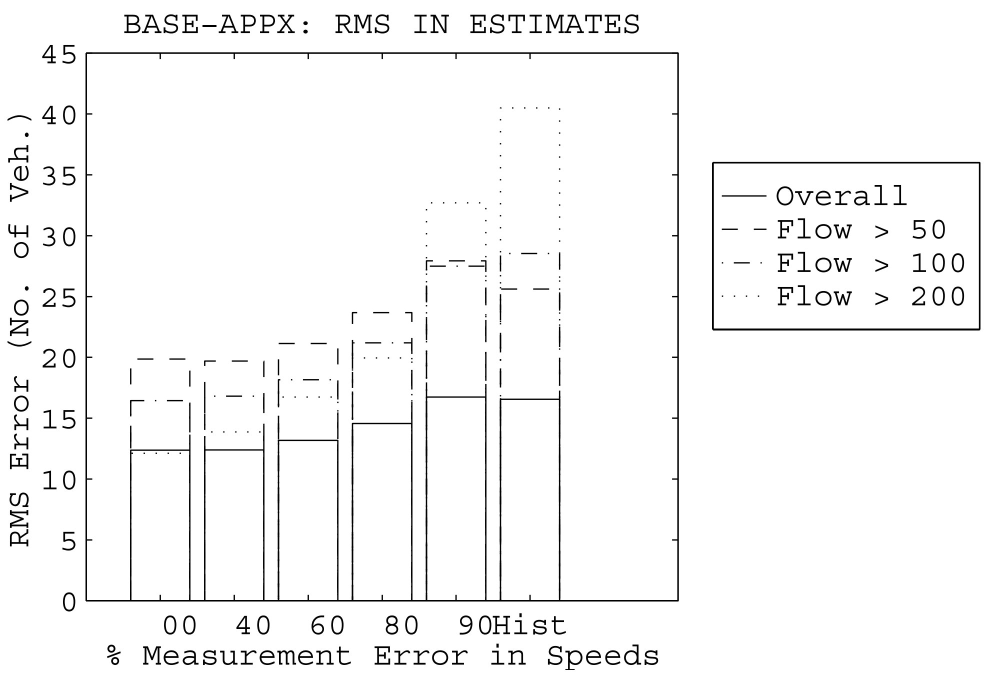


Figure 12: Errors in *Base-Appx* as a function of accuracy of speeds: Beltway

and algorithms.

3. Formulations that are based on modeling departure rates and shares separately perform better than their linear counterparts in predictions but worse in estimation. The non-linear models tend to converge fairly quickly to reasonable values.

4. Predicted estimates tend to converge to the historical values with increasing prediction time-step. There is not much to be gained from three step predictions over historical values.

Of course the above conclusions are specific to these case studies and do not purport to be generally true.

6 Conclusion

In this paper, two approaches for real-time dynamic O-D matrix estimation have been proposed. Both approaches have been evaluated using actual traffic data from three sources. The two original contributions of this work are development of an approximate method appropriate to real-time estimation and a new formulation that exploits differential temporal variation of trips and shares. Preliminary results from the case studies are promising and indicate that the filtering process is quite robust. Future research should focus on comparing methods of updating the historical database, on computational issues, on use of additional sources of information such as probe vehicles or intersection level turning fractions and on implementation issues for large networks.

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