On the Optimal Timing of Benefits with Heterogeneous Workers and Human Capital Depreciation*

Robert Shimer
University of Chicago and NBER
shimer@uchicago.edu

Iván Werning
MIT and NBER
iwerning@mit.edu

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Abstract

This paper studies the optimal timing of UI benefits when workers are heterogeneous, or when human capital depreciates during unemployment. Our model builds on Shimer and Werning (2005) to distinguish unemployment benefits from consumption during unemployment by allowing workers to save and borrow freely—a crucial feature given our focus on the timing of benefits. We show that, unlike in homogeneous-agent economies without skill depreciation, optimal benefits are typically not constant. We investigate the main determinants for decreasing benefits schedules to be optimal.

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1 Introduction

When workers are homogenous and face a stationary search problems, constant UI benefits are optimal or nearly so (Shimer and Werning, 2005). This paper examines how the optimal timing of benefits is affected by relaxing these assumptions. In particular, we model an unemployment agency that chooses a schedule of UI benefits as a function of the length of the unemployment spell and faces either: (i) a group of homogenous workers whose human capital depreciation during unemployment; or (ii) a group of workers that are heterogenous.

Both scenarios are empirically relevant and represent a departure in important directions from previous work on optimal UI, which largely focuses on homogenous workers with stationary search problems (e.g. Shavell and Weiss, 1979; Hopenhayn and Nicolini, 1997; Werning, 2002; Shimer and Werning, 2005). Our work also departs from most research on optimal UI by distinguishing consumption while unemployed from benefits by giving workers financial freedom. In a previous paper we argued that this is crucial for understanding the design of UI policy: a constant net benefit is optimal, but workers choose to have consumption decline during the unemployment spell (Shimer and Werning, 2005). Intuitively, when the agency faces a single type of worker with a stationary search problem, a constant UI benefit is optimal because the tradeoff between insurance and incentives is the same each period.

In this paper we show that constant benefits are not optimal when one moves away from the benchmark with identical workers solving stationary search problems. Intuitively, the tradeoff between insurance and incentives is no longer constant when workers' job opportunities deteriorate during the spell, or when the pool of unemployed workers shifts from one type to another. Our model provides a tractable framework to investigate the main determinants for the shape of the schedule. In particular, it is well suited for understanding the impact of depreciation and heterogeneity, precisely because optimal benefits are constant in the benchmark case without these features.

We show that the optimal time-varying path of benefits depend on the particular form that depreciation or heterogeneity take: there are forces for increasing or decreasing benefits. We explore our model numerically to understand these different forces and disentangle the various effects. Our explorations are preliminary and ongoing, and we have initially focused on human capital depreciation.

Some interesting lessons emerge from our exercises. We consider two forms of depreciation. In the first, job opportunities arrive at a constant rate but the wage distribution from which they are drawn deteriorates in a steady and parallel fashion. This case captures skill depreciation and is similar to that modeled by Ljungqvist and Sargent (1998).

In the second form of depreciation, workers continue to sample from the same distribution of wages, but the arrival of such opportunities gets rarer over time. As the search friction rises during their spell, workers become more detached from the labor market. Perhaps this captures the dynamics of a search process where the nearest or more obvious sources of jobs are initially scouted, as these are exhausted the worker turns to more remote job possibilities obtaining fewer contacts per unit time.

For concreteness, we call the first form of depreciation *skill depreciation*, and the second form *search depreciation*.

In our skill-depreciation exercises we find decreasing optimal UI benefits. Decreasing benefits are reasonable since constant benefits would give the long-term unemployed a higher replacement ratio relative to their *potential wages*, inducing these workers to become overly picky or even drop out, as stressed by Ljungqvist and Sargent (1998). In our exercises, the lower potential wages lead the worker's reservation wage to decline steadily during unemployment. However, we find that benefits decline even faster: the ratio of UI benefit to the reservation wage is decreasing.

Interestingly, we also find that skill depreciation creates a force for higher unemployment benefit levels. Intuitively, the shocks to worker's permanent income from remaining unemployed for an additional week, which we seek to insure, become larger: they are no longer simply the missed current earnings, but also include the lower future earnings.

In sharp contrast, we find that search depreciation creates a force for rising benefits and reservation wages. Intuitively, unfortunate workers receive fewer job offers and, as a result, demand more insurance to deal with their heightened duration risk. Long-term unemployed workers have lower exit rates from unemployment, but not because they become more picky about the wages they accept. Indeed, we find that reservation wages steadily fall during the unemployment spell. The reason they are less likely to become employed is that they receive fewer job offers. Intuitively, the moral hazard problem is less severe while risks loom greater, and more insurance is optimal. In particular, we find that benefits eventually rise sharply and limit to high levels.

After briefly discussing some related literature, this rest of the paper is organized into four sections. Section 2 describes the model. Section 3 characterizes the worker's search problem for any given UI benefit schedule. Section 4 studies the optimal UI problem when workers face depreciation during unemployment. Section 5 turns to the case of heterogeneity. When the time is ripe, future drafts will include a concluding section.

Some Related Literature

Ljungqvist and Sargent (1998) emphasize human capital depreciation of unemployed workers to explain higher European unemployment. They model skill loss as stochastic, so their story actually also combines elements of heterogeneity. In particular, during 'tranquil' times human capital depreciates steadily during unemployment generating unimportant amounts of heterogeneity among the unemployed. In contrast, during 'turbulent' times a fraction of workers lose skills immediately at the moment they are laid off, generating significant amounts of heterogeneity.[1]

Pavoni (2003) and Violante and Pavoni (2005) study optimal unemployment insurance in environments with human capital depreciation, and other elements. However, these papers do not focus on the optimal timing of benefits since they assume that the UI agency can control consumption and do not attempt to distinguish consumption from benefits.

2 The Model

We adapt the model from Shimer and Werning (2005, 2006). These papers provide a tractable version of a McCall (1970) search problem enhanced to incorporate risk-averse workers that can save and borrow freely. Time is continuous and infinite $t \in [0, \infty)$. Workers seek to maximize expected discounted utility

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c(t)) dt \tag{1}$$

where $c(t)$ is consumption. We work with Constant Absolute Risk Aversion (CARA) utility functions: $u(c) = -e^{-\gamma c}$ with $c \in \mathbb{R}$. This assumption allows us to solve the model in closed form.[2]

Unemployed workers sample job opportunities at Poisson arrival rates $\lambda(t)$. Jobs are distinguished by their wage w drawn from a distribution $F(w, t)$ with density $f(w, t)$. This introduces two potential forms of human capital depreciation. We assume jobs last forever.

[1] Ljungqvist and Sargent (1998, pg. 548) conclude that "during tranquil times, the depreciation of skills during spells of unemployment [...] is simply too slow to have much effect on the amount of long-term unemployed. The primary cause of long-term unemployment in our turbulent times is the instantaneous loss of skills at layoffs. Our probabilistic specification of this instantaneous loss creates heterogeneity among laid-off workers having the same past earnings."

[2] In Shimer and Werning (2005) we verified that CARA preferences provided a good benchmark: the numerical solution with CRRA preferences was very close to the CARA closed form solution.

Budget Constraints. Workers can save and borrow at the market interest r. Their budget constraints are

$$\dot{a}(t) = ra(t) + y(t) - c(t), \tag{2}$$

where $a(t)$ are assets and $y(t)$ represents current non-interest income: it equals benefits, $B(t)$, during unemployment and the wage, $w(t)$, during employment. Initial assets $a(0) = a_0$ are given. In addition workers must satisfy the No-Ponzi condition $\lim_{t\to\infty} e^{-rt}a(t) \geq 0$.

Heterogeneity. To capture worker heterogeneity we assume that there are N types indexed by $n = 1, 2, \ldots, N$ and index any of the primitives of the worker's search problem by the type, e.g. $\lambda_n(t)$, $F_n(w,t)$, γ^n, and so on. To focus on depreciation we initially assume there is only one type and drop the superscripts.

UI Policy. In this paper we are interested in the optimal timing of UI benefits, and not in larger, more comprehensive, social welfare reforms. This motivates the policy problem we consider, which is to select a schedule of unemployment benefits $\{B(t)\}_{t\geq 0}$ that stipulates the benefit $B(t)$ received by a worker that remains unemployed at t. The duration of unemployment is the only variable for which benefits can be conditioned on.[3]

The optimal UI policy problem we study is to find the best such schedule of benefits. For the case with heterogenous workers one must, in general, specify a welfare criterion. However, to avoid redistributional concerns we allow initial lump-sum transfers between workers types. These transfers are equivalent to redistributions in terms of initial assets a_0. It turns out that, with CARA preferences, the optimal schedule $\{B(t)\}_{t\geq 0}$ is then uniquely pinned down: all Pareto efficient allocations can be achieved with the same schedule by varying the initial lump-sum transfers (initial assets) between workers. Thus, we do not need to specify any particular welfare criterion and our analysis characterizes all Pareto efficient schedules.

3 Worker Behavior

In this section we characterize the behavior of a single unemployed worker confronted with any benefit schedule $\{B(t)\}_{t\geq 0}$.

Let $V(t)$ represent the lifetime utility of an unemployed worker at time t,

$$V(t) \equiv \mathbb{E}_t \int_0^\infty e^{-\rho s} u(c(t+s))ds. \tag{3}$$

[3] We do not consider, for instance, menus of schedules that may self-select and separate worker types; likewise we do not consider constraining workers access to savings.

We now derive a few properties about $V(t)$.

For any job-acceptance policy, workers solve a standard consumption-savings subproblem, maximizing utility in equation (1) subject to the budget constraint equation (2) and the No-Ponzi condition. This subproblem implies the usual intertemporal Euler equation

$$u'(c(t)) = e^{-(\rho-r)s}\mathbb{E}_t u'(c_{t+s})$$

With CARA preferences $u'(c) = \gamma u(c)$, so this implies

$$u(c(t)) = e^{-(\rho-r)s}\mathbb{E}_t u(c_{t+s})$$

Substituting this into equation (3) gives

$$V(t) = \frac{1}{r}u(c(t)) \qquad \text{or equivalently} \qquad c(t) = u^{-1}(rV(t)), \tag{4}$$

which conveniently relates lifetime utility to current consumption.

Lifetime utility $V(t)$ can always be decomposed as

$$V(t) \equiv -v(t)u(ra(t)), \tag{5}$$

where $v(t)$ represents the lifetime utility of a worker with zero assets.

Unemployed workers just wait around for job offers. An unemployed worker accepts a job offer if the wage is higher than the wage $\bar{w}(t)$ which makes her indifferent. Since a worker earning $\bar{w}(t)$ with assets $a(t)$ consumes $ra(t) + \bar{w}(t) + (\rho - r)/\gamma r$,

$$V(t) = \frac{u(ra(t) + \bar{w}(t) + (\rho - r)/\gamma r)}{r} \tag{6}$$

$$\Rightarrow \bar{w}(t) = u^{-1}(rV(t)) - ra(t) - \frac{\rho - r}{\gamma r} = u^{-1}(rv(t)) - \frac{\rho - r}{\gamma r} \tag{7}$$

With a reservation rule the lifetime utility during unemployment is a function of time and evolves according to

$$\rho V(t) = u(c(t)) + \dot{V}(t) + \lambda(t)\int_{\bar{w}(t)}^{\infty}\left(\frac{u(ra(t) + w + (\rho - r)/\gamma r)}{r} - V(t)\right)dF(w,t)$$

Using equation (4) it follows that we can write this as

$$\dot{V}(t) = (\rho - r)V(t) - \lambda(t)\int_{\bar{w}(t)}^{\infty}\left(\frac{u(ra(t) + w + (\rho - r)/\gamma r)}{r} - V(t)\right)dF(w,t)$$

Rewriting this in terms of $v(t)$ using equation (5) gives

$$-\dot{v}(t) + \gamma v(t) r \dot{a}(t) = (r - \rho)v(t) - \lambda(t)\int_{\bar{w}(t)}^{\infty}\left(\frac{-u(w + (\rho - r)/\gamma r)}{r} + v(t)\right)dF(w,t) \quad (8)$$

As for $\dot{a}$, the budget constraint equation (2) during unemployment combined with equations (4) and (5) gives

$$\begin{aligned}\dot{a}(t) &= ra(t) + B(t) - c(t) = ra(t) + B(t) - u^{-1}(rV(t)) \\ &= ra(t) + B(t) - u^{-1}(rv(t)) - ra(t) = B(t) - u^{-1}(rv(t)).\end{aligned}$$

Substituting this into equation (8) yields

$$\begin{aligned}\dot{v}(t) = \gamma v(t) r\big(B(t) - u^{-1}(rv(t))\big) - (r - \rho)v(t) \\ + \lambda(t)\int_{\bar{w}(t)}^{\infty}\left(\frac{-u(w + (\rho - r)/\gamma r)}{r} + v(t)\right)dF(w,t). \quad (9)\end{aligned}$$

To transform this into a law of motion for the reservation wage, note that equation (7) implies

$$\dot{\bar{w}}(t) = \frac{1}{u'(u^{-1}(rv(t)))}r\dot{v}(t) = \frac{-1}{\gamma u(u^{-1}(rv(t)))}r\dot{v}(t) = -\frac{\dot{v}(t)}{\gamma v(t)}.$$

Then equation (9) becomes

$$\dot{\bar{w}}(t) = G(\bar{w}, t) - rB(t), \quad (10)$$

where

$$G(\bar{w}, t) \equiv r\bar{w}(t) - \frac{\lambda(t)}{\gamma}\int_{\bar{w}(t)}^{\infty}(1 + u(w - \bar{w}(t)))\,dF(w,t). \quad (11)$$

In a stationary environment, with constant benefits B, a constant arrival rate λ, and a constant wage distribution $F(w)$, equation (10) boils down to the reservation wage equations in Shimer and Werning (2005, 2006).

Relation between $\bar{w}$ and B. We shall use the characterization of the relationship in equation (10) between the reservation wage path $\{\bar{w}(t)\}$ and benefits schedule $\{B(t)\}$ extensively. It is useful to understand what this relation does and does not imply.

Suppose we are in a stationary environment so that $G(\bar{w}, t)$ is independent of time t. Then, at a steady state, where $\dot{\bar{w}} = 0$, we have $B = G(\bar{w}, t)/r$ and since $G_{\bar{w}} > 0$ it follows that there is a positive relation between benefits and the reservation wage. This is very intuitive since a higher benefit level makes the option of waiting for higher wage draws more attractive without making employment any more desirable. As a result, the worker becomes more picky about what jobs to accept.

However, this steady state relationship does not imply that along any dynamic path B and $\bar{w}$ will rise and fall in tandem. For example, suppose the reservation wage is monotonically increasing. Perhaps, it is reasonable to expect benefits to rise also. This will usually be the case as long as the reservation wage does not rise too fast. That is, benefits may decrease over a range where $\bar{w}$ rises quickly, so that $\dot{\bar{w}}$ is very high. Intuitively, if the reservation wage is rising sharply, this indicates that the unemployed worker's lifetime utility is doing the same; this may be the case because current benefits are temporarily low.

Transversality Condition. In addition one needs to impose the transversality condition that

$$\lim_{t\to\infty} e^{-\rho t}\mu(t)V(t) = 0$$
$$\Rightarrow \lim_{t\to\infty} e^{-\rho t}\mu(t)v(t)u(ra(t)) = 0$$

where

$$\mu(t) \equiv \exp\left(-\int_0^t H(\bar{w}(s), s)ds\right) \tag{12}$$

is the probability of being unemployed at time t and

$$H(\bar{w}, s) \equiv \lambda(s)(1 - F(\bar{w}, s)) \tag{13}$$

is the hazard rate of accepting a job.

4 Optimal Policy: Human Capital Depreciation

In this section we consider the case of a single worker type that faces a non-stationary search problem, with λ or F are changing over time.

4.1 Policy Problem

We imagine an unemployment insurance agency that wishes to maximize unemployed workers' lifetime utility subject to the constraint that it must break even on average. Imagine the agency charging the worker an upfront fee C, equal to the expected present value of benefits. Then using equation (6) the worker's utility is

$$V(0) = \frac{u(r(a_0 - C) + \bar{w}(0) + (\rho - r)/\gamma r)}{r}$$

Thus, the agency chooses a path of benefits to maximize $\bar{w}(0) - rC$, where the expected cost of the UI system is

$$C \equiv \int_0^\infty e^{-rt} B(t)\mu(t)dt$$

and reservation wages solve equation (10) and the transversality condition.

Use equation (10) to eliminate $B(t)$ from the objective function

$$\bar{w}(0) - rC = \bar{w}(0) + \int_0^\infty e^{-rt}(\dot{\bar{w}}(t) - G(\bar{w}(t), t))\mu(t)dt, \tag{14}$$

Use integration-by-parts to eliminate the term of the integral involving $\dot{\bar{w}}(t)$:

$$\begin{aligned}\int_0^\infty \dot{\bar{w}}(t)\mu(t)e^{-rt}dt &= -\bar{w}(0) - \int_0^\infty \bar{w}(t)(\dot{\mu}(t) - r\mu(t))e^{-rt}dt \\ &= -\bar{w}(0) + \int_0^\infty \bar{w}(t)\big(r + H(\bar{w}(t), t)\big)\mu(t)e^{-rt}dt,\end{aligned}$$

since $\mu(0) = 1$. Substitute this back equation (14) to simplify the objective function $\bar{w}(0) - rC$. The planner must choose a sequence of reservation wages to solve

$$\max_{\{\bar{w}\}} \int_0^\infty \Big(\bar{w}(t)\big(r + H(\bar{w}(t), t)\big) - G(\bar{w}(t), t)\Big)\mu(t)e^{-rt}dt$$

s.t. $\dot{\mu}(t) = -H(\bar{w}(t), t)\mu(t)$.

4.2 Constant and Non-Constant Benefits

An interesting property of optimal benefits that follows from our reformulation is that the schedule is entirely forward looking: only future values of λ and F are relevant. The next result then follows immediately from this observation.[4]

Proposition 1 (Shimer-Werning, 2005) *With a single worker type facing a stationary problem $\lambda(t) = \lambda$ and $F(w,t) = F(w)$ for all $t \geq 0$ the optimal benefit schedule is flat: $B(t) = \bar{B}$ for some $\bar{B} > 0$.*

Indeed, we can tackle the more general problem by writing the problem recursively. Let $\Phi(\mu, t)$ be the value, solving the Bellman equation

$$r\Phi(\mu,t) = \max_{\bar{w}} \Big(\big(\bar{w}(r + H(\bar{w},t)) - G(\bar{w},t)\big)\mu - \Phi_\mu(\mu,t)H(\bar{w},t)\mu + \Phi_t(\mu,t) \Big)$$

Note that the value function is linear in μ and so we can define $\phi(t) = \Phi(\mu,t)/\mu$ solving

$$r\phi(t) = \max_{\bar{w}} \Big(\bar{w}(r + H(\bar{w},t)) - G(\bar{w},t) - \phi(t)H(\bar{w},t) + \dot{\phi}(t) \Big)$$

Equivalently, we have an ordinary differential equation for $\phi(t)$:

$$\dot{\phi}(t) = M(\phi(t), t) \tag{15}$$

where the law of motion function M is given by

$$M(\phi, t) \equiv \min_{\bar{w}} \Big((r + H(\bar{w},t))(\phi - \bar{w}) + G(\bar{w},t) \Big). \tag{16}$$

Note that the envelope condition implies that the law of motion function $M(\phi,t)$ is increasing in ϕ. Moreover, since the cross-partial derivative of $\bar{w}$ and ϕ is negative in the objective function, it follows that the optimal $\bar{w}$ is increasing in ϕ.

To characterize optimal unemployment insurance, we simply need to solve this ordinary differential equation (15) so that the transversality condition holds. For example when primitives settle down in the long-run one can solve backwards from the long run steady-state, e.g. an asymptotic steady state of the economy. The optimal reservation wage solves the right hand side of equation (16) at each date. And the net benefits that implement this reservation wage are found by inverting equation (10) for $B(t)$.

We first illustrate how Proposition 1 translates to solution to the ODE system. Consider first the case where primitives are constant so that $\lambda(t) = \lambda$ and $F(w,t) = F(w)$ for all $t \geq 0$.

[4] This result is proven in Shimer and Werning (2005) in a discrete time version of the model using a different argument. That paper shows that the planner does not want to distort savings. In contrast, here we have simply assumed that the policy problem does not consider introducing such distortions.

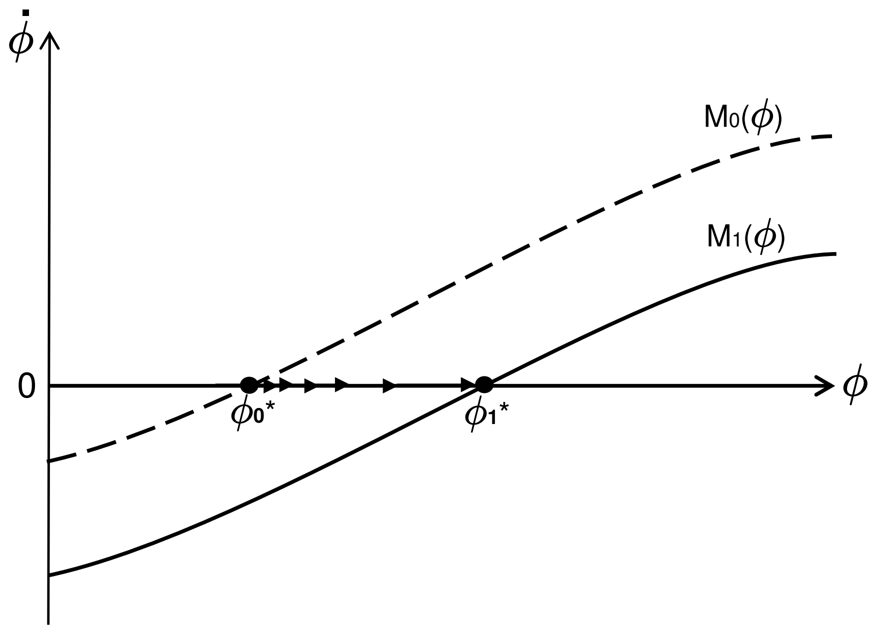

Figure 1: Law of Motion for ϕ.

It follows immediately that the law of motion is independent of time: $M(\psi, t) = M(\psi)$. Then since this function is increasing there exists a unique steady-state value: ψ^* such that $0 = M(\psi^*)$. Moreover, the unique solution must then be $\psi(t) = \psi^*$ since any other solution to the ODE is explosive and would not satisfy the transversality condition. With a constant ψ it follows immediately that benefits are constant.

A simple non-stationary case, illustrated in Figure 1, is when primitives are constant up to some time T, at which point they switch forever after. That is, we have $\lambda(t) = \lambda_0$ and $F(w, t) = F_0(w)$ for all $t < T$ and $\lambda(t) = \lambda_1$ and $F(w, t) = F_1(w)$ for all $t \geq T$. This implies that ϕ evolves according to $M_0(\psi)$ for $t < T$, and then $M_1(\psi)$ for $t \geq T$.

The optimal solution must reach the steady-state point ϕ_1^* of M_1 at $t = T$. Thus, the initial value $\phi(0)$ must start somewhere to the right of point ϕ_0^* and increase—accelerating with the explosive dynamics of M_0—until it reaches ϕ_1^* exactly at time $t = T$, at which point it remains constant there. The larger is T the closer $\phi(0)$ must be to ϕ_0^*; indeed, as $T \to \infty$ then $\phi(0)$ limits to ϕ_0^*.

The implications for benefits $B(t)$ are immediate translations of those derived for $\phi(t)$. Let B_i^* denote the optimal constant benefits for the stationary problem with λ_i and $F_i(w)$. Then benefits $B(t)$ converge to the optimal constant benefit B_1^* in the long run as $t \to \infty$ and they start somewhere near B_0^*. This result is generalized in the next proposition, where we imagine time extending indefinitely on both sides.

Proposition 2 *Suppose that we have $\lambda(t)$ and $F(w,t)$ defined for all $t \in \mathbb{R}$ with well defined limits $\lim_{t\to-\infty} \lambda(t) = \lambda_0$ and $\lim_{t\to-\infty} F(w,t) = F_0(w)$ and $\lim_{t\to\infty} \lambda(t) = \lambda_1$ and $\lim_{t\to\infty} F(w,t) = F_1(w)$. Then $B(t)$ is such that $\lim_{t\to-\infty} B(t) = B_0^*$ and $\lim_{t\to\infty} B(t) = B_1^*$, where B_i^* is defined as the optimal constant benefit levels for the economies with constant primitives at λ_i and $F_i(w)$.*

4.3 An Aside: Q-Theory Analogy

Our model can be mapped into the adjustment cost model of investment with constant returns to scale which Hayashi (1982) used to related investment to "Tobin's Q".

The investment model can be formulated as

$$\max \int_0^\infty \pi(i(t),t)K(t)dt$$

s.t. $\dot{K}(t) = i(t)K(t)$, where $i(t) = I(t)/K(t)$ is the investment rate. There are constant returns to scale in the net profit function (which includes static profits net of investment costs) and constant returns in investment. Note that no claim of concavity is required for this formulation. By writing this recursively, one can show that the value function $V(K)$ is linear, and so the marginal and average value of capital, q, often referred to as "Tobin's Q", solves

$$rq = \max_i \{\pi(i,t) + iq\} + \dot{q}.$$

The important result in this theory is that the investment rate i is a known function of the value of the firm divided by capital q. The entire future variations in productivity are captured by the forward looking average value of the firm q.

This model maps directly into our framework, with μ playing the role of K, with $\bar{w}$ playing the role of investment i, and for a particular π function.

4.4 Numerical Explorations

This section describes the outcome of two numerical experiments. We first consider skill depreciation, then search depreciation. The purpose of these explorations is not definite quantitative conclusions. Our goal is to understand the qualitative workings of the model and perhaps get a tentative feel for their quantitative importance.

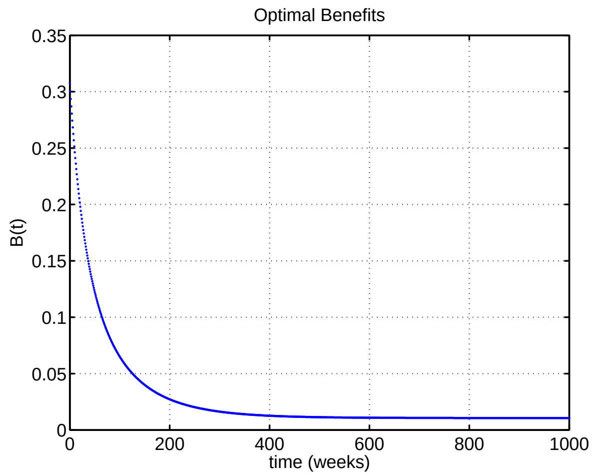

Figure 2: Optimal Benefits with Skill Depreciation.

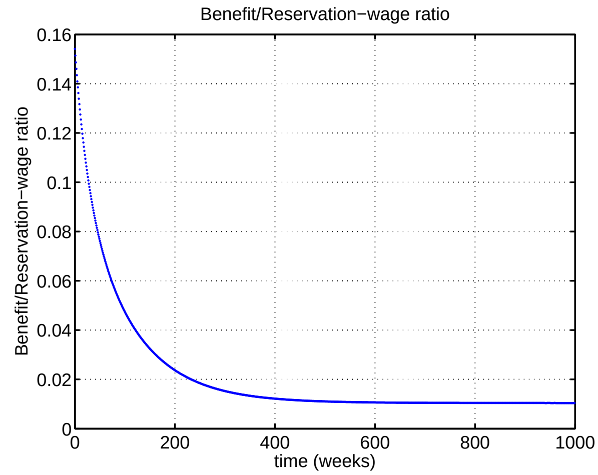

Figure 3: Ratio of benefits to the reservation wage: $B(t)/\bar{w}(t)$.

Our baseline parameterization is close to that in Shimer and Werning (2005). We set $\gamma = 1$, $r = \rho = .001$ and $\lambda = 1$ and interpret a period to be a week, with an implied annual interest rate of 5.3%. The distribution is assumed to be Fréchet: $F(w) = \exp(-w^{-\theta})$. We set $\theta = 103.5$.

This baseline calibration has wages concentrated near 1 and delivers an expected duration of around 10 weeks, which is in line with unemployment durations in the United States. The optimal constant benefit level in this economy turns out to be very low, around 0.01. The desire to insurance is small enough, while the moral-hazard problem severe enough, that low benefits result at the optimum. As discussed by Shimer and Werning (2005), liquidity, in contrast, is important: workers are able to smooth their shocks, spreading their impact over time, by dissaving or borrowing.

Skill Depreciation

In this first exercise we keep $\lambda(t) = 1$ constant and instead assume that the distribution of wages shifts downwards in a parallel fashion. Our specification is inspired by the depreciation process used in Ljungqvist and Sargent (1998). Specifically we let $F(w, t) = F(w - \exp(-\delta_F \cdot t))$ for all $t < T$ and $F(w, t) = F(w - \exp(-\delta_F \cdot T))$ for $t \geq T$, where $F(w)$ is the baseline Fréchet distribution defined above. Thus, at $t = 0$ the wage distribution is simply a rightward shift of the baseline distribution. Over time the distribution shifts to the left, converging back to the baseline distribution. We set the speed of convergence to $\delta_F = 0.01$.

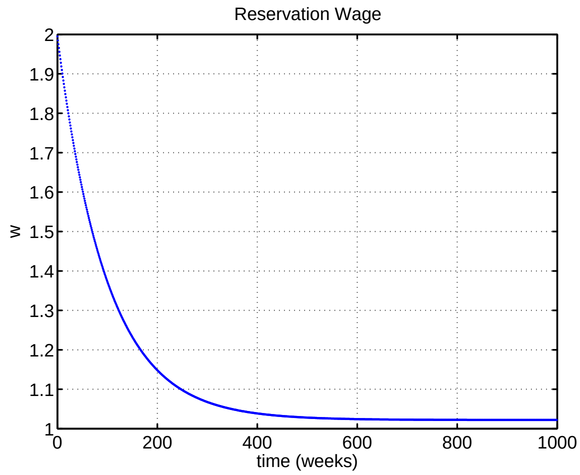

Figure 4: Reservation Wage.

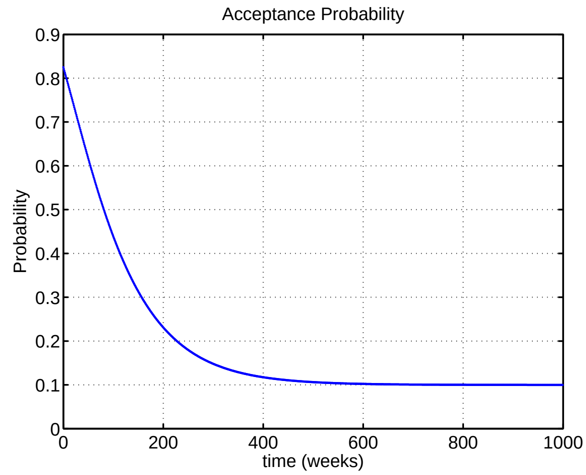

Figure 5: Acceptance Probability.

Our approach is to solve the ODE system in equation (15). We first solve the system's steady state for $t \geq T$. We then solve the ODE backwards up to $t = 0$. This gives us $\phi(t)$. We then compute $\bar{w}(t)$ and solve equation (10) for $B(t) = \frac{1}{r}(G(\bar{w}(t), t) - \dot{\bar{w}}(t))$.[5]

Figure 2 shows the outcome of this exercise for the optimal schedule of benefits. The schedule is decreasing with unemployment duration, starting at benefits just above 0.30 and falling to the steady-state value of 0.01—equal to the value of benefits at the baseline. Figure 4 shows that these benefits induce the reservation wage to fall during the unemployment spell. The rate at which the reservation wage drops, however, does not keep up with the rate of decline in the distribution of wages. This is shown in Figure 5 where we plot the acceptance probability of job opportunities, $1 - F(\bar{w}(t))$. The optimal schedule induces workers to become pickier: the probability of accepting a job offer falls from around 80% to 10%. Note that in this case, since $\lambda(t) = 1$ the acceptance probability equals the hazard rate of out unemployment, $\lambda(t)(1 - F(\bar{w}(t)))$. Benefits, however, all even faster: Figure 3 plots the ratio $B(t)/\bar{w}(t)$, it is decreasing.

From this exercise, we conclude that skill depreciation seems to provide a force for a decreasing benefit schedule in our model.

[5] The numerical details are as followed. We employed Matlab's `ode45` routine to solve the ODE, along with the `fminbnd` routine for the required optimization. We computed $\phi(t)$ and then backed out the implied reservation wage $\bar{w}(t)$. We then fit a piecewise cubic shape-preserving spline through $\bar{w}(t)$ (using Matlab's `interp1` routine with the `pchip` option) to obtain the derivative $\dot{\bar{w}}(t)$ from it.

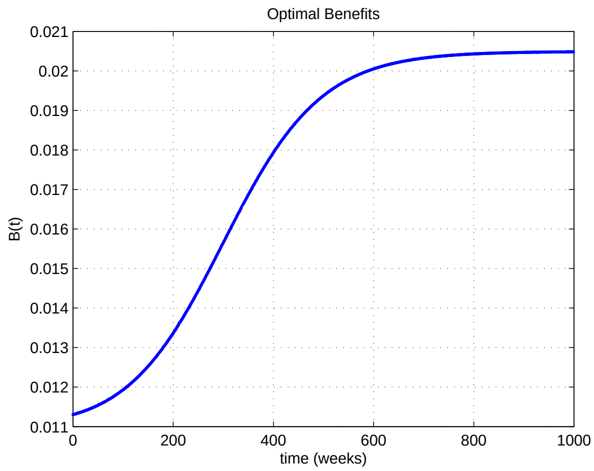

Figure 6: Optimal Benefits with Search Depreciation.

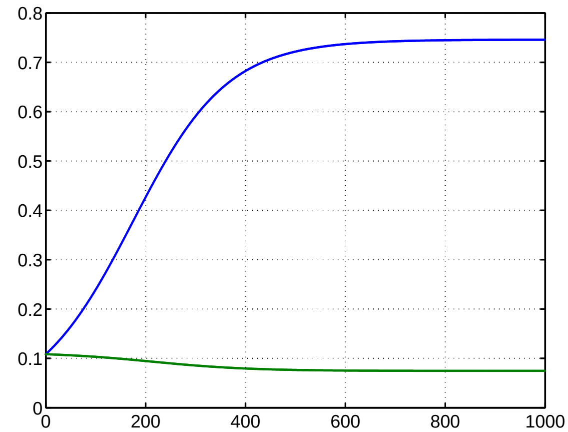

Figure 7: Acceptance Probability and Hazard Rate with Search Depreciation.

Search Depreciation

In this second exercise we assume that the wage distribution is fixed at the baseline's Fréchet but that the arrival rate falls continuously over time. Specifically, we let $\lambda(t) = \bar{\lambda}_0 + \bar{\lambda}_1 \exp(-\delta_\lambda \cdot t)$ for $t \leq T$ and constant thereafter; we set $\delta_\lambda = 0.01$, $\bar{\lambda}_0 = .9$ and $\bar{\lambda}_1 = .1$.

Figure 6 shows the results of this exercise for optimal benefits. We find an increasing schedule, which is in line with Proposition 2, since a lower arrival rate increases the duration risk of unemployment prompting higher benefits. The level of benefits is quite low in this case throughout, so the increase in benefits is not very spectacular. Figure 7 shows that even though benefits rise the acceptance probability (blue line) rises with duration as job opportunities become rarer. However, the resulting hazard rate out of unemployment $\lambda(t)(1 - F(\bar{w}(t)))$, also shown (green line), comes out to be slightly declining.

We have found that the limiting long-run benefit level is quite sensitive to the long-run value λ_1 for low enough values, and become quite large for λ_1 near zero. To illustrate this Figure 8 shows optimal benefits when $\lambda_1 = .01$. Note that benefits rise only moderately for about 6 years—similar to what we found in Figure 6 for higher λ_1. However, when $\lambda(t)$ gets very close to zero benefits rise sharply and they asymptote to a very high level around 0.73. Of course, with these parameters only an insignificant fraction of workers make it this far into long-term unemployment. Nevertheless, this illustrates that size of the increase in benefits generally depends on the parameters.

From this exercise, we conclude that search depreciation seems to provide a force for

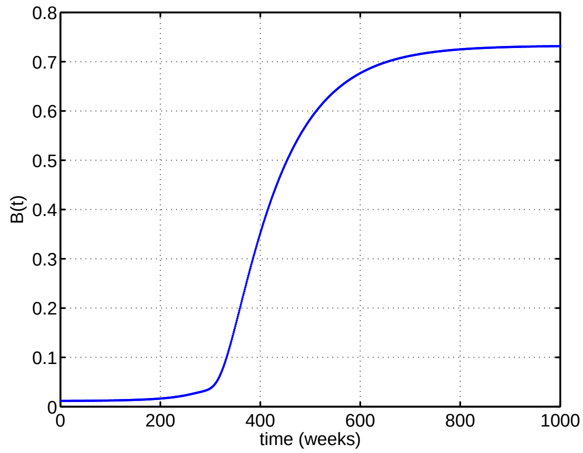

Figure 8: Optimal Benefits with Search Depreciation when $\lambda_1 = .01$.

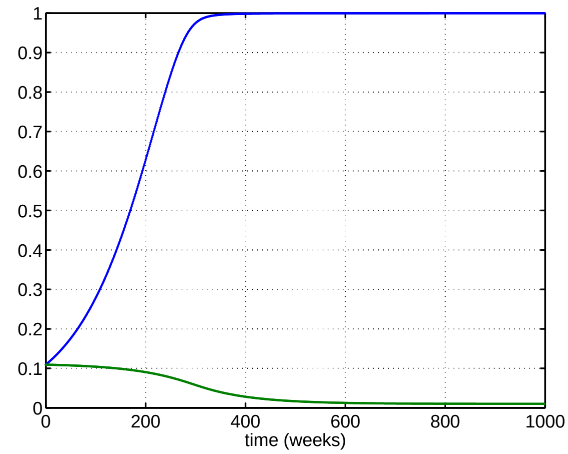

Figure 9: Acceptance Probability and Hazard Rates when $\lambda_1 = .01$.

increasing benefits in our model.

5 Optimal Policy: Heterogeneity

In this section we formulate the planning problem for the case where the agency faces heterogeneous workers indexed by $n = 1, 2, \ldots, N$. To bring out the role of heterogeity we suppose that for each worker type the arrival rate of offers λ^n and the distribution of wages F^n does not vary with time t.

For any given common benefit schedule the pool of unemployed worker types typically varies over time. That is, worker types that tend to have lower hazard rates become more prevalent as time passes. This is an important effect we wish to focus on. For now, we only formulate the problem. We shall study it in future versions of this draft.

5.1 Problem Statement

The agency selects a single benefit schedule to maximize the sum of the reservation wages net of the discounted cost the program. The problem can be formulated as the optimal control problem:

$$\max_B \sum_{n=1}^{N} \left(\bar{w}^n - \int_0^\infty e^{-rt} B(t) \mu^n(t) dt \right)$$

subject to,

$$\dot{\mu}^n(t) = -H(\bar{w}^n(t))$$
$$\dot{\bar{w}}^n(t) = G^n(\bar{w}^n(t)) - rB(t)$$

with $\mu^n(0)$ given.

One can attach the cost minimization problem as an optimal control problem or set it up recursively and derive its Hamilton-Jacobi-Bellman equation. For the case with two types one can show that the problem can be reduced to a two-dimensional state variable system: the state keeps track of the ratio of types remaining in the unemployment pool $\mu^2(t)/\mu^1(t)$ and the reservation wage of one of the two types, say, $\bar{w}^2(t)$.

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