Pareto Efficient Income Taxation

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NBER Public Economics meeting
Q: Good shape for tax schedule?
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  - positive: redistribution vs. efficiency
  - normative: Utilitarian social welfare function
Q: Good shape for tax schedule?

  - positive: redistribution vs. efficiency
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- this paper: Pareto efficient taxation
  - positive: redistribution vs. efficiency
  - normative: Utilitarian social welfare function

Pareto Efficiency
Old Motivation: “New New New...”

- Why not Utilitarian? ($\sum_i U^i$)
  - **practical**: cardinality $U^i \rightarrow W(U^i)$ (or even $W^i(U^i)$)
    ... which Utilitarian?
  - **conceptual**: political process:
    social classes $\rightarrow$ Coasian bargain
    ...but $\max \sum U^i$?
  - **philosophical**: other notions of fairness and social justice
Old Motivation: “New New New...”

Why not Utilitarian? \( (\sum_i U^i) \)

- **practical:** cardinality \( U^i \rightarrow W(U^i) \) (or even \( W^i(U^i) \))
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- **conceptual:** political process:
  social classes \( \rightarrow \) Coasian bargain
  ...but \( \max \sum U^i \) ?

- **philosophical:** other notions of fairness and social justice

- Pareto efficiency \( \rightarrow \) weaker criterion
Pareto Frontier

- Introduction
- Motivation
- Contribution
- Results

Model

Main Results

Applications

Conclusions

Pareto Efficient Income Taxation
Pareto Frontier
Pareto Frontier

\[ V_L \] \[ V_H \]
Pareto Frontier
Contribution

- invert Mirrlees model...
- ...express in tractable way
- ...use it: some applications
Results

#0 restrictions generalize “zero-tax-at-the-top”

#1 Any $T(Y)$ . . .
  ▶ efficient for many $f(\theta)$
  ▶ inefficient for many $f(\theta)$
  . . . anything goes

#2 Given $T_0(Y) \rightarrow g(Y) \rightarrow f(\theta)$ (Saez, 2001)
  ▶ efficient set of $T(Y)$: large
  ▶ inefficient set of $T(Y)$: large

#3 Simple test for efficiency of $T_0(Y)$
Results

#4 Simple formulas...
- bound on top tax rate
- efficiency of a flat tax

#5 Increasing progressivity
  ➡ maintains Pareto efficiency

#6 observable heterogeneity
  ➡ not conditioning can be efficient
Setup

Positive side of Mirrlees (1971)

- continuum of types $\theta \sim F(\theta)$
- additive preferences

\[ U(c, Y, \theta) = u(c) - \theta h(Y) \]

(e.g. $Y = w \cdot n$ and $h(n) = \alpha n^\eta$)
Setup

Positive side of Mirrlees (1971)

- continuum of types $\theta \sim F(\theta)$
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  \[ U(c, Y, \theta) = u(c) - \theta h(Y) \]
  
  (e.g. $Y = w \cdot n$ and $h(n) = \alpha n^\eta$)

- given $T(Y)$
  
  \[ v(\theta) \equiv \max_Y U(Y - T(Y), Y, \theta) \]
Positive side of Mirrlees (1971)

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- given $T(Y)$

  \[ v(\theta) \equiv \max_Y U(Y - T(Y), Y, \theta) \]

- Government budget

\[ \int T(Y(\theta)) \, dF(\theta) \geq G \]
Positive side of Mirrlees (1971)

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\[ U(c, Y, \theta) = u(c) - \theta h(Y) \]

(e.g. $Y = w \cdot n$ and $h(n) = \alpha n^\eta$)

- given $T(Y)$

\[ v(\theta) \equiv \max_Y U(Y - T(Y), Y, \theta) \]

- Resource feasible

\[ \int (Y(\theta) - c(\theta)) \, dF(\theta) \geq G \]
Positive side of Mirrlees (1971)

- continuum of types $\theta \sim F(\theta)$
- additive preferences

$$U(c, Y, \theta) = u(c) - \theta h(Y)$$

(e.g. $Y = w \cdot n$ and $h(n) = \alpha n^\eta$)

- given $T(Y)$

$$v'(\theta) = U_\theta(Y(\theta) - T(Y(\theta)), Y(\theta), \theta)$$

- Resource feasible

$$\int (Y(\theta) - c(\theta)) \, dF(\theta) \geq G$$
Setup

Positive side of Mirrlees (1971)

- continuum of types $\theta \sim F(\theta)$
- additive preferences
  \[ U(c, Y, \theta) = u(c) - \theta h(Y) \]
  (e.g. $Y = w \cdot n$ and $h(n) = \alpha n^\eta$)
- given $T(Y)$
  \[ v'(\theta) = -h(Y(\theta)) \]
- Resource feasible
  \[ \int (Y(\theta) - c(\theta)) \, dF(\theta) \geq G \]
**Setup**

Positive side of Mirrlees (1971)

- continuum of types \( \theta \sim F(\theta) \)
- additive preferences
  \[
  U(c, Y, \theta) = u(c) - \theta h(Y)
  \]
  (e.g. \( Y = w \cdot n \) and \( h(n) = \alpha n^\eta \))
- given \( T(Y) \)
  \[
  v'(\theta) = -h(Y(\theta))
  \]

- Resource feasible
  \[
  \int (Y(\theta) - e(v(\theta), Y(\theta), \theta)) \, dF(\theta) \geq G
  \]
Planning Problem

Dual Pareto Problem

maximize net resources

subject to,

\( \tilde{v}(\theta) \geq v(\theta) \)

incentives
Dual Pareto Problem

\[
\max_{\tilde{Y}, \tilde{v}} \int (\tilde{Y}(\theta) - e(\tilde{v}(\theta), \tilde{Y}(\theta), \theta)) \, dF(\theta)
\]

subject to,

\[\tilde{v}(\theta) \geq v(\theta)\]

incentives
Planning Problem

Dual Pareto Problem

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\max_{\tilde{Y}, \tilde{v}} \int (\tilde{Y}(\theta) - e(\tilde{v}(\theta), \tilde{Y}(\theta), \theta)) \, dF(\theta)
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subject to,

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\tilde{v}(\theta) \geq v(\theta)
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\tilde{v}'(\theta) = -h(\tilde{Y}(\theta))
\]
Planning Problem

Dual Pareto Problem

$$\max_{\tilde{Y}, \tilde{v}} \int (\tilde{Y}(\theta) - e(\tilde{v}(\theta), \tilde{Y}(\theta), \theta)) \, dF(\theta)$$

subject to,

$$\tilde{v}(\theta) \geq v(\theta)$$

$$\tilde{v}'(\theta) = -h(\tilde{Y}(\theta))$$

$$\tilde{Y}(\theta) \text{ nonincreasing}$$
Efficiency Conditions

Lagrangian

\[ \mathcal{L} = \int \left( \tilde{Y}(\theta) - e(\tilde{v}(\theta), \tilde{Y}(\theta), \theta) \right) dF(\theta) \]

\[ - \int \left( \tilde{v}'(\theta) + h(\tilde{Y}(\theta)) \right) \mu(\theta) d\theta \]
Efficiency Conditions

Lagrangian (integrating by parts)

\[ \mathcal{L} = \int \left( \tilde{Y}(\theta) - e(\tilde{v}(\theta), \tilde{Y}(\theta), \theta) \right) dF(\theta) - \tilde{v}(\bar{\theta}) \mu(\bar{\theta}) + \mu(\theta) \tilde{v}(\theta) \]

\[ + \int \tilde{v}(\theta) \mu'(\theta) d\theta - \int h(\tilde{Y}(\theta)) \mu(\theta) d\theta \]
Efficiency Conditions

Lagrangian (integrating by parts)

\[ \mathcal{L} = \int (\tilde{Y}(\theta) - e(\tilde{v}(\theta), \tilde{Y}(\theta), \theta)) \, dF(\theta) - \tilde{v}(\bar{\theta})\mu(\bar{\theta}) + \mu(\theta)\tilde{v}(\theta) + \int \tilde{v}(\theta)\mu'(\theta) \, d\theta - \int h(\tilde{Y}(\theta))\mu(\theta) \, d\theta \]

First-order conditions

\[ (1 - e_Y(v(\theta), Y(\theta), \theta)) f(\theta) = \mu(\theta)h'(Y(\theta)) \quad [Y(\theta)] \]
Efficiency Conditions

Lagrangian (integrating by parts)

\[ \mathcal{L} = \int (\tilde{Y}(\theta) - e(\tilde{v}(\theta), \tilde{Y}(\theta), \theta)) \, dF(\theta) - \tilde{v}(\bar{\theta}) \mu(\bar{\theta}) + \mu(\theta) \tilde{v}(\theta) + \int \tilde{v}(\theta) \mu'(\theta) \, d\theta - \int h(\tilde{Y}(\theta)) \mu(\theta) \, d\theta \]

First-order conditions

\[ \tau(\theta) f(\theta) = \mu(\theta) h'(Y(\theta)) \]
Efficiency Conditions

Lagrangian (integrating by parts)

\[ \mathcal{L} = \int (\tilde{Y}(\theta) - e(\tilde{v}(\theta), \tilde{Y}(\theta), \theta)) \, dF(\theta) - \tilde{v}(\bar{\theta}) \mu(\bar{\theta}) + \mu(\theta) \tilde{v}(\theta) \]

\[ + \int \tilde{v}(\theta) \mu'(\theta) \, d\theta - \int h(\tilde{Y}(\theta)) \mu(\theta) \, d\theta \]

First-order conditions

\[ \mu(\theta) = \tau(\theta) \frac{f(\theta)}{h'(Y(\theta))} \]
Efficiency Conditions

Lagrangian (integrating by parts)

\[
\mathcal{L} = \int \left( \tilde{Y}(\theta) - e(\tilde{v}(\theta), \tilde{Y}(\theta), \theta) \right) dF(\theta) - \tilde{v}(\bar{\theta})\mu(\bar{\theta}) + \mu(\theta)\tilde{v}(\theta) \\
+ \int \tilde{v}(\theta)\mu'(\theta)d\theta - \int h(\tilde{Y}(\theta))\mu(\theta) d\theta
\]

First-order conditions

\[
\mu(\theta) = \tau(\theta)\frac{f(\theta)}{h'(\tilde{Y}(\theta))}
\]

\[
\mu'(\theta) \leq e_v(\nu(\theta), Y(\theta), \theta) f(\theta)
\]
Efficiency Conditions

Lagrangian (integrating by parts)

\[ \mathcal{L} = \int \left( \tilde{Y}(\theta) - e(\tilde{v}(\theta), \tilde{Y}(\theta), \theta) \right) dF(\theta) - \tilde{v}(\tilde{\theta}) \mu(\tilde{\theta}) + \mu(\theta)\tilde{v}(\theta) \]

\[ + \int \tilde{v}(\theta)\mu'(\theta) d\theta - \int h(\tilde{Y}(\theta))\mu(\theta) d\theta \]

First-order conditions

\[ \mu(\theta) = \tau(\theta) \frac{f(\theta)}{h'(Y(\theta))} \]

\[ \mu'(\theta) \leq e_v(v(\theta), Y(\theta), \theta) f(\theta) \]

\[ \tau(\theta) \left( \theta \frac{\tau'(\theta)}{\tau(\theta)} + \frac{d \log f(\theta)}{d \log \theta} - \frac{d \log h'(Y(\theta))}{d \log \theta} \right) \leq 1 - \tau(\theta) \]
Efficiency Conditions

Proposition. \( T(Y) \) is Pareto efficient if and only if

\[

t(\theta) \left( \frac{\tau'(\theta)}{\tau(\theta)} + \frac{d \log f(\theta)}{d \log \theta} - \frac{d \log h'(Y(\theta))}{d \log \theta} \right) \leq 1 - t(\theta)
\]

\[
\tau(\bar{\theta}) \geq 0 \quad \text{and} \quad \tau(\theta) \leq 0.
\]
**Efficiency Conditions**

**Proposition.** \( T(Y) \) is Pareto efficient if and only if

\[
\tau(\theta) \left( \theta \frac{\tau'(\theta)}{\tau(\theta)} + \frac{d \log f(\theta)}{d \log \theta} - \frac{d \log h'(Y(\theta))}{d \log \theta} \right) \leq 1 - \tau(\theta)
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\tau(\bar{\theta}) \geq 0 \quad \text{and} \quad \tau(\theta) \leq 0.
\]

- note: “zero-tax-at-top” \( \rightarrow \) special case
Efficiency Conditions

Proposition. $T(Y)$ is Pareto efficient if and only if

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\tau(\theta) \left( \theta \frac{\tau'(\theta)}{\tau(\theta)} + \frac{d \log f(\theta)}{d \log \theta} - \frac{d \log h'(Y(\theta))}{d \log \theta} \right) \leq 1 - \tau(\theta)
$$

$$
\tau(\bar{\theta}) \geq 0 \quad \text{and} \quad \tau(\theta) \leq 0.
$$

note: “zero-tax-at-top” special case

more general condition:

$$
\frac{\tau(\theta)f(\theta)}{h'(Y(\theta))} + \int_{\theta}^{\bar{\theta}} \frac{1}{u'(c(\tilde{\theta}))} f(\tilde{\theta}) \, d\tilde{\theta}
$$

is nonincreasing.
Intuition

\[ \hat{T}(Y) \equiv \begin{cases} T(Y(\hat{\theta})) - \varepsilon & Y = Y(\hat{\theta}) \\ T(Y) & Y \neq Y(\hat{\theta}) \end{cases} \]

Proposition. \( \hat{T} \succ T \)

\[ \tau(\theta) \left( \theta \frac{\tau'(\theta)}{\tau(\theta)} + 2 \frac{d \log f(\theta)}{d \log \theta} - \frac{d \log h'(Y(\theta))}{d \log \theta} \right) \leq 3(1 - \tau(\theta)) \]

is violated at \( \hat{\theta} \)
Simple Tax Reform

Introduction

Model

Main Results
- Intuition
- Anything Goes
- Identification and Test
- Graphical Test
- Empirical Strategy
- Quantifying Inefficiencies

Applications

Conclusions
Simple Tax Reform

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\[
\frac{g'(Y)}{g(Y)} \text{ small } \left( \frac{f''(\theta)}{f(\theta)} \text{ large} \right) \rightarrow \text{ inefficiency}
\]
lower taxes $\rightarrow$ increase revenue

Pareto improvements $\leftrightarrow$ “Laffer” effect

**Proposition.** $T_1(Y) \succ T_0(Y) \rightarrow T_1(Y) \leq T_0(Y)$
\[ \tau(\theta) \left( \frac{\theta}{\tau(\theta)} \frac{\tau'(\theta)}{\tau(\theta)} + \frac{d \log f(\theta)}{d \log \theta} - \frac{d \log h'(Y(\theta))}{d \log \theta} \right) \leq 1 - \tau(\theta) \]

**Proposition.** For any \( T(Y) \)

- exists set \( \{f(\theta)\} \) → Pareto efficient
- exists set \( \{f(\theta)\} \) → Pareto inefficient
Proposition. For any $T(Y)$

- exists set $\{f(\theta)\}$ → Pareto efficient
- exists set $\{f(\theta)\}$ → Pareto inefficient

Without empirical knowledge

 anything goes
\[
\tau(\theta) \left( \theta \frac{\tau'(\theta)}{\tau(\theta)} + \frac{d \log f(\theta)}{d \log \theta} - \frac{d \log h'(Y(\theta))}{d \log \theta} \right) \leq 1 - \tau(\theta)
\]

**Proposition.** For any \( T(Y) \)

- exists set \( \{f(\theta)\} \) \( \rightarrow \) Pareto efficient
- exists set \( \{f(\theta)\} \) \( \rightarrow \) Pareto inefficient

- without empirical knowledge \( \rightarrow \) anything goes
- need information on \( f(\theta) \) to restrict \( T(Y) \)
observe $g(Y)$ identify (Saez, 2001)

$$\theta(Y) = (1 - T'(Y)) \frac{u'(Y - T(Y))}{h'(Y)}$$

$$f(\theta(Y)) = \frac{g(Y)}{\theta'(Y)}$$
Identification and Test

- observe \( g(Y) \) identify (Saez, 2001)

\[
\theta(Y) = (1 - T'(Y)) \frac{u'(Y - T(Y))}{h'(Y)}
\]

\[
f(\theta(Y)) = \frac{g(Y)}{\theta'(Y)}
\]

- efficiency test...

\[
\frac{d \log g(Y)}{d \log Y} \geq a(Y)
\]

... for tax schedule in place
define Rawlsian density:

\[ \alpha(Y) = \frac{\exp \left( \int_0^Y a(z) \, dz \right)}{\int_0^\infty \exp \left( \int_0^Y a(z) \, dz \right) \, dz} \]

graphical test:

\[ \frac{g(Y)}{\alpha(Y)} \text{ nondecreasing} \]
Empirical Implementation

- Empirical Strategy needed
  1. current tax function $T(Y)$
  2. distribution of income $g(Y)$
  3. utility function $U(c, Y, \theta)$
Empirical Implementation

- needed
  1. current tax function $T(Y)$
  2. distribution of income $g(Y)$
  3. utility function $U(c, Y, \theta)$

- in principle: #1 and #2 easy
  #3 usual deal
Empirical Implementation

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- Diamond (1998) and Saez (2001)
Empirical Implementation

- needed
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  #3 usual deal

- Diamond (1998) and Saez (2001)

- some challenges...
  1. econometric: need to estimate $g'(Y)$ and $g(Y)$
  2. conceptual: static model
    lifetime $T(Y)$ and $g(Y)$ (Fullerton and Rogers)
Output Density

- IRS’s SOI Public Use Files for Individual tax returns
  - lifetime $g(Y)$?
  - lifetime $T(Y)$ schedule?

- $Y^i = \frac{1}{n} \sum Y^i_t$

- smooth density estimate assumed $T(Y) = .30 \times Y$
Output Density

IRS’s SOI Public Use Files for Individual tax returns

- lifetime \( g(Y) \)?
- lifetime \( T(Y) \) schedule?

\[
Y^i = \frac{1}{n} \sum Y_t^i
\]

Smooth density estimate assumed \( T(Y) = .30 \times Y \)

Figure 1: Density of income \( g(Y) \).

Figure 2: Implied elasticity \( Y g'(Y) \).
Output Density

- IRS’s SOI Public Use Files for Individual tax returns
  - lifetime $g(Y)$?
  - lifetime $T(Y)$ schedule?

- $Y_i = \frac{1}{n} \sum Y_t^i$

- smooth density estimate assumed $T(Y) = 0.30 \times Y$

---

IRS’s SOI Public Use Files for Individual tax returns

- lifetime $g(Y)$?
- lifetime $T(Y)$ schedule?

- $Y_i = \frac{1}{n} \sum Y_t^i$

- smooth density estimate assumed $T(Y) = 0.30 \times Y$
Quantifying Inefficiencies

- efficiency test \(\rightarrow\) qualitative

- quantitative...

\[
\Delta \equiv \int (\tilde{Y}^*(\theta) - \tilde{c}^*(\theta)) \, dF(\theta) - \int (Y(\theta) - c(\theta)) \, dF(\theta)
\]

- does not count welfare improvements

\[
\tilde{v}(\theta) > v(\theta)
\]
Top Tax Rate

\[ u(c) = \frac{c^{1-\sigma}}{1 - \sigma} \quad \text{and} \quad h(Y) = \alpha Y^\eta \]

\[ \bar{\tau} \equiv \lim_{\theta \to 0} \tau(\theta) = \lim_{Y \to \infty} T'(Y) \]

exists
Top Tax Rate

- $u(c) = c^{1-\sigma}/(1 - \sigma)$ and $h(Y) = \alpha Y^\eta$

- suppose top tax rate

  $\bar{\tau} \equiv \lim_{\theta \to 0} \tau(\theta) = \lim_{Y \to \infty} T'(Y)$

  exists

- efficiency condition bound

  $\bar{\tau} \leq \frac{\sigma + \eta - 1}{\varphi + \eta - 2}$.

  where $\varphi = -\lim_{T \to \infty} d \log g(Y)/d \log Y$. 

Pareto Efficient Income Taxation
Top Tax Rate

\[ u(c) = \frac{c^{1-\sigma}}{1-\sigma} \quad \text{and} \quad h(Y) = \alpha Y^\eta \]

suppose top tax rate

\[ \bar{\tau} \equiv \lim_{\theta \to 0} \tau(\theta) = \lim_{Y \to \infty} T'(Y) \]

exists

efficiency condition bound

\[ \bar{\tau} \leq \frac{\sigma + \eta - 1}{\varphi + \eta - 2}. \]

where \( \varphi = -\lim_{T \to \infty} \frac{d \log g(Y)}{d \log Y} \).

Saez (2001): \( \varphi = 3 \)
Top Tax Rate

The graph illustrates the upper bound on the top tax rate (\(\tau\)) as a function of the elasticity \(1/\eta+1\). The curve shows how the upper bound on the top tax rate decreases as the elasticity increases.
Flat Tax

- linear tax → necessary condition

\[ \bar{\tau} \leq \frac{\sigma + \eta - 1}{-\frac{d \log g(Y)}{d \log Y}} + \eta - 2 \]

- linear tax → sufficient condition

\[ \bar{\tau} \leq \frac{\eta - 1}{-\frac{d \log g(Y)}{d \log Y}} + \eta - 1 \]
Quasi-linear $u(c) = c$

result: can always increase progressivity
Heterogeneity

- groups $= 1, \ldots, N$
  - $f^i(\theta)$ and $U^i(c, Y, \theta)$

- unobservable $i$
  - single $T(Y)$
  - average efficiency condition

- observable $i$
  - multiple $T^i(Y)$
  - $N$ efficiency conditions

- observation:
  - $T^i(Y) = T(Y)$ may be Pareto efficient
  - never optimal for Utilitarian
Conclusions

- Pareto efficiency → simple condition
- generalizes zero-tax-at-the-top result
- Pareto inefficient → Laffer effects
- flat taxes may be optimal...
- ...more progressivity always efficient