Empirical Validation of Changing the Curvature of a Jump Rope with Additional Weights

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12/23/12

2.671 Measurement and Instrumentation

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Abstract

The shape of the standard uniform density jump rope exhibits a "V" shape that leaves little space for the feet, increasing the likelihood of the rope getting caught on the feet. This report presents empirical evidence that adding small weights along the length of the jump rope dramatically alters the rope's shape to much more like a "U." High speed video analysis shows that the addition of small weights during testing increased the horizontal span of rope at the feet level by 8.7 ± 2.0 cm. The data from the unweighted rope are fit to two models: the catenary model based on gravity and the sinus amplitudinis model based on centripetal forces. The sinus amplitudinis model was best overall, the catenary model captured the residuals at the bottom, and neither model accounted for the negative curvature near the hands due to transitional effects.

1. Introduction

The standard curvature of a jump rope is not ideal for jumping. As shown in Figure 1, the jump rope swings very close to bottom of the feet as the jump rope almost makes a "V" shape. The sharp curvature at the bottom 1) increases the likelihood of having the rope get caught on the feet which 2) forces the jump roper to have to jump higher. Ideally, the shape of a jump rope would be more like a "U" shape. The jump roper would have more space horizontally for the feet and there would be less need for jumping high.

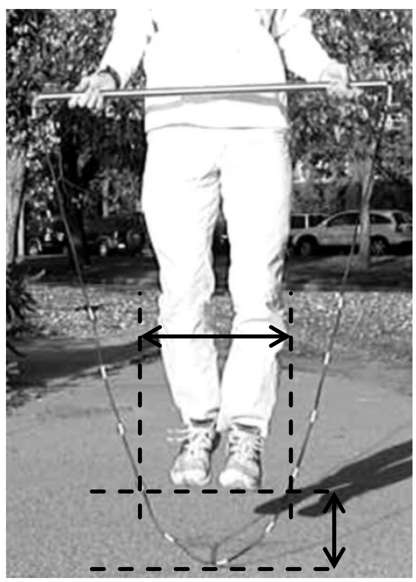

Figure 1: The shape of the conventional jump rope with uniform mass distribution is very much like a "V." The width of the jump rope at the feet level is small and the jump roper has to jump high to avoid contact with the rope.

In 2001, the idea to add small weights along a jump rope was first patented. The inventor David Loew called the weights "counterweights" mentioning their purpose simply "to define the dynamic shape of said rope when said rope is swung."[1] No further description was given to justify how significant the shape of the rope changes with the addition of weights. This report tests the concept of how a jump rope's curvature can be changed using one weight on each side of rope, as shown in Figure 2.

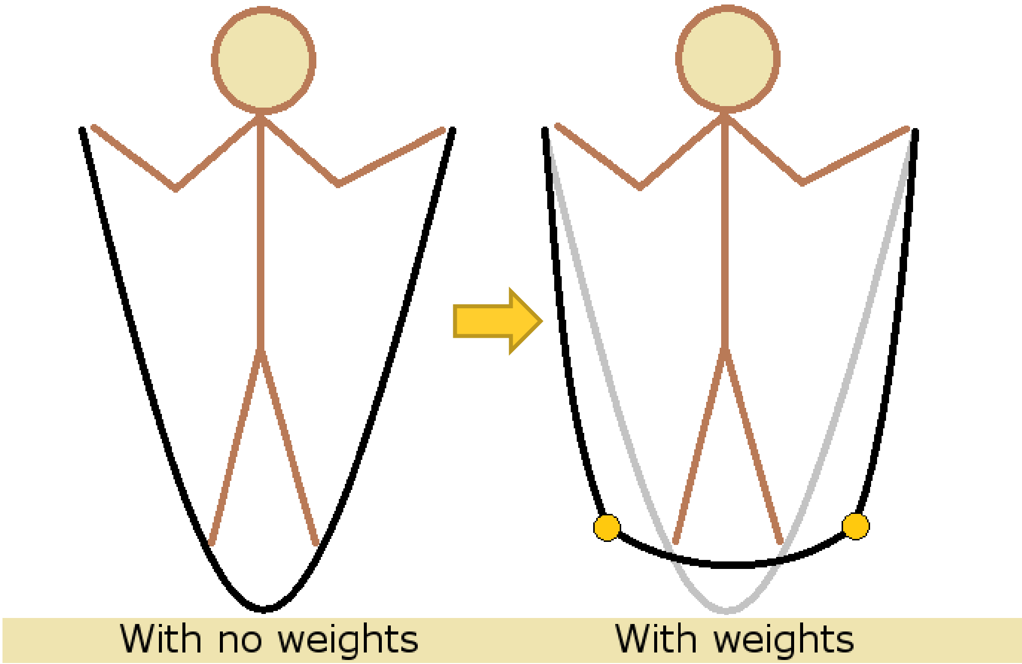

Figure 2: The addition of two weights reshapes the jump rope such that it is easier to jump. Notice how there is a wider area for foot space with the jump rope and the section between the tassels is flatter than without the tassels.

The curvature after weights is expected to be improved in a number of ways. First, the wider base of the jump rope allows more tolerance for the rope to swing horizontally. Generally jump rope users keep their feet together, but lateral movement of the rope is common as the rope never reaches equilibrium. More horizontal distance at the foot level will result in fewer collisions with the rope.

The curvature after weights is also expected to allow the jump roper to not have to jump as high. The curvature is expected to reward consistency rather than simply high jumps. This will be a major advantage for jump ropers who aim to jump faster, jump longer, or are just learning.

The data from the unweighted case will also be compared to two historical models of the shape of a rope: a hanging model due to only gravity and a swinging model based only on centripetal forces.

This report presents the theoretical shape of a jump rope in section 2, how the tests were set up in section 3, the empirical results in section 4, and the conclusion in section 5.

2. Historical Models of Jump Rope Curvature

2.1 A Simple Hanging Rope

The simplest curvature for a jump rope is when the rope is under a uniform gravitational field. This shape was subject to great historical debate, even incorrectly labeled a parabola by Galileo.[2] However, it can be shown mathematically that the shape of a hanging rope in a uniform gravitational field is what now called a catenary.[3] The debate seems reasonable as a catenary is very close to the shape of a parabola. The mathematical model for a catenary is given in

Equation 1. The parameters A, B, C, and D shift and scale the catenary. The variable x is the horizontal position and the output y(x) is the vertical position.

$$y(x) = A\left(e^{B(x-C)} + e^{-B(x-C)}\right) + D \qquad 1$$

The catenary model helps to provide an intuitive understanding that relates to adding weights. The preface to this understanding is that tension forces are always parallel to the rope. Due to the direction of tension forces, weights can be simply treated as concentrated segments of rope. This means that weights effectively juxtapose two segments of the catenary function that otherwise would be separated. Hence weights in this view are quite literally like a knot in the rope that accounts for the rope's kink as shown in Figure 3. Although the knot, kink, and juxtaposition analogy does not perfectly extend to a swinging rope, this method of viewing a concentrated weight is still novel and useful for analysis in the limiting case of small weights near the bottom of a swinging rope.

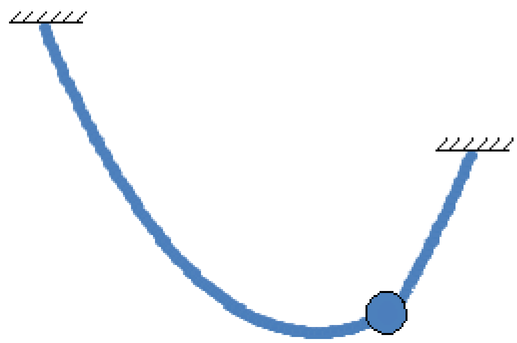

Figure 3: A basic hanging rope with a weight. The weight juxtaposes two segments of the catenary, but it does not affect any other part of the overall shape of the rope. If the rope were rotating, its overall shape would change with the addition of weights.

2.2 A Spinning Rope

A more math intensive model for the jump rope considers not only gravitational forces but also centripetal forces. Arne Nordmark and Hanno Essen in *The Skipping Rope Curve* use a model that only considers centripetal forces.[4] Their model ignores gravitational and air damping forces. One could argue that at least gravitational forces are negligible near the bottom of rope in comparison to centripetal forces if the angular frequency is sufficient. Nonetheless, Arne and Hanno's derivation lacks an empirical comparison for which this article provides.

Arne and Hanno show that the "exact" solution (if gravity and air drag are negligible) is given by a sinus amplitudinis function (sn) as shown in Equation 2:

$$y(x) = |y_m|sn\left(\frac{\sqrt{1+\frac{y_m{}^2}{4L^2}}x}{L} + B,\ k\right) \qquad 2$$

where y_m is the lowest point, B is a shift left and right, L is

$$L = \sqrt{\frac{S_m}{g\lambda}} \qquad 3$$

where S_m is the tension at the bottom of the rope, g is the acceleration due to gravity, λ is the linearly density, and k is

$$k = \sqrt{\frac{1}{1+\frac{4L^2}{ym^2}}} \qquad 4$$

However, Arne and Hanno's derivation is not practically useful because it is generally unpractical to measure S_m, the tension at the bottom of the rope. For fitting purposes, the sn function used is given in Equation 5. The A, B, C, D, and E are variables that affect the shape, change the scaling, or shift the position.

$$y(x) = A \times sn(B(x - C), D) + E \qquad 5$$

For a complete derivation of this sn model based on centripetal forces see Arne and Hanno's article.

3. Empirical High Speed Video Recording of Jump Roping

In order to change the shape of the rope from a "V" shape to a "U" shape, an empirical test was conducted. The testing used high speed video footage of a jump rope before and after the addition of weights to compare their shapes.

3.1 The Jump Rope

The jump rope was a Valeo Deluxe Speed Rope purchased from Amazon.com. The jump rope is made of rubber and comes with bearings. See Figure 4.

Figure 4: The Valeo jump rope.[5] Made from rubber, the jump rope had slight springback and tight packaging caused some bends along the length of the rope. The rubber rope stretched some, but for all analysis, this minor extension and the springback are neglected.

Preliminary testing showed that it can be difficult to maintain a constant distance between the hands while jump roping. To control this variable, the rope was attached to two ends of a metal rod. The bearings of the jump roped were crimped with the lip of the metal rod as shown in Figure 5.

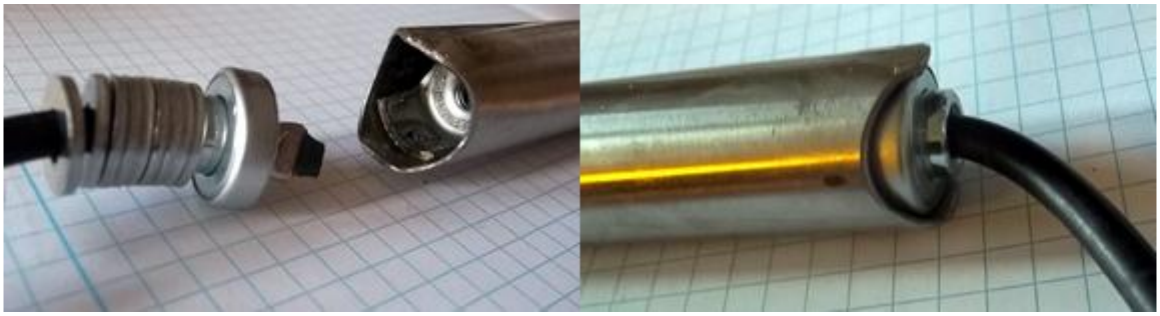

Figure 5: (Left) A view of the end of the metal pipe and the bearing that went inside. (Right) The bearing is inserted and crimped into place. Notice that the wire inside the tube is very short and the bearing is crimped into place by bending the metal lip of the pipe. This configuration held the bearing securely in place while allowing free rotation of the wire.

To add weight to the jump rope, washers (as shown in Figure 6 Left) were added to the jump rope. The washers were quarter inch flat zinc plated washers. 40 of the thicker washers were selected and then put on the rope. During testing, the washers were secured in place using Gardner Bender 4 inch 18 lb. cable ties that had extra length cut off. 10 washers were placed 1 foot of jump rope length away from the center on each side. The other 20 washers were kept at the top and were not used.

As initial testing showed, it was imperative to add clear markers along the length of the rope. The markers allow for an optimal comparison of the rope's shape with and without weights. The markers were bright pieces of colored papers secured with clear tape. See Figure 6 Right. The markers were aligned every 15.24 cm (6 in) with a Tekton tape measure with 0.16 cm (.0625 in) precision. This task was compromised due to the stretching of the rope and slight bending, but care was taken to try to ensure even spacing along the rope with minimal tension. A total of 11 markers were placed: one in the center and five to each side. The first step to adding markers along the rope was finding the midpoint. The midpoint was determined by hanging the rope with the metal beam such that the beam was level as shown in Figure 7.

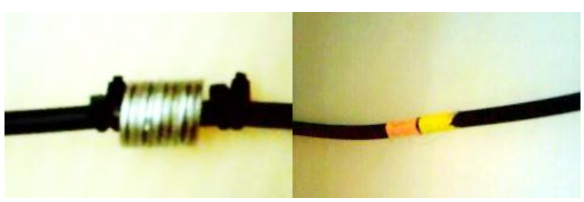

Figure 6: (Left) The weights made of ten washers secured with two clipped twist ties on each side. (Right) The markers attached to the rope.

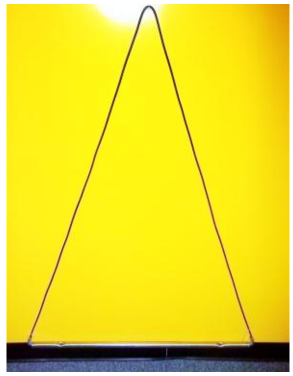

Figure 7: The midpoint of the cord is determined based up leveling the metal beam as it hung from the rope hung over a small pushpin.

3.2 The Empirical Test

The empirical test used video analysis to confirm there is a significant difference in the rope's curvature when weights are added to the jump rope. To capture the video, a Casio Exlim EXFC100 Camera was used to capture 240 frames per second with a resolution of 360 by 480 pixels. The longer dimension was orientated vertically for the experiment.

A number of different settings were tested for the optimal location to conduct the testing. The preliminary tests took place in an MIT classroom, an indoor stairwell (with a high ceiling), and a conference room. Unfortunately, none of these locations provided sufficient lighting for optimal video recording. In the end, tests had to be conducted outside in sunlight.

The camera rested upon a tripod raised to slightly above knee level. The camera was positioned over 10 feet from where the rope was jumped and the zoom feature was used to utilize the most pixels possible. The distance positioned away is important because the video analysis is only valid for small angles of capture. Two representative frames from the video footage are shown in Figure 8.

To help maintain a constant rate of rotation, a song with a constant tempo was played. The song chosen was *Turn It Up* by Grits featuring Scientist and Jade. This song has a tempo of 130 beats per minute. However, with the outside environment, it was difficult to hear the music, and so the rate of jumping varies slightly. The non-weighted case averaged 138 jumps/min and the weighted case averaged 150 jumps/min.

The points of the markers were recorded in Logger Pro's video analysis. In addition to the 11 markers, the endpoints of the metal beam (only used for centering) and the top tangent edges of the rope (used in the fitting) were recorded.

4. Results and Discussion

The first result was a change in the rope's shape with the addition of weights. Further discussion about the true curvature of an unweighted rope is then discussed.

4.1 The Empirical Change in the Rope's Shape from Weights

As shown in Figure 8, the weighted distribution empirically shows a significant change in the curvature of the rope. The non-weighted distribution looks much more like a "V;" whereas the weighted distribution looks much more like a "U." The addition of weights brought the center point up 3.3 ± 2.7 cm and the weights spread the distance of the 60.93 cm (2 foot) center span of rope length an extra 8.7 ± 2.0 cm horizontally. It can be confidently (statistically) said that the rope was flatter and there was a wider foot space. Note that these distances can have large implications because there are normally only a few centimeters between the foot and the rope. Thus the change in the rope's shape has great practical significance.

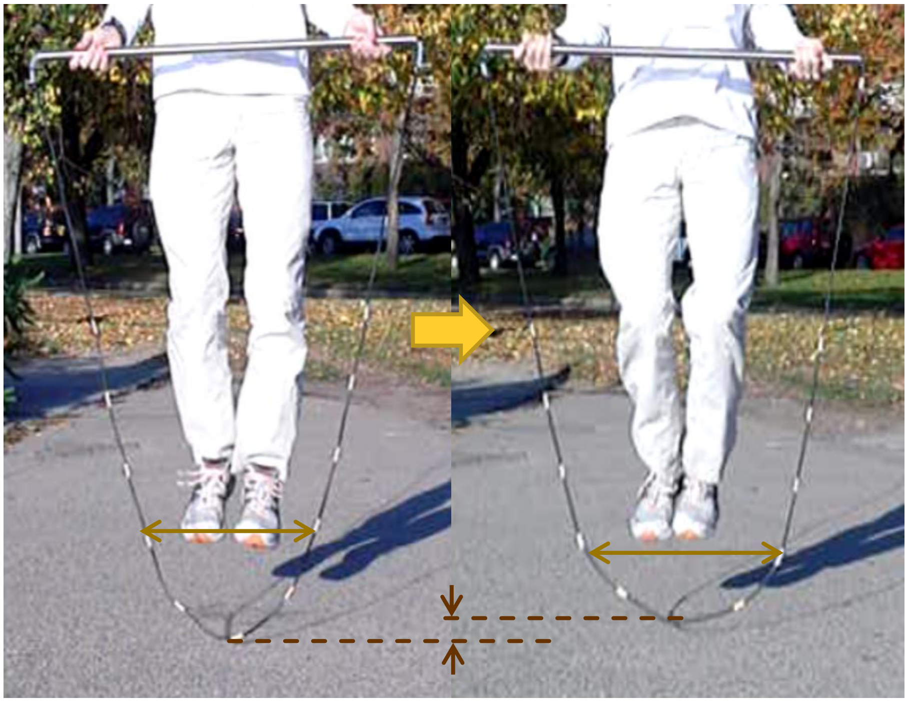

Figure 8: (Left) A representative frame without weights. (Right) A representative frame with weights. Both frames are zoomed in slightly and centered for comparison. Notice how there is a visibly significant difference in the curvature of the rope at the bottom between the two cases, with a moderately pronounced kink in the weighted case. The weighted distribution is more ideal.

Both of the frames in Figure 8 had to be scaled and shifted to make a valid comparison due to the jumper's position. To compensate for the variation in jumper position, the data is rescaled and then repositioned. The data is rescaled about an origin near the midpoint of the metal beam. The vertical position of the origin is the midpoint in-between the ends of the metal beam. The horizontal position of the origin is based upon the average horizontal position of all the points. However, as Figure 8 displays, the center marker fell off when using weights. Therefore, all other points were used to position the data horizontally, and the "center mark" weighted data is a best approximation of its true location. The data is all scaled using the length of the metal beam which is 89.54 ± .15 cm. The unweighted and weighted data sets are plotted in Figure 9.

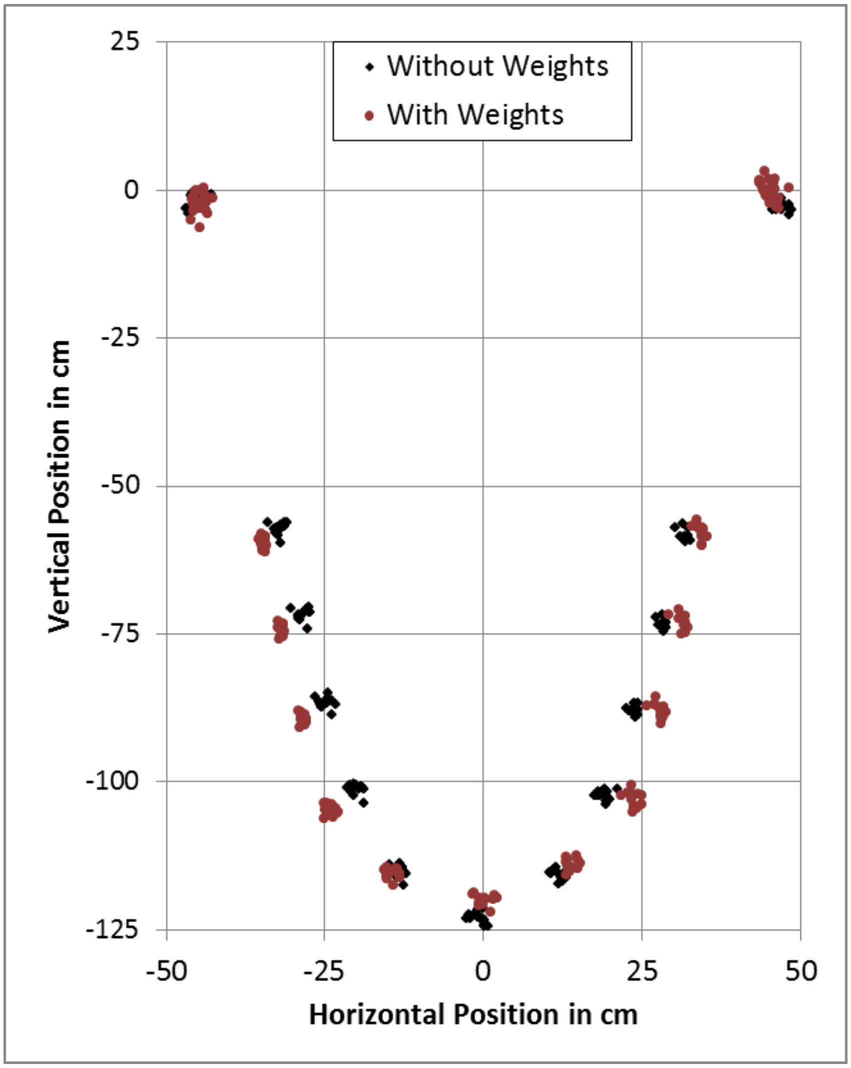

Figure 9: A plot of all data points showing the curvature of the jump rope without and with weights. The data is centered horizontally, shifted vertically about the center of the metal rod (not shown), and properly scaled. The weights are placed two clusters away from the center. Notice how weights cause the jump rope to be flatter at the bottom and have slightly less depth. There are 13 frames for without weights and 13 frames for with weights.

To appreciate the significance of the weights, it is important to know the masses of the weights and the relative density of the rope. After testing, the masses of the washers with the twist ties weighed 0.208 ± 0.014 N and 0.194 ± 0.014 N for each side. This was measured using a Vernier Dual-Range Force Sensor.

The length of the rope was 276.86 ± 0.64 cm. Thus, the overall linear density of the rope (with the practically negligible markers) had a linear force of 0.125 ± 0.003N per 30.48cm (1 foot). This means that overall the weights combined were only about 160% as heavy than the rope between them. One could thus make the claim that this rather drastic effect did not take much more weight than what would normally be used for a jump rope. In fact, decreasing the density of the rope between the weights would let the weights have less mass and yet still have similar effects.

Although there are many sources of variation, the variation can be reasonably approximated as part of the natural variation in the data points. Sources of variation include scaling, which is different for each frame. Plus there was quantization previously when clicking pixels. Additionally, sometimes the metal beam was not perfectly level. Also there was difficulty hearing the tempo at which to jump, so the non-weighted case averaged 138 jumps/min and the weighted case was at 150 jumps/min. Rather than try to calculate the uncertainty and weight each data point, it is simpler to just view the final data set with all the variation combined. The above uncertainty in the measurements is based using the final scaled and shifted data.

4.2 Comparison of Unweighted Jump Rope Shape to Models

The second goal of this experiment was to test and analyze the validity of two models for the curvature of a swinging rope: one based purely on gravitational forces and the other based purely on centripetal forces. In Figure 10, the unweighted data are plotted and the two models are compared. The gravitational model is a catenary from Equation 1. The centripetal model is a sinus amplitudinis (sn) function from Equation 5.

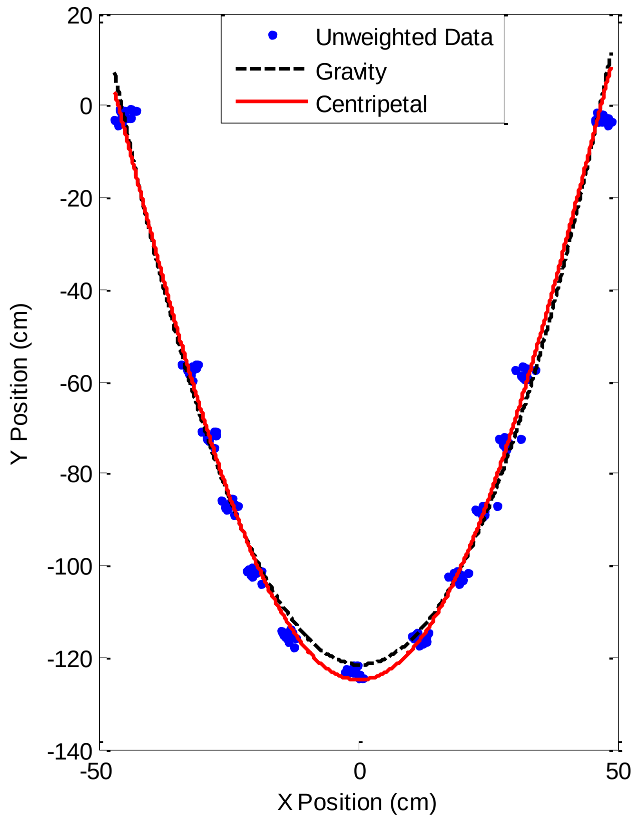

Figure 10: The unweighted data with both the gravitational and centripetal model fitted to the data. The centripetal model is best overall. The "U" shape of the gravitational model accounts for the slightly more bowed out curve at the bottom of the rope that the centripetal model does not account for. Notable is also the slight negative curvature at the top of the rope due to transitional effects for which neither model accounts.

As Figure 10 shows, overall the centripetal model is better than the gravitational model. The centripetal model most closely exhibits the characteristic "V" shape of a swinging jump rope; the gravitational model is too bowed out as a "U" shape to properly fit the data.

However, the true shape of the rope is better represented by a combination of the models. The residuals from the catenary model show that at the bottom, the catenary model is too much like a V shape as gravity helps to shape the rope with slightly less curvature at the bottom. An improved model would account for both gravity and centripetal forces.

However, neither model nor the combination of the models accurately captures the full dynamics of the rope. This is because the rope is never in equilibrium as the direction of gravity changes relative to the rope's spinning reference frame. The effective change in the direction of gravity causes transitional effects. In particular, the transitional effects include negative curvature near the top of the rope. This negative curvature is visible in Figure 10 and slightly on Figure 8 Left. The video footage in slow motion shows this transitional effect more clearly.

5. Conclusion

The addition of small weights significantly improved the curvature of the jump rope. With high speed video capturing, the shape of the jump rope was recorded for both without weights and with weights. The distribution showed that the addition of weights helped to provide a wider region for the feet and flatten the curvature at the bottom. The addition of weights brought the center point up 3.3 ± 2.7 cm and the weights spread the distance of the 60.93 cm (2 foot) center span of rope length an extra 8.7 ± 2.0 cm horizontally.

The unweighted data were compared to two models: one based on gravity and one based on centripetal forces. Overall, the centripetal force model was best, but the gravitational model accounted for the more bowed shape at the bottom of the rope. Neither model accounts for the negative curvature observed near the handles that are due to transitional effects.

Future research should include empirical studies of the effects of the addition of weights for users. It would be desirable to know quantitatively how much the addition of weights changes the frequency that jump ropes gets caught on jumpers' feet and if jumpers stay more engaged.

Acknowledgments

The author Jonathan Abbott would like to thank Prof. Azadi for suggestions on the experiment, Dr. Hughey for help calculating uncertainty, and Professor Leonard for suggesting relevant articles.

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