Online Problems in Routing and Scheduling

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1 Introduction

An online problem is one that must be "solved" without knowing the future or without having complete information. One common approach to evaluating strategies for such problems is to assume a specific stochastic model of the unknown and use various probabilistic tools. Another approach, sometimes called competitive analysis, and related to min-max strategies in game theory, evaluates strategies under worst-case scenarios. In this talk, after reviewing the state of the art for online problems in routing and scheduling problems, we present recent results on online traveling salesman problems, online traveling repairman problems, and other variants. Let us briefly describe here results associated with the online traveling salesman problems as contained in Jaillet and Wagner [6].

2 Online TSP

The Traveling Salesman Problem (TSP) is one of the most intensely studied problems in optimization. In one of its simplest forms we are given a metric space and a set of points in the space, representing cities. Given an origin city, the task is to find a tour of minimum total length, beginning and ending at the origin, that traverses each city at least once. A vast number of different versions of this problem have been studied.

The assumption that the instance is completely known a priori is unrealistic in many real world situations. Consider online versions of the TSP where the instance is not revealed all at once. There are many different frameworks for revealing problem instances incrementally. Let us describe two different situations. In the first, new cities are discovered as the salesman reaches a given city. These revealed cities could be, for example, the neighbors of the last visited city or they could be some other location independent set. The offline version (where the entire instance is known a priori) would be the classical TSP. In the second framework, the cities are revealed dynamically over time, independent of the salesman’s location. The natural offline problem in that case would be the TSP with release dates and will be the one we will consider in this talk.

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2.1 Literature Review

The literature for the TSP is vast. The interested reader is referred to the books by Lawler et al. [11] and Korte and Vygen [9] for comprehensive coverage of results concerning the classical TSP. Probabilistic versions of the TSP, where a different approach is used to represent limited knowledge of the problem instance, have also attracted interest (e.g., see the works by Jaillet [5] and Bertsimas [3]).

A systematic study of online algorithms was given by Sleator and Tarjan [13], who suggested comparing an online algorithm with an optimal offline algorithm. Karlin et al. [8] introduced the notion of a competitive ratio. More details related to these works will be presented in the next section. Online algorithms have received much attention in the computer science community. These algorithms have been used to analyze paging in computer memory systems, distributed data management, navigation problems in robotics and multiprocessor scheduling (see Albers [1] for some details and references).

Research concerning online versions of the TSP have been introduced recently. Kalyanasundaram and Pruhs [7] examined the case where new cities are revealed locally during the traversal of a tour (i.e., an arrival at a city reveals any adjacent cities that must also be visited). Most related to our paper is the stream of works which started with the paper by Ausiello et al. [2]. In this paper, the authors studied the version in which cities are revealed dynamically over time (not necessarily adjacent to the salesman’s current location); they analyzed this version of the online TSP on the real line and on general metric spaces, developing online algorithms for both cases and achieving an optimal online algorithm for general metric spaces (with a competitive ratio of 2). Lipmann [12] developed an optimal online algorithm for the real line (with a competitive ratio of approximately 1.64). Analyses have also been performed by Blom et al. [4] and Krumke et al. [10]. Blom et al. [4] gave an optimal online algorithm for the nonnegative real line (with a competitive ratio of $\frac{3}{2}$). All of the aforementioned works only consider the case where a revealed city is ready for immediate service; i.e., all the disclosure dates equal their respective release dates. We will specifically relax this assumption and design optimal algorithms which also performs well on average.

2.2 Contributions

All of the aforementioned works only consider the case where a revealed city is ready for immediate service; i.e., all the disclosure dates equal their respective release dates. We will specifically relax this assumption and design optimal algorithms which also performs well on average. Cities are revealed dynamically over time at their disclosure dates. These disclosure dates provide a bridge between the classical online problem considered in the literature and the offline problem – when all the disclosure dates are zero, we have the offline problem; when all the disclosure dates are equal to their respective release dates, we have the online problem considered in the literature.

We give an optimal online algorithm for this generalized problem and give evidence, both theoretical and computational, that it outperforms other optimal online algorithms, on average. We also consider the situation where all cities receive the same amount of advance notice; i.e., the difference between a given city’s release date and disclosure date is the same for all cities.

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In this case, we show that our new algorithm is optimal and that an existing algorithm is not. We also provide a refined online analysis of this algorithm. This analysis gives new insights into the structure of the best problem instances (i.e., instances that induce an online cost that is not much larger than the offline cost) for our generalized problem as well as for the problem already considered in the literature.

In a second part, we prove some convergence results under some very general probabilistic assumptions. These results state that the online costs of the algorithms considered in the first part of the paper converge to the optimal offline cost, in probability, as the number of cities $n \to \infty$. We extend these results to other previously proposed online algorithms that solve online Traveling Salesman Problems on the real line as well as $\mathbb{R}^d$, for any $d \geq 2$. We conclude with computational studies that perform two tasks for the online TSP version where the underlying metric space is the non-negative real line: (1) show the above convergence computationally and (2) show that our new algorithm outperforms an existing optimal algorithm, on average.

3 Other routing problems

We will also present results on variants of the Online TSP such as the Online Traveling Repairman Problems. In all cases we consider the fundamental question of the value of advance information in designing and analyzing online competitive algorithms.

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References


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