Superdiffusive Dispersion and Mixing of Swarms with Reactive Levy Walks

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***Abstract*—A common swarm task is to disperse evenly through an environment from an initial tightly packed formation. Due to communication and sensing limitations, it is often necessary to execute this task with little or no communication between swarm members. Prior approaches based on repulsive forces or uniform random walks motion, unfortunately, converge slowly or pass through bad transient states before converging. A simple integrate-and-fire mechanism, however, can generate modified Levy flights, thereby producing a near-optimal rapid and monotonic dispersion with no communication. This mechanism also produces rapid mixing of swarms and is a plausibly evolvable generator for the Levy flight behaviors observed in biological organisms.**

***Index Terms*—spatial computing; swarm; dispersion; deployment; coverage; mixing; Levy flight; Levy walk; anomalous diffusion**

I. Introduction

Dispersion is one of the basic maneuvers needed for a wide variety of swarm applications: beginning from an initially tightly packed formation, the individuals comprising a swarm spread out evenly through their environment. Moreover, due to communication and sensing limitations, it is often necessary to execute this task with little or no communication between swarm members.

To be effective in many applications, swarm dispersion must balance between two contradictory goals: on the one hand, swarm individuals need to spread outward quickly in order to disperse well. At the same time, however, it is often the case that the swarm needs to at the same time maintain a continuous coverage over its initial deployment area. For example, if the swarm is being used for information gathering or for communication then deploying the swarm should not leave any large transient gaps in covering the immediate area around the people deploying it.

Prior methods of low-communication dispersion have generally been based on simple physics models, such as repulsive forces or diffusion through uniform random walks. These methods, however, are unable to spread quickly unless carefully tuned for the expected scale of the swarm and its environment.

Another physical model, however, offers a possible solution to this problem: anomalous diffusion, such as is observed in certain forms of plasmas and turbulent flows (e.g., [1], [2]), operates much more quickly than ordinary diffusion. In fact, particles in certain conditions move according to random processes with heavy-tailed probability distributions, such that they are guaranteed to cover space quickly, yet still have a high probability of remaining close to their origin. These processes—Levy flights and Levy walks—have previously been applied to balancing explore/exploit trade-offs in search, both for design of engineered systems (e.g., [3], [4]) and for modeling the behavior of biological organisms (e.g., [5], [6]).

This paper develops a new process variant, reactive Levy walks, and apply this method to the problems of swarm dispersion and mixing. Reactive Levy walks are asymptotically faster than prior approaches and provide both speed and continuous central coverage, while requiring minimal sensing and no communication between swarm individuals. We will also see that this process can be realized as a parallel integrate-and-fire mechanism, which is suitable for simple electronic or biological implementations and is incrementally evolvable.

A. Related Work

Levy flights and Levy walks are scale-free particle motion processes originally formulated as models in the study of chaotic physical phenomena [7] (precise definitions will be given in the next section). Although these two random processes are distinct, they are often frequently used and referred to interchangeably in the literature (this sort of imprecise thinking can lead to significant problems, as in the recent challenges to some analyses of Levy motions by animals [8], [9], [10]).

Levy motions have since been applied to modeling a number of physical processes, including diffusion under turbulence [2], the passage of photons through hot gases [11], and plasma physics [1]. More recently, it has been proposed that animals use Levy walks in their foraging patterns [5], [6], and evidence of such behavior has been reported for an extremely wide range of organisms, from amoebas [6], to bumblebees, deer, and albatross [5], from predatory fish, turtles, and penguins [12], [13], to spider monkeys [14]. Even humans appear to evidence Levy statistics in our movements [15], [16].

Turning from modeling to motion planning, Levy motions have been used for generating search patterns for robots or other agents a number of times [3], [4], [17], as well as for routing [18]. These systems have all considered small numbers of agents in sparse environments, a problem with significantly different constraints and requirements than dispersion.

With regards to swarm dispersion (or coverage, which is closely related), much of the prior work has considered systems out of scope of our current investigation due to reliance on high levels of communication, centralized planning or ability to leave markers such as pheromones in their shared (real or virtual) environment. More local and self-organizing approaches typically fall into two categories of nature-inspired models. One set are primarily based on uniform random walks, either unbiased or biased (e.g., [19], [20], [21]). The other, more common strategy uses repulsive forces in a variety of combinations and models such as flocking (e.g., [22]), potential fields (e.g., [23]), and gradient descent (e.g., [24]). There are also a number of modified or hybrid strategies, such as combining biased random walks and diffusion-limited aggregation [25] or springs and random walks [26]. Asymptotic analyses, however, reveal that both uniform random walks [27] and force-driven dispersion (seen as a distributed consensus process [28]), will generally perform poorly for large swarms.

II. Reactive Levy Walks

A Levy flight [7] is a random movement process similar to a random walk: a particle makes a sequence of moves, where each move is in a random direction. Unlike a random walk, however, where the lengths of the moves are identical, Levy flight moves have a random length generated from a heavy-tailed probability distribution, such that the probability of moving a distance of d is:

$$p(d \geq l) \propto l^{-1} \quad (1)$$

Each move of a Levy flight thus moves an unbounded expected distance, modeling a scale-free *superdiffusive* motion of particles—that is, where particles have an expected displacement over time much further than predicted by uniform random-walk models of diffusion.

Levy walks are Levy flights where the particle moves at a constant velocity. This maintains the scale-free property of distribution while restricting to more physically realizable motions. This model (and its generalization to coupled continuous-time random walks [29], [30], which encompasses both Levy walks and uniform random walks, as well as a spectrum of other related processes) is used to model a number of natural phenomena, as described in the previous section.

To apply this to the problem of swarm dispersion, let us introduce a new process, which modifies Levy walks with the inclusion of reactivity to the environment. Members of a dispersing swarm operate (at least initially) in a highly constrained environment, where each member's movements are obstructed by the other nearby members of the swarm. Thus, individuals moving in accordance with a pure Levy walk may enter a mutually blocking configuration for unboundedly long periods of time. The same problem occurs when an individual encounters an obstacle in the environment. This problem can be alleviated by assuming that each individual has a proximity sensor that can detect the close presence of swarm members or other obstacles (e.g., in robots, this could be bump sensors, sonar ranging, LIDAR, etc.). A reactive Levy walk is then identical to an ordinary Levy walk, except that the current move is aborted and a new random move started whenever the proximity sensor is triggered.

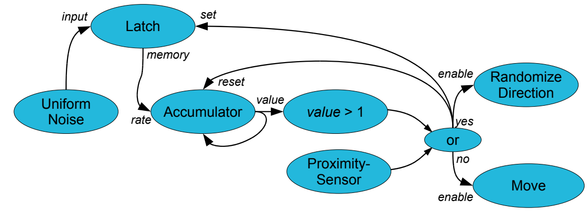


Fig. 1. Reactive Levy walk implemented as a parallel circuit that can be easily implemented as an electronic, neural, or biochemical network.

Algorithm 1: Reactive Levy Walk

repeat
  // Choose a Random Motion
  $\hat{i}$:= **random-direction()**
  $\alpha := U[0, 1]$ // Unit uniform random number
  accumulator := 0
  // Move until threshold is reached
  repeat every Δt **seconds**
    move($v\hat{i}$)
    accumulator := *accumulator* + $\alpha \cdot \Delta t$
  until *accumulator* $\geq T$ **or proximity-sensor()**

Pseudocode for a reactive Levy walk is given in Algorithm 1. In addition to adding the reactivity term, this formulation also specifies the scale-free distribution using a variable-rate accumulator rather than the standard probability computation given in Equation 1. A particle executing a reactive Levy walk begins by selecting a random direction $\hat{i}$, a random rate of accumulation α uniformly between 0 and 1, and setting its accumulator to zero. While the accumulator value rises at a rate of α per second (quantized in steps of Δt in this implementation), the particle moves in direction $\hat{i}$ at velocity v. The particle resets its accumulator and selects a new direction and rate whenever either its accumulator reaches T (taking α^{-1} seconds) or its proximity sensor is triggered. Note that, as for standard Levy walks, changing the parameters of a reactive Levy walk affects its behavior by only a constant factor and preserve the scale-free property so long as $v > 0$ and $T > 0$.

Neglecting the reactivity term, this inverted formulation of Levy walking has equivalent statistics to a standard Levy walk, but can be implemented with a simple parallel circuit, as shown in Figure 1, that can be easily realized using either digital or analog electronics or a neural or biochemical network. We will discuss the implications of this circuit implementation further in Section VI.

When executed by the members of a tightly packed swarm, we may predict that a reactive Levy walk will produce different behaviors on the edges and in the interior. On the edges, individuals are not significantly constrained by proximity, and are able to move superdiffusively, rapidly dispersing the swarm. In the interior, where individuals are constantly in close proximity to one another, the reactive Levy walk reduces to a constrained random walk. Individuals in this region move at best diffusively, effectively marking time while they wait to the dispersion of the edges to allow them to move more freely.

Similarly, reactive Levy walks allow a dispersing swarm to cope more gracefully with obstacles and boundaries. Without reactivity, individuals encountering an obstacle or boundary can become stuck indefinitely before choosing a new direction; with reactivity they change direction as soon as a limit of motion is detected.

III. Asymptotic Levy Walk Dispersion

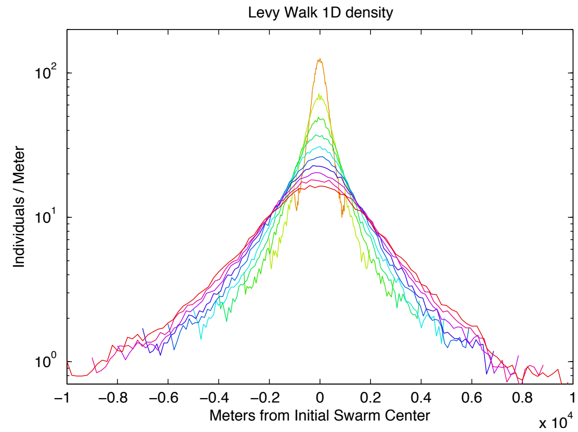
In comparing reactive Levy walks against prior methods of dispersion, let us begin by considering the asymptotic case, where the swarm is dispersing through a very large environment with little constraint. In this case, the reactivity term plays little role, and we can approximate behavior with a pure Levy walk.

A. One-Dimensional Levy Walks

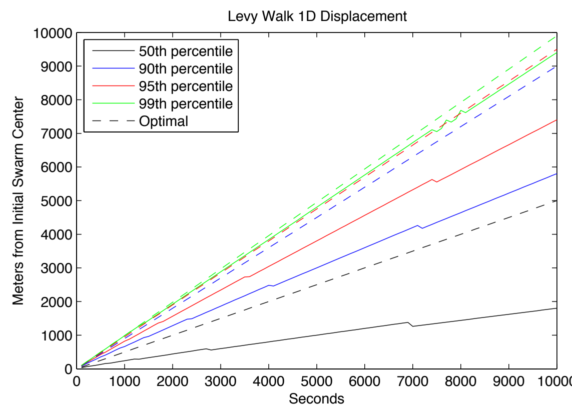
Let us begin by considering the evolution of a swarm's distribution with a one-dimensional Levy walk. We determine this empirically with 100 trials simulating the evolution of a swarm with 1,000 individuals for 10,000 seconds, moving with a speed of $v = 1$ meter per second, a threshold of $T = 1$, and time quantization $\Delta t = 1$ second with random initial phase. Each swarm is initially uniformly randomly distributed over a 10 meter interval, and the positions of the swarm members recorded every 100 simulated seconds.

Figure 2 shows the evolution of the swarm's distribution over time. While the density of the swarm remains highest at the center, it spreads rapidly outward over time: Figure 2(a) shows the evolving density of individuals per meter over time, with colors indicating time ranging linearly from $t = 1,000$ (orange) to $t = 10,000$ seconds (red). If we consider the absolute displacement of individuals from the initial mean, another way to measure the spread of a swarm over time is by tracking the displacement of the nth percentile distance. Figure 2(b) shows that the swarm disperses at a linear rate. The central portions of the distribution disperse more slowly than the edges, but all are k-competitive with an optimal dispersion in which the swarm spreads perfectly evenly outward through space at rate v, with its distribution forming a perfect disc.

From the behavior of a swarm dispersion via Levy walks in one dimension, we can predict swarm dispersion in two or more dimensions. With an isotropic random choice of direction and a constant velocity, the distribution of a dispersing swarm must be symmetric and should follow the same overall form, but with rate of dispersion in along any given axis decreased by a constant factor depending on the number of dimensions.



(a) Density



(b) Displacement

Fig. 2. A swarm dispersing in one dimension via Levy walks spreads rapidly through space (a), dispersing at a linear rate as indicated by tracking the magnitude of the nth percentile displacement from the initial mean (b). Colors in (a) indicate time, ranging linearly from $t = 1,000$ (orange) to $t = 10,000$ seconds (red).

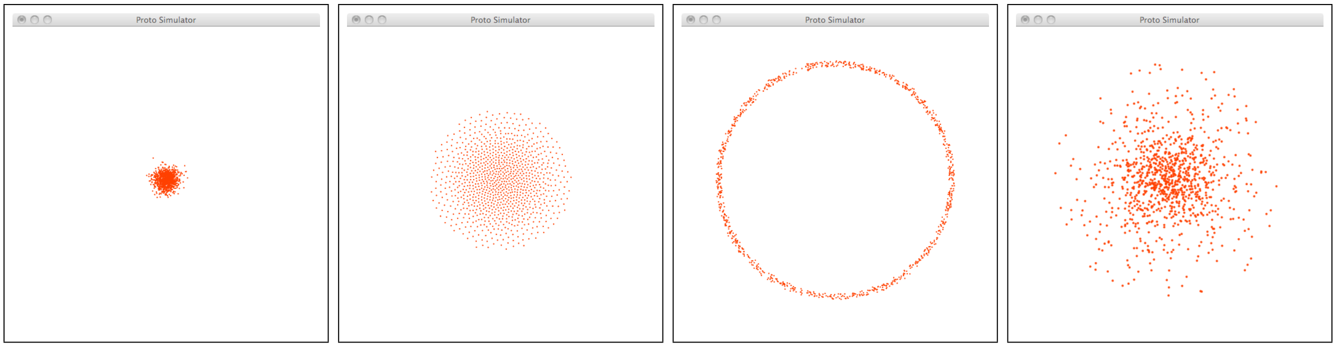
B. Two-Dimensional Dispersion

We will compare reactice Levy walks against two representative prior methods of dispersion—random walk and repulsive forces—and a simple reactive strategy of turning upon contact. In particular, we implement these as follows:

- Random walk moves at v meters per second in a random direction, selecting a new random direction every Δt seconds.
- Repulsive forces applies a force inversely proportional to the distance d_{ij} from each particle i to its neighbors $j \in nbrs(i)$, where the neighbors are the set of individuals up to r meters away:

$$v(i) = k \sum_{j \in nbrs(i)} d_{ij}^{-1} \tag{2}$$

  Note that although the combination of forces and exponents on various repulsive force approaches in the literature varies, the asymptotic behavior will be similar.
- Turn-on-contact is the simplest possible reactive strategy: it selects a random direction and moves at v meters



(a) Random Walk (b) Repulsive Forces (c) Turn on Contact (d) Levy Walk

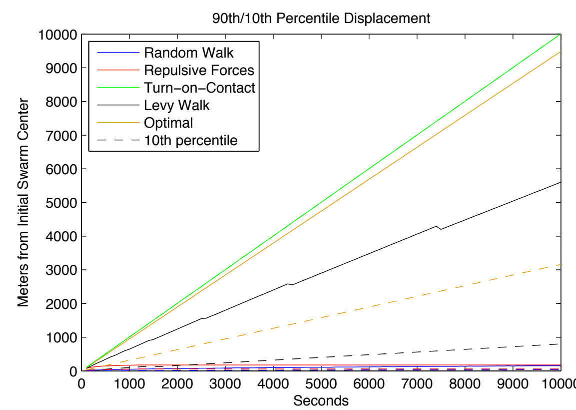
Fig. 3. Snapshots of swarms of 1000 individuals after $t = 200$ seconds of dispersion from the center of a 500x500 meter region via random walk (a), repulsive forces (b), turn-on-contact (c), and Levy walk (d).

per second in that direction until its proximity sensor is triggered, at which point it selects a new random direction of travel.

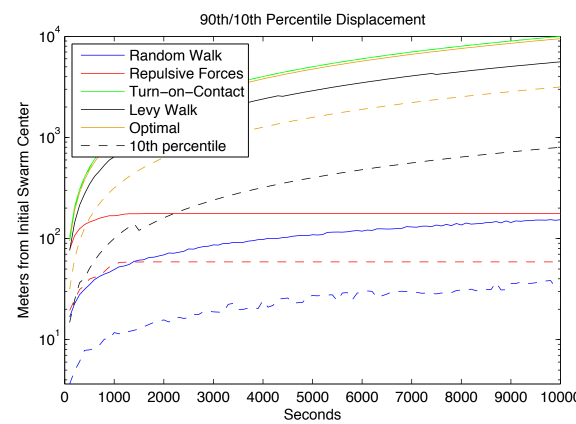
To compare these four methods, we consider the trajectories of swarms of 1000 individuals dispersing in two dimensions. Figure 3 shows examples of these four methods operating with $v = 1$, $\Delta t = 1$, and repulsive force $k = 0.05$ and $r = 10$, after dispersing for $t = 100$ seconds from an initial uniform random distribution across a 10-meter square. Even after such a short period of time, the contrast between the various approaches is stark: random walk has barely begun to disperse, while turn-on-contact has moved outward the maximum distance possible, leaving the center entirely empty. Only repulsive forces comes close to matching the even dispersion of Levy walk, and that method is already rapidly decelerating its dispersion as repulsive forces decrease on the individuals at the edge of the swarm.

For a more systematic comparison, we consider the trajectories of 100 trials of each method on a swarm of 1000 individuals dispersing for 10,000 seconds with the same parameters and initial distribution, with the positions of the swarm members recorded every 100 simulated seconds. Figure 4 compares the behavior of the four methods, as well as the optimal dispersion of a perfectly even swarm density spreading outward in a disc at rate v. The trends are the same as in the examples in Figure 3: only Levy walks provide a rapid and even dispersion over space. Random walk disperses slowly, at a rate proportional to $\sqrt{t}$, and repulsive forces follow the same pattern before stopping altogether as the individuals of the swarm reach distance r from one another and lose contact with their neighbors. Turn-on-contact suffers from the opposite problem, moving away from the center too rapidly and leaving it empty (and thus actually displacing its 90th percentile "faster than optimal") until individuals' paths can be randomized by contact with obstacles or the boundary of the space.

Thus, we see that for dispersion in large open regions, a (reactive) Levy walk spreads a swarm evenly through space at a near-optimal rate. By contrast, the other methods produce either much slower or highly uneven dispersion.

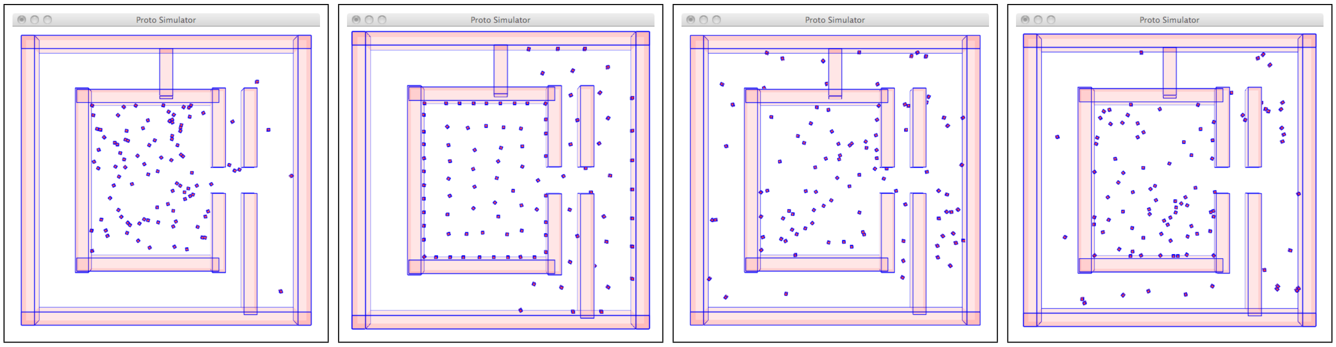


(a) Linear Scale



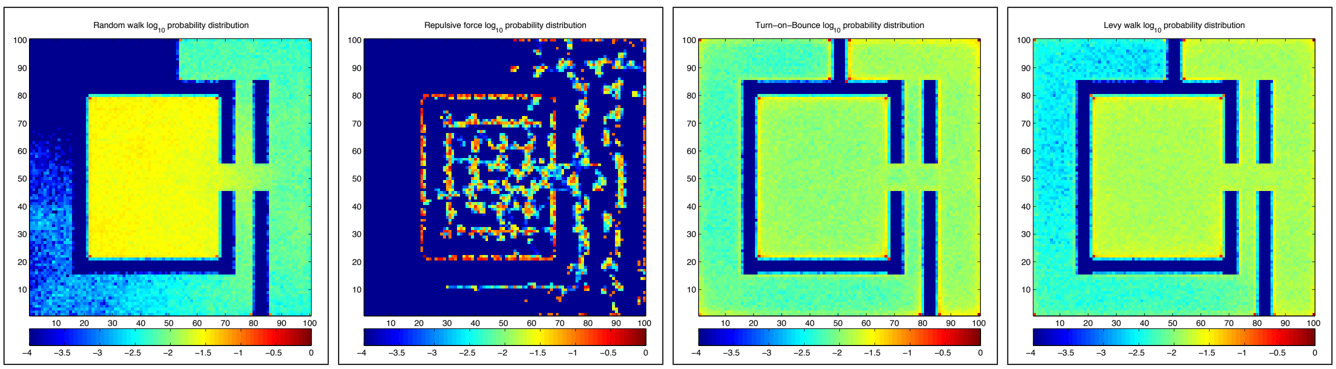
(b) Log Scale

Fig. 4. The 90th percentile of individual x-coordinate displacement disperses at a near-optimal linear rate for Levy flights and for turn-on-contact (whose "faster-than-optimal" line is due to the empty center of the distribution). Repulsive forces begin quickly, but rapidly slow to a stop as neighbor forces decrease; random walk continually expands, but much more slowly than Levy flights. The 10th percentile lines follow a similar trend, except that of turn-on-contact, which is not visible because it precisely overlaps its 90th percentile line, indicating the empty center of the distribution.



(a) Random Walk (b) Repulsive Forces (c) Turn on Contact (d) Reactive Levy Walk

Fig. 5. Snapshots of swarms of 100 individuals after $t = 2000$ seconds of dispersion from the center of a 100x100 meter "maze" of barriers via random walk (a), repulsive forces (b), turn-on-contact (c), and Levy walk (d).



(a) Random Walk (b) Repulsive Forces (c) Turn on Contact (d) Reactive Levy Walk

Fig. 6. Mean density of individuals per square meter from $t = 5,000$ to $t = 10,000$ seconds of dispersion of a swarm of 100 individuals from the center of a 100x100 meter "maze" of barriers via random walk (a), repulsive forces (b), turn-on-contact (c), and Levy walk (d). Warmer colors indicate logarithmically greater density.

IV. Reactivity and Constrained Dispersion

In a highly constrained space, an individual moving as directed by a pure Levy walk will often become stuck for long periods. Adding reactivity simply means that an individual begins a new walk step whenever it encounters an obstacle.

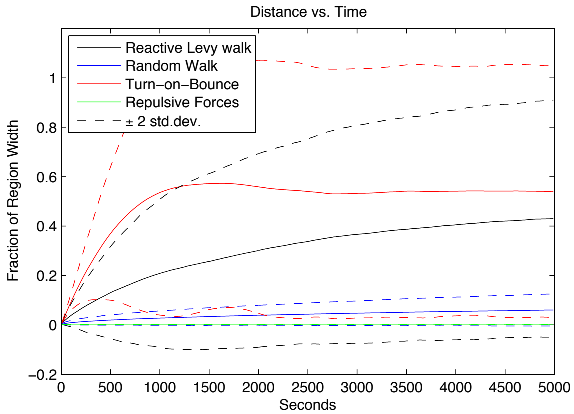
We evaluate the performance of reactive Levy walks in a constrained environment by simulating a swarm of 100 individuals dispersing through a 100-by-100 meter maze-like environment with a large central "room" and narrow "corridors" leading outward and around it (Figure 5). Each individual in the swarm is a 1-meter cube, and the movements and physical interactions of individuals with one another and the maze are simulated in Proto [31], [32] using the ODE [33] Newtonian physics engine. The swarm begins packed as tightly as possible, in physical contact in a 10 meter square. Dispersion is then run for 10,000 seconds using the same methods and parameters as before, 10 trials per method, with the positions of the swarm members recorded every 10 simulated seconds.

The results of this simulation are presented in Figure 5, which shows example snapshots of swarms after $t = 2000$ seconds of dispersion, and Figure 6, which shows the mean density of individuals per square meter over the second half of the simulation, from $t = 5,000$ to $t = 10,000$. As in the asymptotic case, random walk performs poorly, spreading slowly enough that it never actually reaches all parts of the maze. Repulsive forces perform even worse, quickly reaching a stable equilibrium where individuals on the edges no longer move outward, even though many others in the swarm are under significant stress. Both reactive Levy walks and turn-on-contact effectively disperse individuals through the entire maze, with turn-on-contact slightly outperforming reactive Levy walks. Note that in this highly constrained environment, turn-on-contact is much more even in its distribution, since the movements of swarm members are rapidly decorrelated by their frequent encounters with walls.

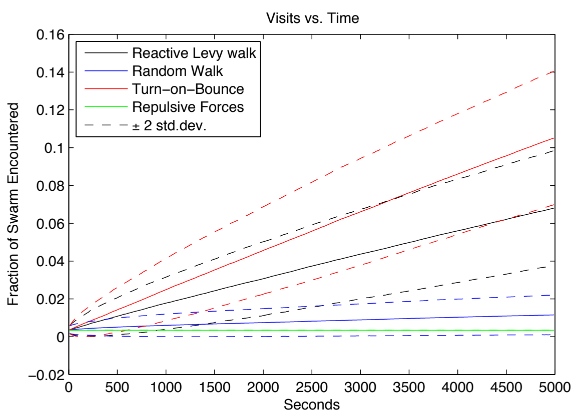
We thus see that, alone of the four methods we consider, reactive Levy walks produce fast and smooth dispersion in both highly constrained and unconstrained environments.

V. Swarm Mixing

Mixing of a swarm is a similar maneuver to dispersion, but starting with the swarm already distributed over space rather than tightly packed. Uses include data ferrying, blending of mission and maintenance tasks, and enhancing robustness through diversity. Dispersion algorithms should generally work well for mixing also, since both dispersion and mixing require individuals to move relatively long distances with low correla-



(a) Distance Displaced



(b) Swarm Visited

Fig. 7. Both reactive Levy walk and turn-on-contact effectively mix a swarm, as shown by measurements of distance displaced (a) and fraction of swarm visited (b) in a swarm of 300 sparsely distributed individuals.

tion in their movements. Based on the results in prior sections, we should thus expect the efficacy of mixing to be good for the reactive Levy walk and turn-on-contact methods, but poor for random walk (where individuals traverse space more slowly) and for repulsive forces (where there is high correlation in movement).

For an empirical test, we consider the same algorithms and parameters as before, but now consider a swarm of 300 individuals beginning distributed uniformly randomly through a 1000-by-1000 meter space with no internal obstacles. Dispersion is then run for 5,000 seconds using the same methods and parameters as before, 10 trials per method, with the positions of the swarm members recorded every 10 simulated seconds.

To evaluate the mixing efficacy of the four methods under consideration, we will use two measures: the displacement of an individual from its initial position, and the number of other individuals that an individual "visits" over time, i.e., that it comes within some threshold d meters of. Figure 7 shows the evolution of the displacement and visit metrics over time for the four methods, using a threshold of $d = 10$ meters for visit proximity. As expected, reactive Levy walk and turn-on-contact both perform well, while random walk and dispersion

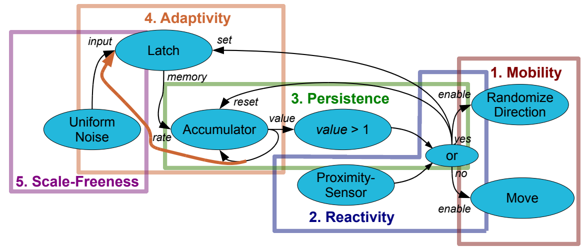


Fig. 8. A circuit implementing a reactive Levy Walk is evolvable through the incremental accumulation of common biological control motifs, each of which is expected to increase the efficacy of the original search or dispersion behavior.

do not.

VI. Evolvability of Reactive Levy Walkers

It has previously been observed that a number of animals move in Levy flight or Levy walk patterns, particularly when hunting or foraging [5]. Examples cross many diverse clades, including amoebas [6], bees [5], penguins [12], sharks [13], and spider monkeys [14]. Several possible biological implementations for generating Levy patterns have previously been proposed: critical connectivity in neural connections [34], positive feedback loops in memory [35], and filtration of fractal noise [36]. All of these prior proposals are rather complex, however, and none has a clear path for evolvability.

When considered as an abstract mathematical process of heavy-tailed number generation, as in the usual formulation, there is indeed no obvious means of biological implementation, let alone evolvability. The implementation of a reactive Levy walk as an adjustable integrate-and-fire circuit, however, as shown in Figure 1, can easily be mapped to a biological implementation as either a neural circuit or a biochemical network (e.g., a genetic regulatory network or protein cascade) by modulating basic mobility capabilities using a collection of standard motifs. Moreover, the inclusion of reactivity increases the evolvability of this system, developing a behavior switch that can later be modulated to create long-range movement.

Figure 8 reproduces the parallel circuit diagram from Figure 1, now decomposed into five common biological motifs and ordered in a possible sequence of assembly by an evolutionary process. Let us consider these in sequence:

1) **Mobility:** We begin with an assumption of two basal motions: forward movement and random change of direction. At a base level, these operate independently and in parallel, producing a random walk. The scale of this random walk is determined by the speed of forward motion and the frequency of direction change.
2) **Reactivity:** The basic motions may become connected by a reactive switch that selects between them, frequently instantiated in biological systems by a biased mutual inhibition motif. In this case, either direction of inhibition may be advantageous and develop first: it is usually better not to waste energy moving when stuck against an obstacle and also often better not to turn when moving is is uninhibited. At this stage, the system moves

over long ranges, but more regularly, implementing the turn-on-contact strategy.

3) **Persistence:** Accumulate-and-fire networks are another common biological motif, often implemented as a linkage between self-activating and self-inhibiting feedback loops. Once reactivity establishes a switch between the two motions, an accumulate-and-fire network can expand the range of exploration by modulating the same signals to produce a controlled mixture of persistent motion and random turning that can be tuned to an optimal ratio by selection.
4) **Adaptivity:** Rate modulation, yet another common motif, adjusts the mixture of motion versus turning by the value of a persistent latch. At this stage, the evolution of the system might be scaffolded by the latch using the value of the accumulator as its input (orange arrow in Figure 8). This results in shorter movements when the proximity sensor is triggered more often, adapting for better exploration of confined spaces.
5) **Scale-Freeness:** Finally, the input for the adaptive latch may be replaced by a random noise source. Noise is an inherent property of biological systems, and cells have previously been shown to exploit this noise [37], [38], [39]. This final stage is advantageous because it at last produces scale-free movement, allowing search or dispersion over arbitrarily large or small areas.

Thus, we see that there is a path by which a reactive Levy walk could evolve by four incrementally adaptive steps, each implemented by a common biological network motif.

This is, of course, only a possible hypothesis for how biological Levy motions may be implemented and have arisen. This hypothesis has two significant arguments in its favor, however: the simplicity of the mechanism and the existence of an incremental path for evolving the full mechanism from a base mobility capability. Furthermore, the relatively simple incremental path for evolvability of this system suggests that if it occurs in nature at all, it is likely to have evolved independently multiple times. Ultimately, however, confirmation or refutation of this hypothesis will require physiological or genetic studies.

VII. Contributions

Reactive Levy walks are an effective method for dispersion and mixing of swarms. This method is much faster than prior approaches and provides both speed and continuous central coverage, while requiring minimal sensing and no communication between swarm individuals. We have also seen that if continuous central coverage is not required, a simple reactive turn-on-contact strategy also provides fast dispersion and mixing. Furthermore, when formulated as an integrate-and-fire mechanism, reactive Levy walks can be implemented with a simple parallel circuit that is both biologically plausible and evolvable.

One key direction for future investigation is tuning of the Levy walk distribution: modification of the exponent or scaling constants will produce a heavy-tailed distribution with different weighting of the mixture of short-range and long-range movements. Another important investigation will be to determine how much various existing applications can be improved by replacing the use of prior less effective dispersion methods. Finally, it remains to be proved whether the integrate-and-fire mechanism and evolvability are borne out in actual natural biological organisms.

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