Laplacian-Based Consensus on Spatial Computers

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AAMAS 2010

BBN TECHNOLOGIES

Laplacian-Based Consensus

- Averaging consensus:

$$x_i(k+1) = x_i(k) + \epsilon \sum_{j \in N_i} w(e_{i,j}) \cdot (x_j(k) - x_i(k))$$

- Many multi-agent applications: flocking, swarming, sensor fusion, formation control, ...
- Fast convergence: $\delta(k) = (1-\varepsilon\lambda_2)^k \, ||\delta(0)||$

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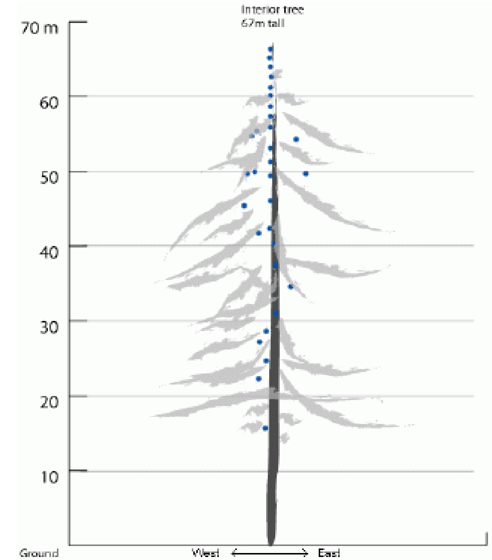
2nd eigenvalue of graph Laplacian
Extremely small on spatial computers

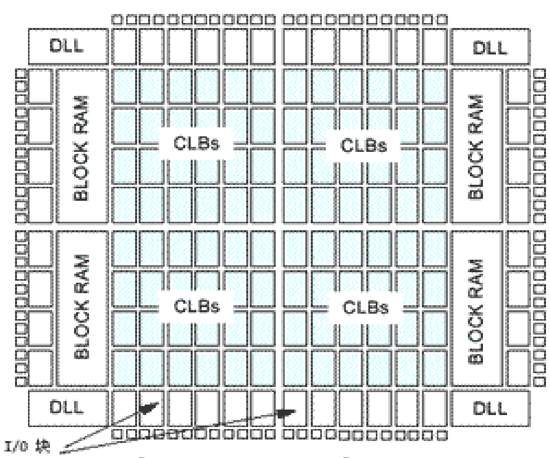
Spatial Computers

Robot Swarms

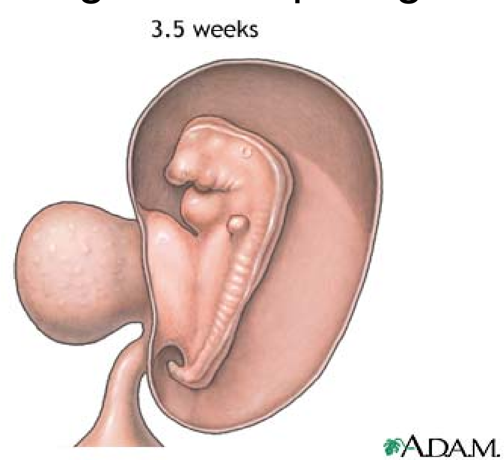
Biological Computing



Sensor Networks



Reconfigurable Computing



Cells during Morphogenesis

Modular Robotics

Spatial Computer: Formal Definition

- Given:
  - Graph $G=\{V,E\}$ of n devices
  - Non-negative weight $w(e_{i,j})$ for each edge
  - Riemannian manifold M with distance function d
  - Mapping $p: V \rightarrow M$ of devices to manifold points
- Spatial computer if $w(e_{i,j}) = O(1/d(p(i),p(j)))$

Analysis: How fast is convergence?

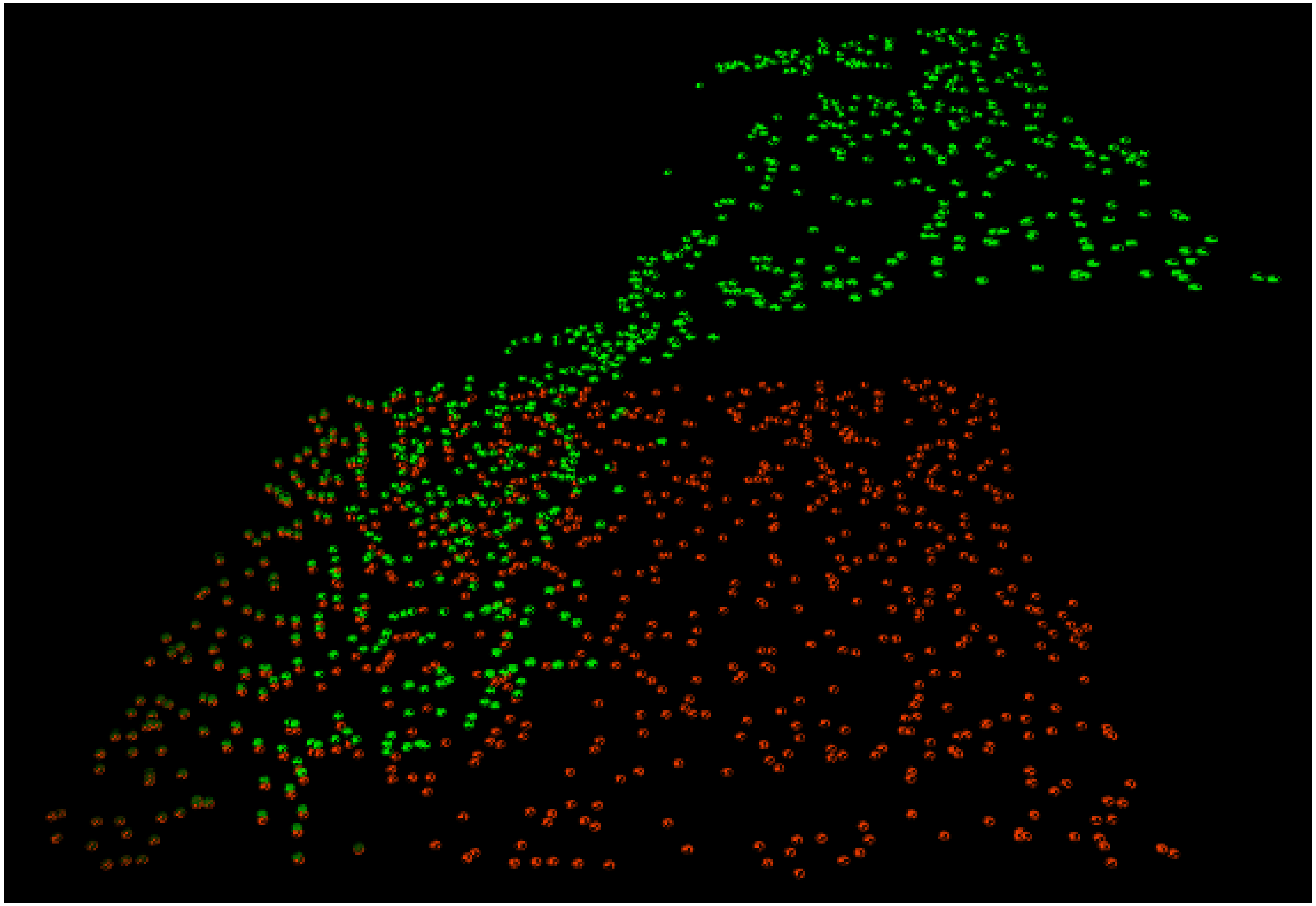
$$\delta(k) = (1-\varepsilon\lambda_2)^k \,||\delta(0)||$$

- Convergence requires: $\epsilon < 1/\Delta$
- Available bounds for λ_2 are very loose:

$$4/n \cdot diam \leq \lambda_2$$

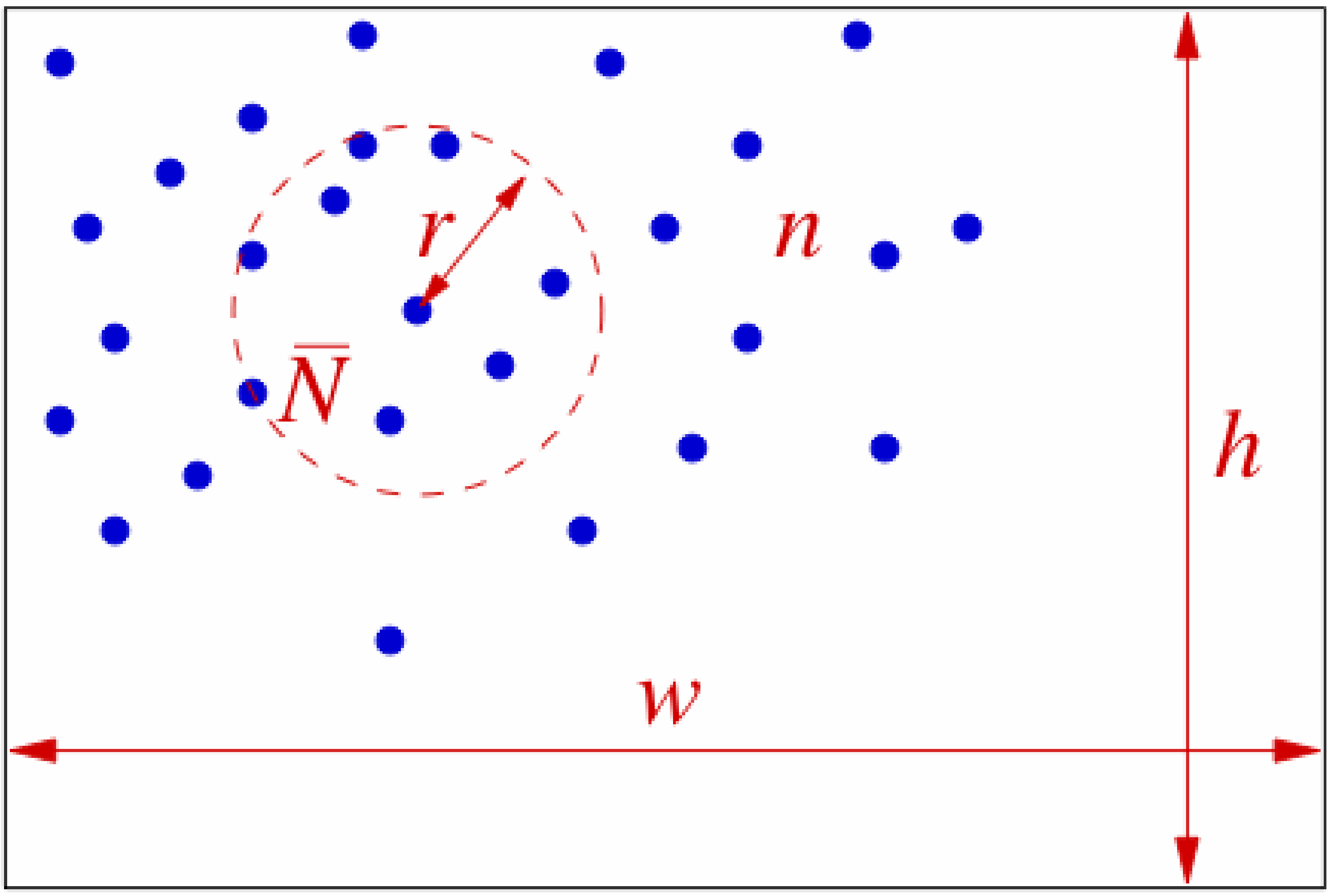
$$\lambda_2 \leq \Delta - 2\sqrt{(\Delta-1)} + ((2\sqrt{(\Delta-1)} - 1)/\lfloor diam/2 \rfloor)$$

In simulation...



```
proto -r 6.3 -n 800 -dim 100 100 '(delta 0.02 50)' -s 1 -l -step -m -T
```

Analysis: Parameter Space

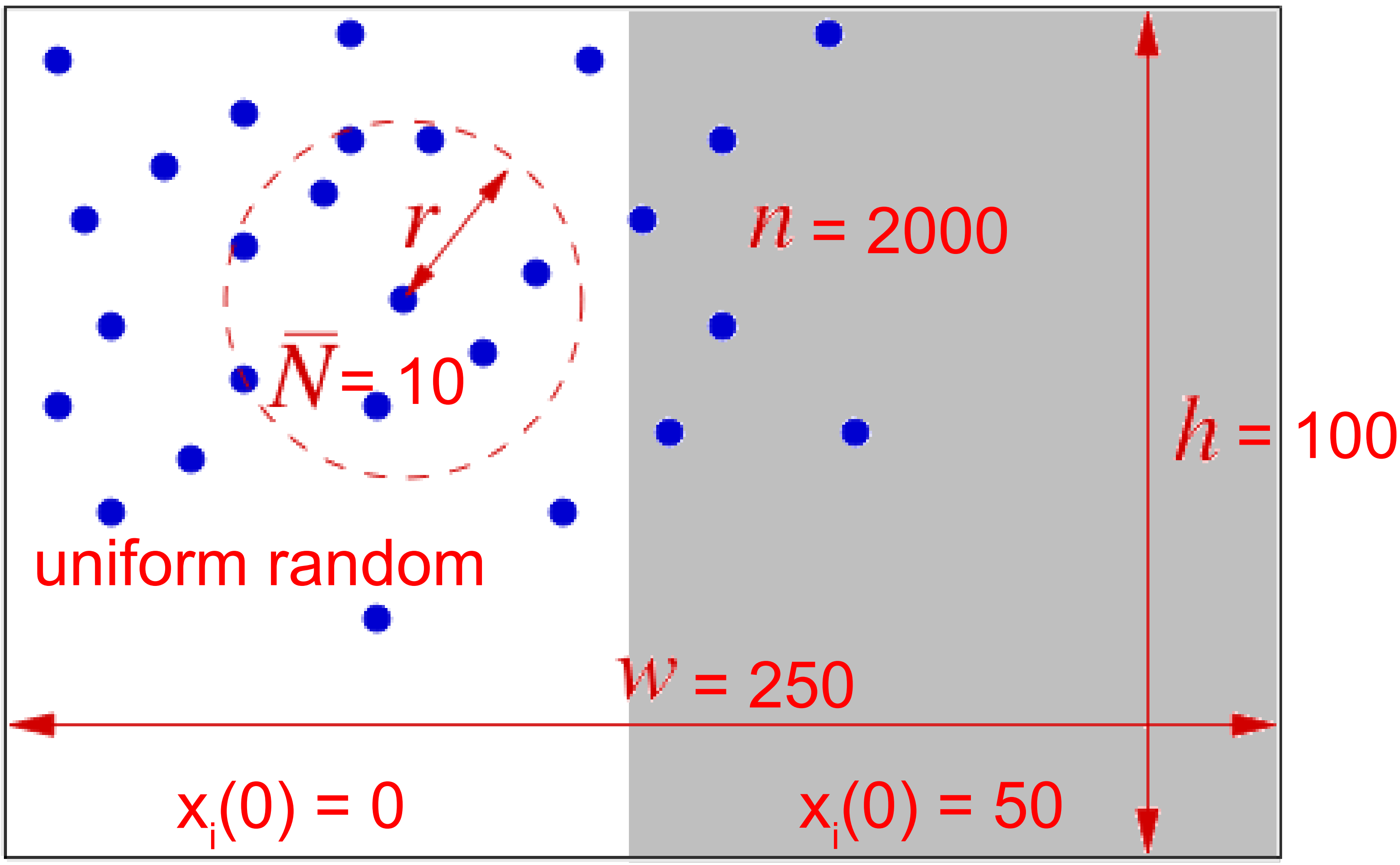


Analysis: How fast is convergence?

- Available bounds for λ_2 are very loose
- But... on a spatial computer with $\overline{N}$>6, Laplacian consensus approximates physical diffusion.
- Convergence to a fixed error level:
  - $O(diameter^2 \cdot \ln(\delta(0)) / \overline{N}\varepsilon)$
  - But ε < 1/degree <= $1/\overline{N}$
  - Thus: $O(diameter^2 \cdot \ln(\delta(0)))$

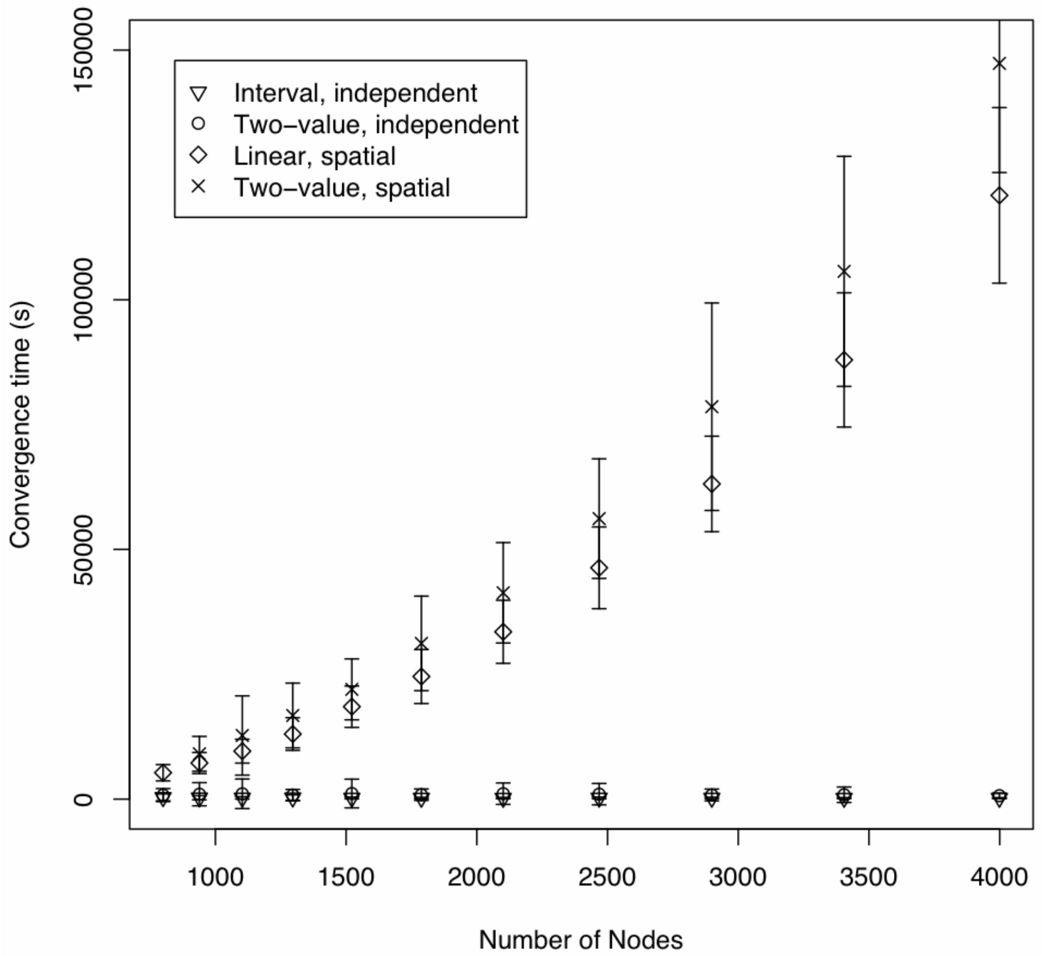
How bad is the constant term?

Empirical Evaluation



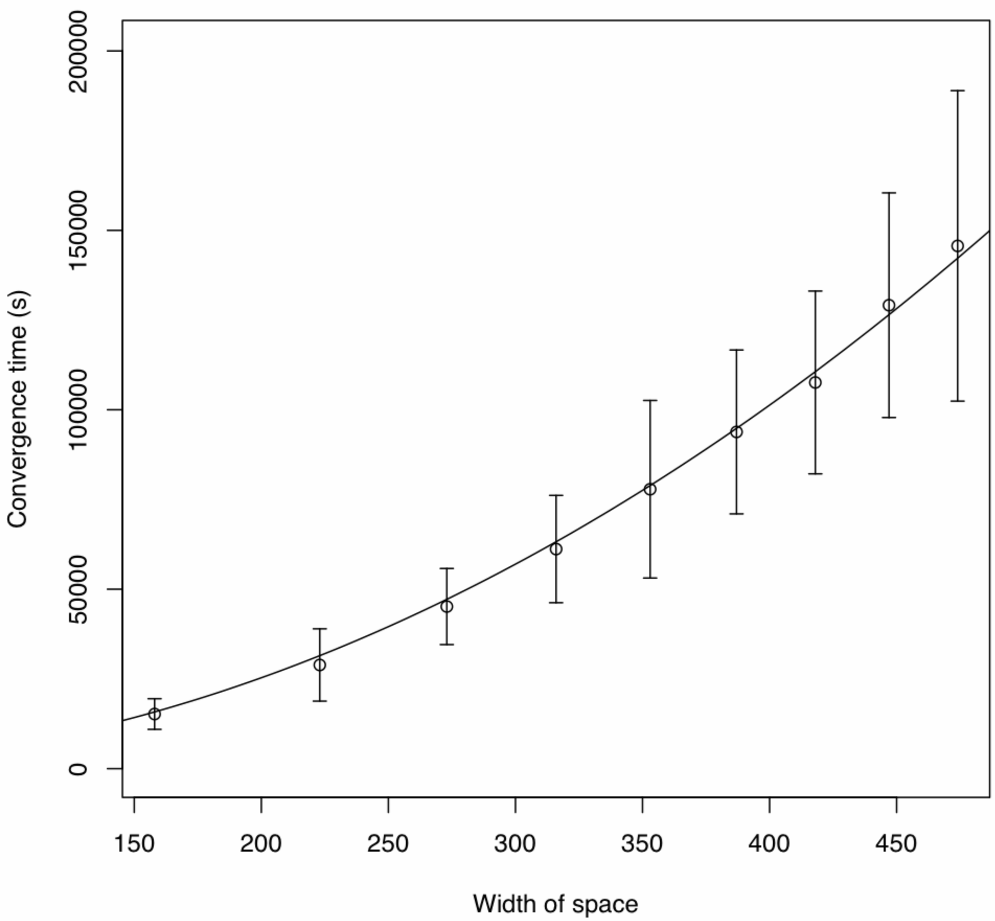
- Synchronous execution, ε = 0.02
- Converged when 95% of devices at mean +/-2

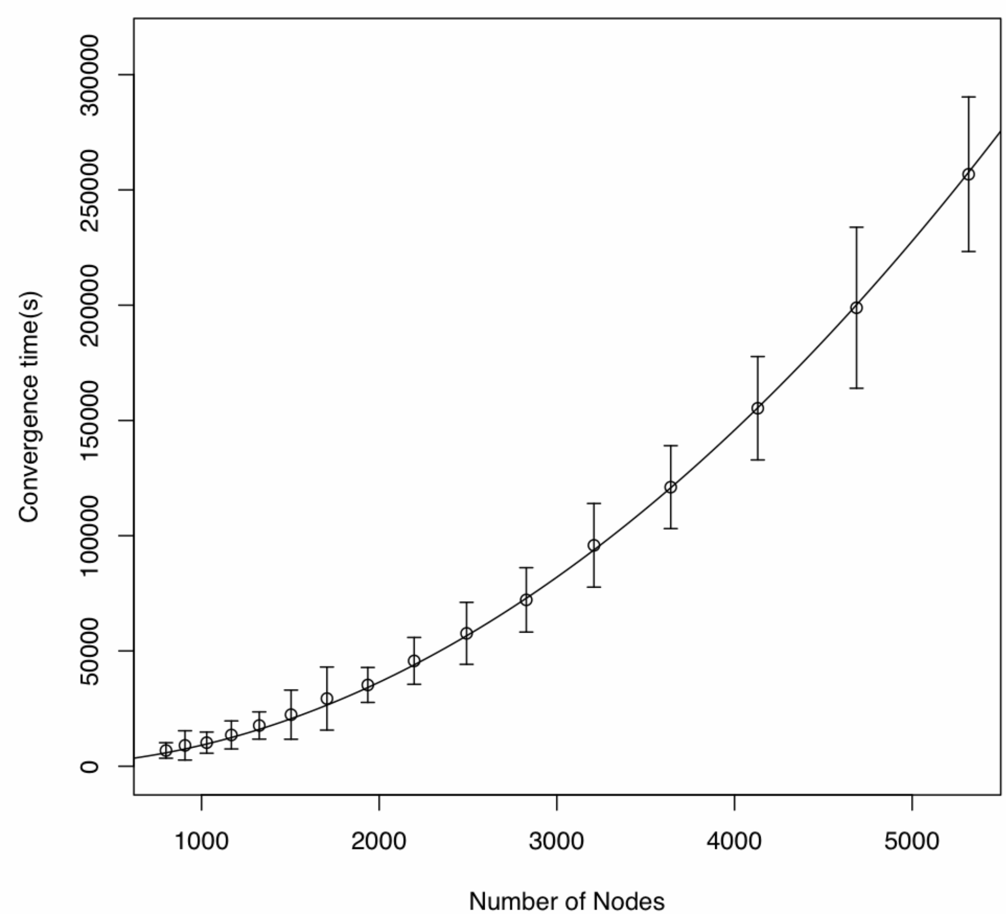
Spatial Correlations Matter



- Random initial values → fast convergence

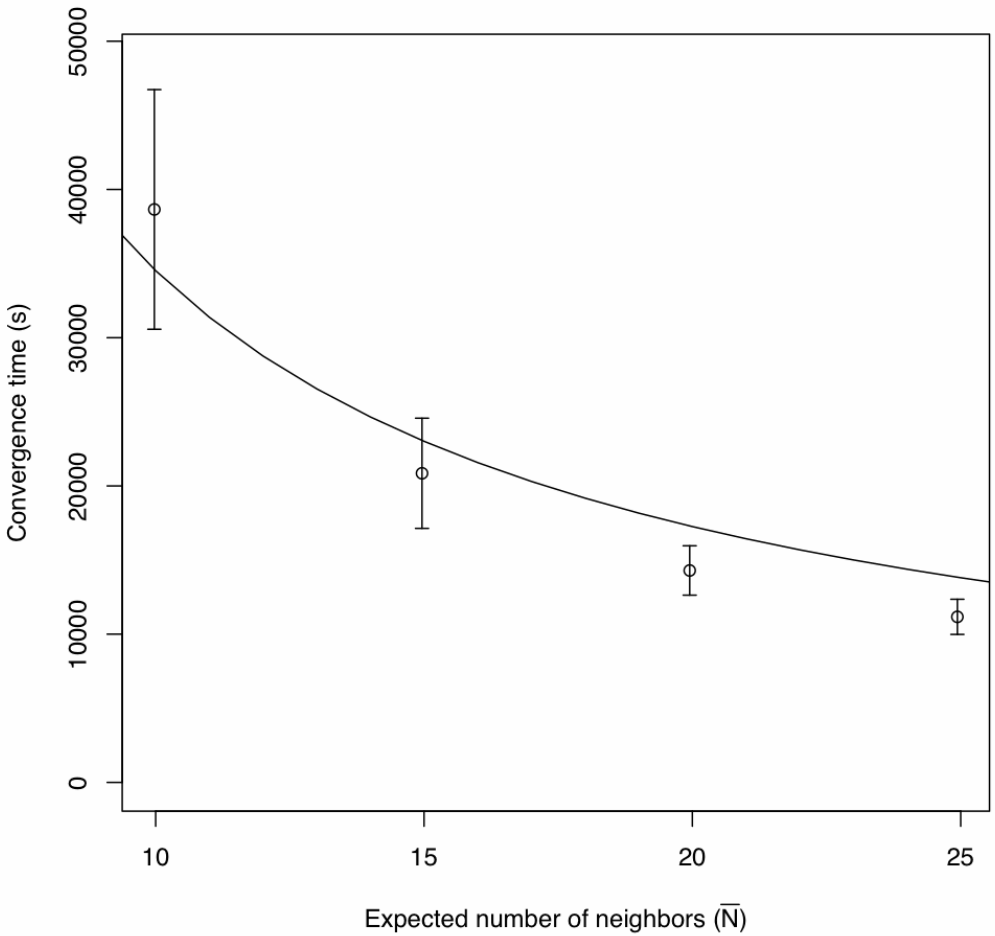
Quadratic Scaling w. Diameter

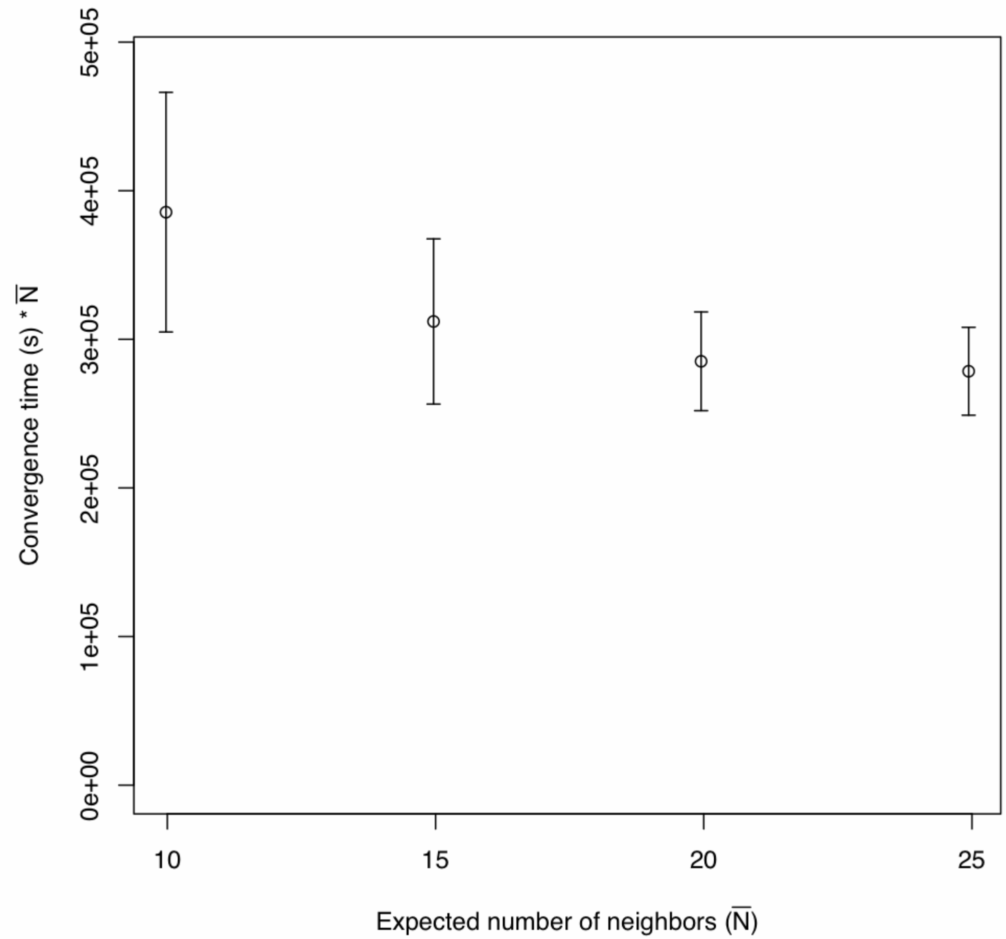




- Convergence time dominated by width

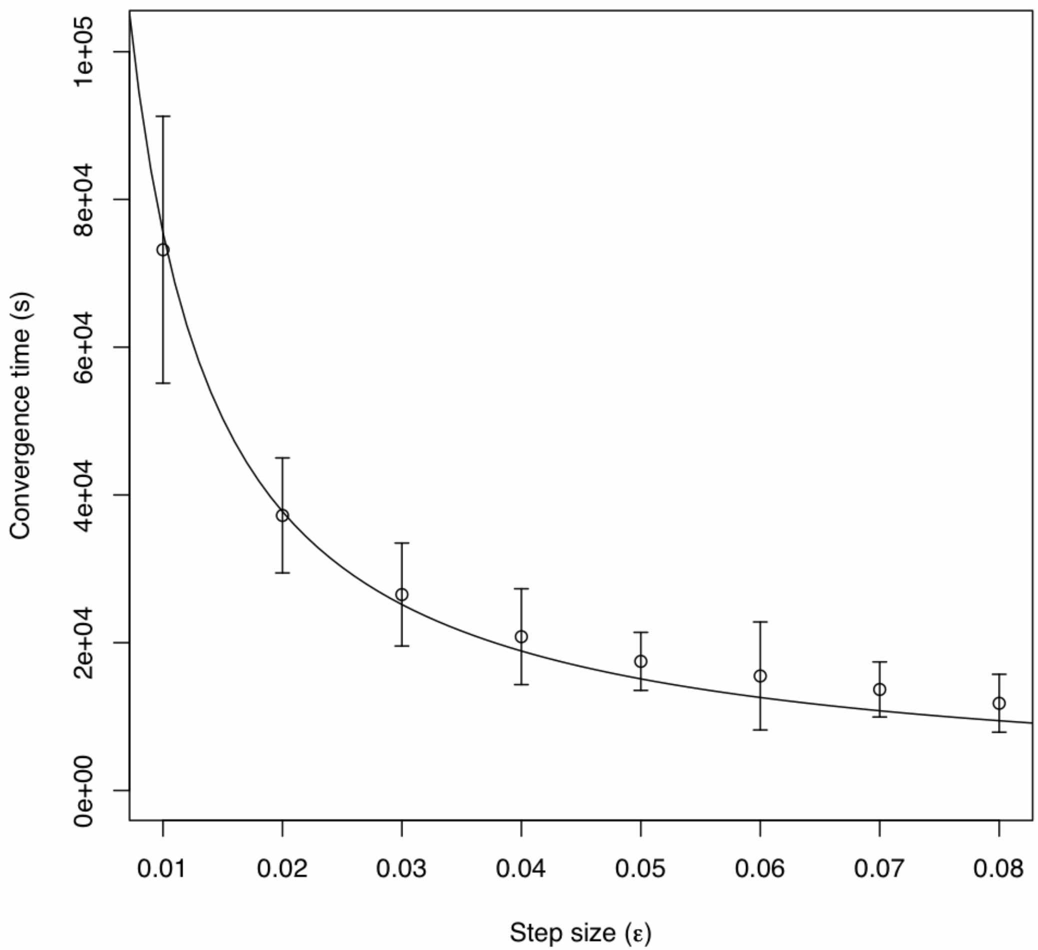
Inverse Scaling w. Num Neighbors





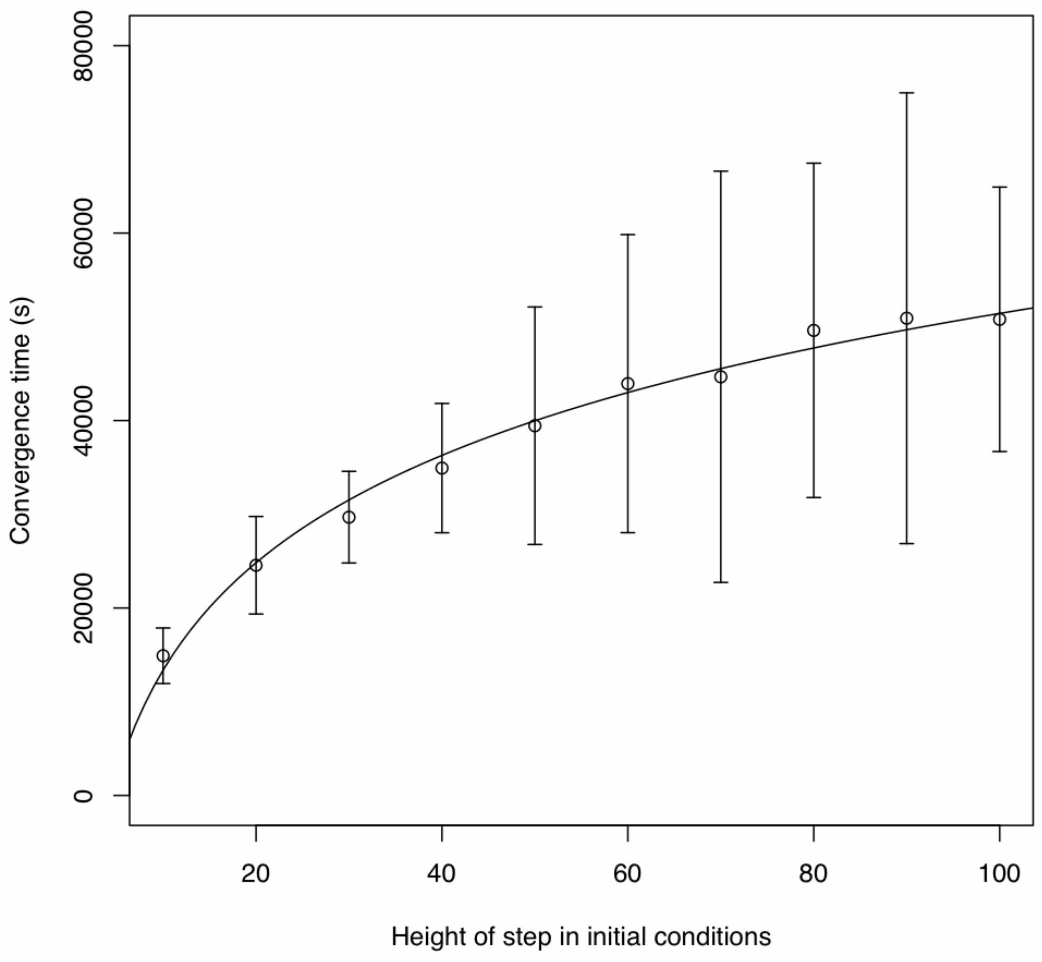
- Secondary improvement from straighter path

Inverse Scaling w. Step Size



- Breaks down as system approaches instability

Logarithmic Scaling w. Initial Difference



Contributions

- Laplacian-based averaging consensus scales poorly on spatial computers:
  - $O(\text{diameter}^2 \cdot \ln(\delta(0)))$
  - Empirical survey shows convergence time constant is high as well