# Complexity Analysis

## Enhancing Methodological Rigor for Computational Cognitive Science

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# Why do we throw away our tools?

*Given computational models, use tools for analyzing computation*

# Computational Complexity

- Asymptotic computational complexity
  - Resource costs vs. scale *n*

- O(f(n)): $\exists\ n',k$ such that $n>n' \rightarrow \text{cost} < kf(n)$
- Also: Ω, Θ, expected, amortized...

- Establish analytically or empirically

# Biological Plausibility

$\sim 10^{11}$ neurons

$\sim 10^{15}$ synapses

$\sim 10^{13}$ "clock cycles"/life

Three way comparison:

- Cost of *n* items
- # items for a mind
- Resource budget

*Is it affordable within a few orders of magnitude?*

# Example Plausible Bounds

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| **Memory to store a set of *n* items** | **$10^{14}$ synapses** | **working a physics problem** | **a few models** | $\boldsymbol{O(c^n)}$ |
| | | **relations in a social network** | **hundreds of friends** | $\boldsymbol{O(n^5)}$ |
| | | **word meanings** | **vocabulary of thousands** | $\boldsymbol{O(n^3)}$ |

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| **Time to learn about a set of *n* items** | **$10^9$ seconds of experience** | **arithmetic** | **a few relations** | $\boldsymbol{O(c^n)}$ |
| | | **word meanings** | **vocabulary of thousands** | $\boldsymbol{O(n^2)}$ |
| | | **possible body movements** | **billions of positions** | $\boldsymbol{O(n)}$ |

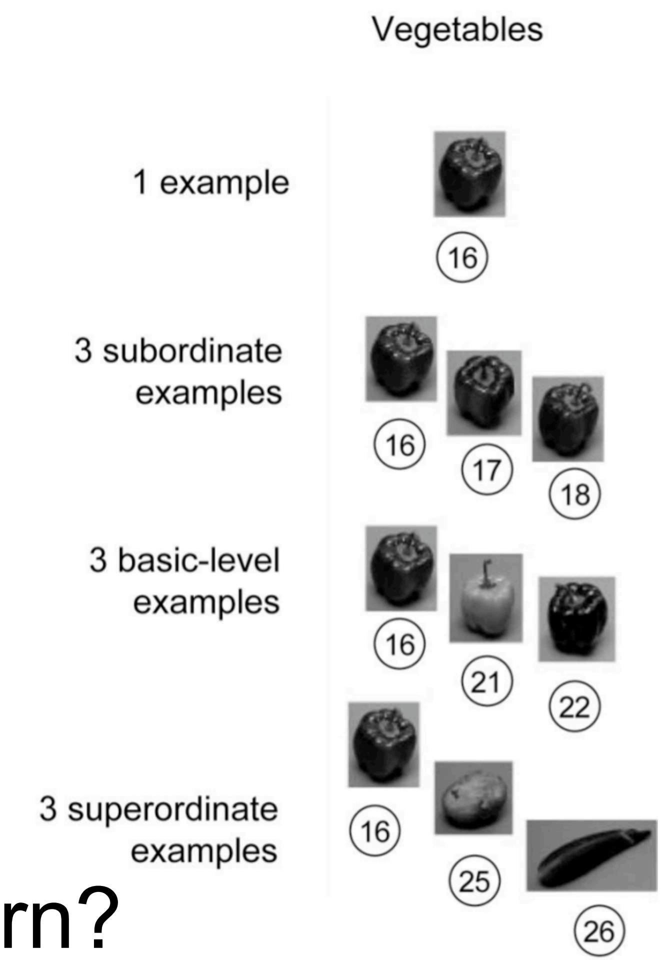
# What if the model is too expensive?

- Find lower complexity equivalent model
  - e.g. bubble sort vs. heap sort vs. parallel bubble
- Tighten complexity bound
  - e.g. expected fast graph coloring
- Change model requirements
  - e.g. consensus vs. approximate consensus
- Reuse costly components
  - e.g. cognitive substrate approach

# Improving Models: Word Learning

Bayesian categorization:

- *n* examples, |*X*| in a category
- Assuming O(1) similarity calc
    - *O(n)* space
    - $O(|X|n^2)$ time per example
    - $O(|X|n^3)$ time to learn words




(Xu & Tenenbaum, 2007)

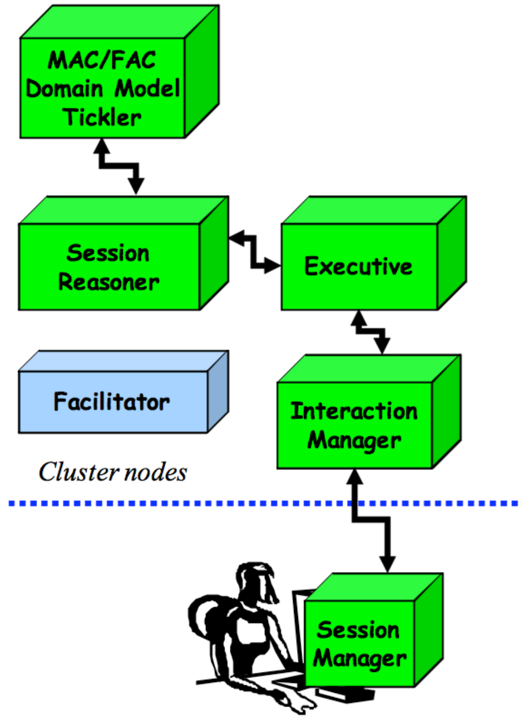
$10^6$ words/life → ~$10^5$ lives to learn?

Incrementalize → ~0.1 lives

# New Predictions: Analogical Reasoning

Companions Cognitive Architecture

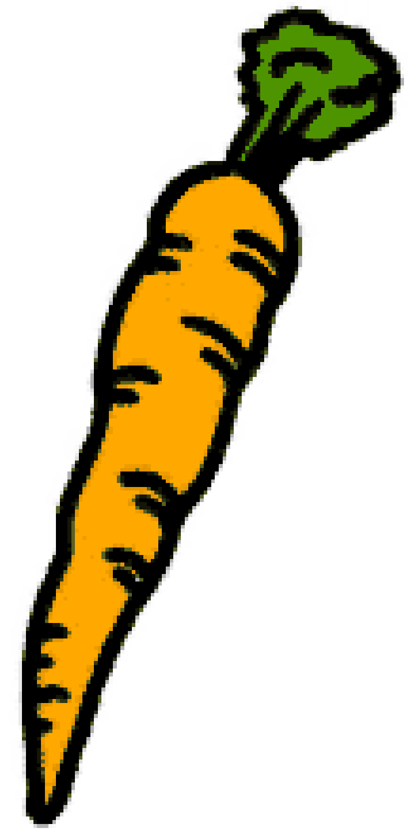
- *k* examples, *n* elements per exa.
  - Retrieval: $O(\log k)$ time, $O(k)$ circuit
  - Mapping: $O(n^2)$ time
- $k = \sim 10^9$, $n = \sim 10^3$ *(Forbus, 2009)*
  - Space OK if noise tolerant
  - Time: $10^6$ is too long, but a greedy algorithm is fast. Do humans fail like the greedy algorithm?




(Forbus et al, 2009)

# Benefits of Complexity Analysis

- Scaling
  - e.g. 21 objects → language
- Robustness
  - Few confounds scale equivalently
- Composability
  - Experimental grounding is portable
  - e.g. varying defn. of “word sense”
- Longevity
  - Flexible coupling to neuroscience



# Must we really?

- Complexity analysis : Model =
  Significance analysis : Experiment

- Omitting complexity analysis is a scientific problem that…
  - … misleads non-computational colleagues
  - … creates erroneous models
  - … stifles research on open problems

# Four Questions for Every Paper

- What resource limitations are pertinent?
- What is the order of growth for each resource?
- What is the scale of the model?
- How does the model compare against current estimates of biological resource limits?

*Bounds can always be tightened later*

# In Summary:

- Computational cognitive models *demand* analysis of computational complexity
  - Analysis can be theoretical or empirical
  - Adding complexity analysis to papers is easy
- Even loose estimates of biological scale and resources can be strong constraints on models
- Application examples:
  - Exposed challenges to word learning model
  - New behavior predictions for analogical reasoning