Financial Relationships and the Limits to Arbitrage

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Main Contributions

- Presents a new (yet complementary) *foundation* for the **limits to arbitrage**.
- Highlights the role and the *limits* of relationships between financial institutions in affecting the price formation process in securities markets.
  - ↪ Underexplored topic in asset pricing/market microstructure (see, e.g. Allen (2001)).
- Show that these arguments are *robust* to reasonable assumptions about the contracting space.
- Argue that the underlying framework of the paper is applicable to *broader* environments that involve **informable finance**.

Existing Literature

- **Essence of the Theory:** *"So there's an arbitrage. So what? This desk has lost a lot of money on arbitrages. Arbitrages aren't particularly good trades."* (Quote from Liu-Longstaff (2003))

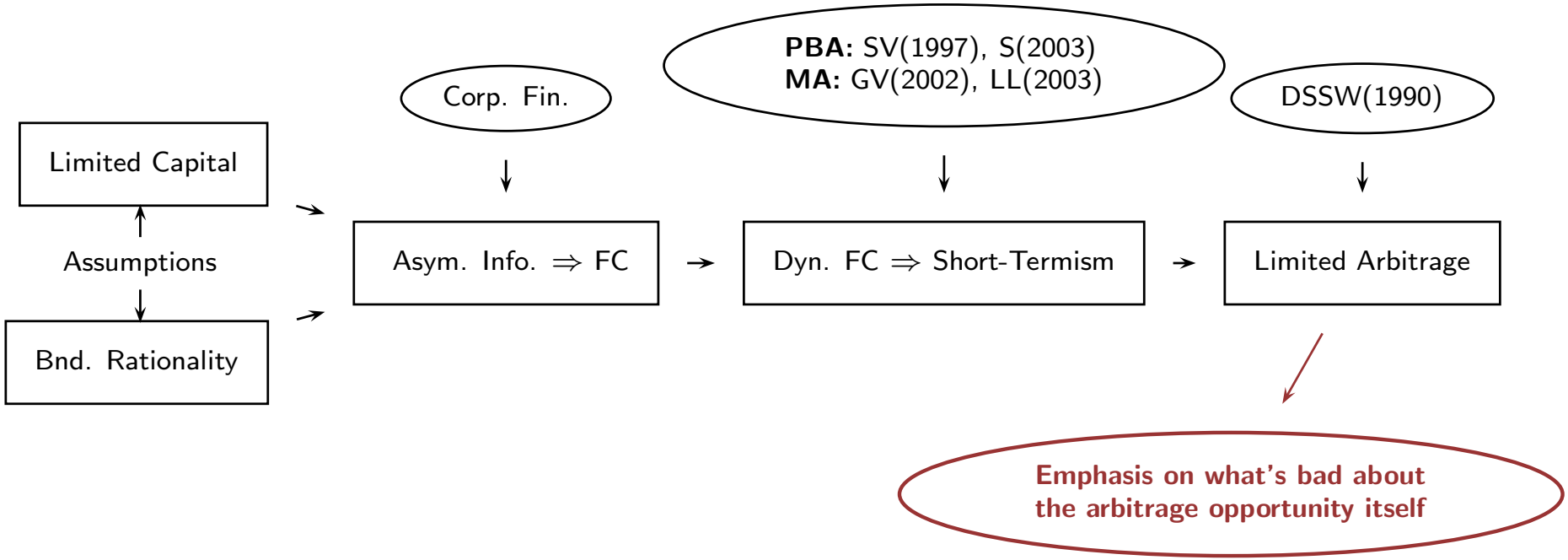


Figure 2: Existing Models of Limited Arbitrage.

New Perspective

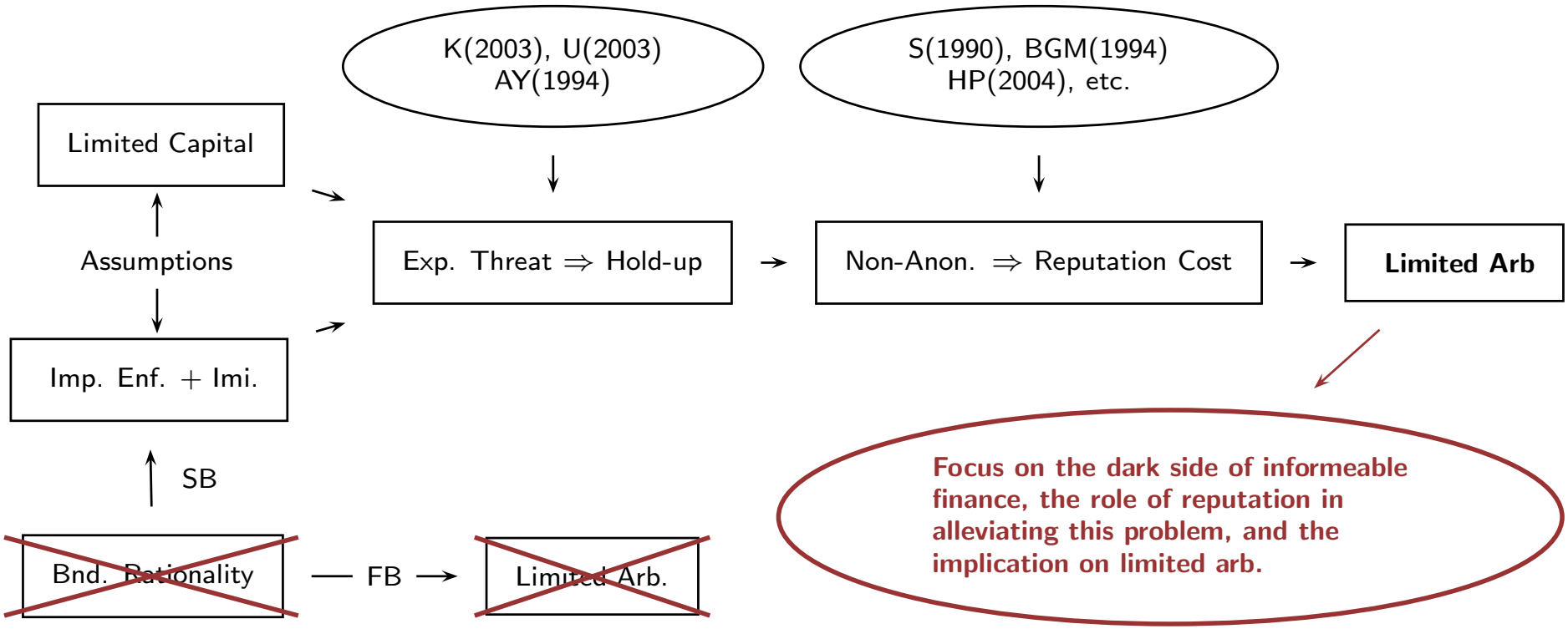


Figure 3: Our Model of Limited Arbitrage.

Related Literature

- **Agency Between Financial Institutions:**
  - ↪ **Predatory Trading:** Brunnermeier-Pedersen (2006).
  - ↪ **Front-Running:** Ko (2002, 2003).
- **Relational Contracting Literature:**
  - ↪ **Stationarity Result:** Levin (2003).
  - ↪ **Stochastic Repeated Game:** Rotemberg-Saloner (1986).
  - ↪ **Implicit vs. Explicit Contracting:** Baker-Gibbons-Murphy (1994).
  - ↪ **In Finance:** Sharpe (1990), Chemmanur-Fulghieri (1994s), etc.
- **Sale and Protection of Ideas Under Weak Intellectual Property Rights:**
  - ↪ **Fundamental Paradox:** Arrow (1962).
  - ↪ **Circulation, Sharing or Sale of Ideas:** Anton-Yao (1994, 2002), Biais-Perotti (2003), and Hellman-Perotti (2004).
  - ↪ **Information Leakage:** Rajan-Zingales (2001), Baccara-Razin (2003).
  - ↪ **Process of Innovation and Control Rights:** Aghion-Tirole (1994).
  - ↪ **Bank vs. Venture Capital:** Ueda (2003).

Some Headlines

- **On Relationship Between Prime Brokers and Hedge Funds:** *"... the cosiness of these dealings and the conflicts of interest they can engender are a growing concern. Partly because they act as a gateway to the entire range of services investment banks offer, prime brokers gain access to privileged market-sensitive information from their clients - information that must be kept from other clients and from the investment bank's proprietary trading departments."* (Financial Times, 07/14/2003)
- **On LTCM's Fear of Being Screwed:** *"Having worked at a major Wall Street bank, John Merriweather felt that investment banks were rife with leaks and couldn't be trusted not to swipe his trades for themselves. Indeed, most of them were plying similar strategies. Thus, as a precaution, Long-Term would place orders for each leg of a trade with a different broker. Morgan would see on leg, Merrill Lynch another, and Goldman yet another, but no one would see them all. Even Long-Term's lawyer was kept in the dark; he would hear the partner's say "trading strategy three," as though Long-Term were developing a nuclear arsenal."* (Lowenstein, p.48)

The Basic Setup

- **2 Securities:** 1 and 2 $\Rightarrow E[v_1] = E[v_2] = \overline{v}$.
- **Arb's Info:** Knows that $v_1 = v_2$.
- **Demands:**
  - $\hookrightarrow$ Noise trading in each market: X_{Ni}.
  - $\hookrightarrow$ Long-run trader demand: $X_{LTi} = \frac{1}{\lambda}(\overline{v} - p_i)$.
  - $\hookrightarrow$ Arb and Bank: Restricted to *convergence strategies*.
- **Prices:** Determined by market clearing...
  - $\hookrightarrow$ $\Delta p = \lambda[\Delta X_N - 2(X_A + X_B)]$.

  where $X_{NH} - X_{NL} \equiv \Delta X_N$.

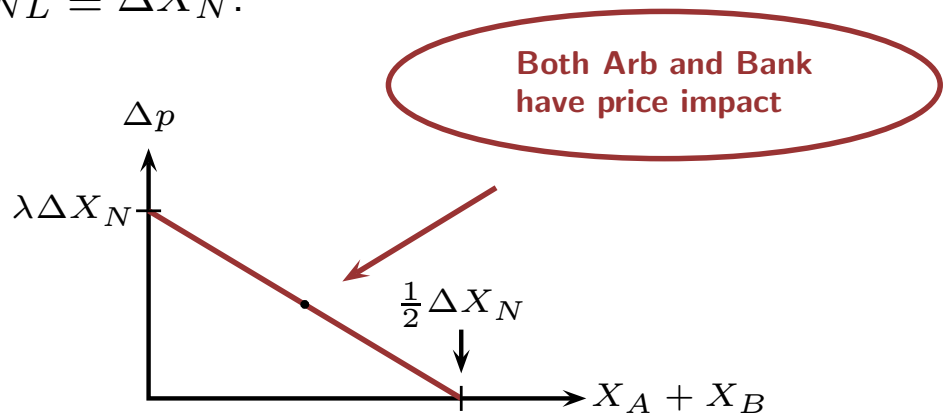


Figure 4: Price Spreads.

The Basic Setup (Continued)

- **Default FC:** Without communication of strategic information, assume Bank *can't* assess the credit risk of Arb's strategy and imposes the FC: $X_A \leq M_L$
  - ↪ Could have an explicit model of adverse selection that generates this constraint. Left out for simplicity.
- **Communication:** Arb can choose to reveal or not reveal details of his strategy. Communication is *costless*.
  - ↪ Rule out *partial revelation* of strategy which could be used as a signalling device (e.g. Anton-Yao (2002)).
  - ↪ Bank then decides whether or not to relax the Arb's FC and also chooses how much to trade on the Arb's idea.
- **Simplification:**
  - ↪ Assume Bank doesn't learn from prices. Reasonable if securities 1 and 2 are part of a *large* group of potential pairs combinations.
  - ↪ For now, ignore the possibility of *explicit* ex-ante contracts between Bank and Arb. More on this later...

The Stage Game

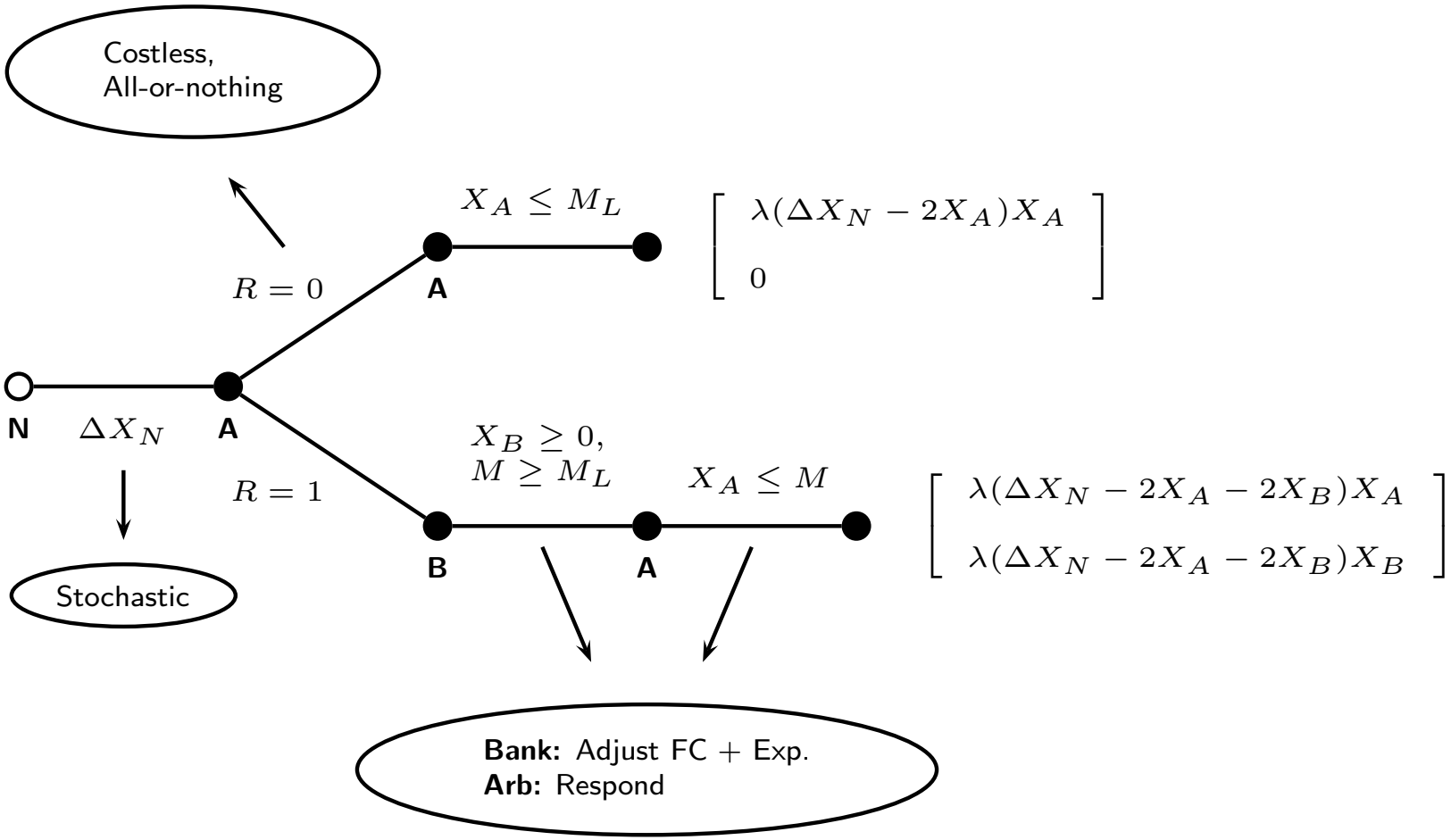


Figure 5: An Arbitrage Opportunity Cycle.

The Stage Game (Continued)

- **What's an Arbitrage Opportunity Cycle?**
  - ↪ Each arbitrage opportunity is a *new* idea.
    - ∗ So no competition between Arbs.
    - ∗ However, new idea becomes broadly known after one cycle. Profitability quickly erodes.
    - ∗ Hence, if Bank decides to hold-up, it only benefits for one cycle.
    - ∗ Also, keep in mind that ΔX_N is stochastic. Arbitrage opportunity may be slight, average, amazing, etc.
  - ↪ Arb has limited reputation so uninformed financiers perceive that it has fairly high credit risk.
    - ∗ FC can bind and become an issue if opportunity is attractive enough.

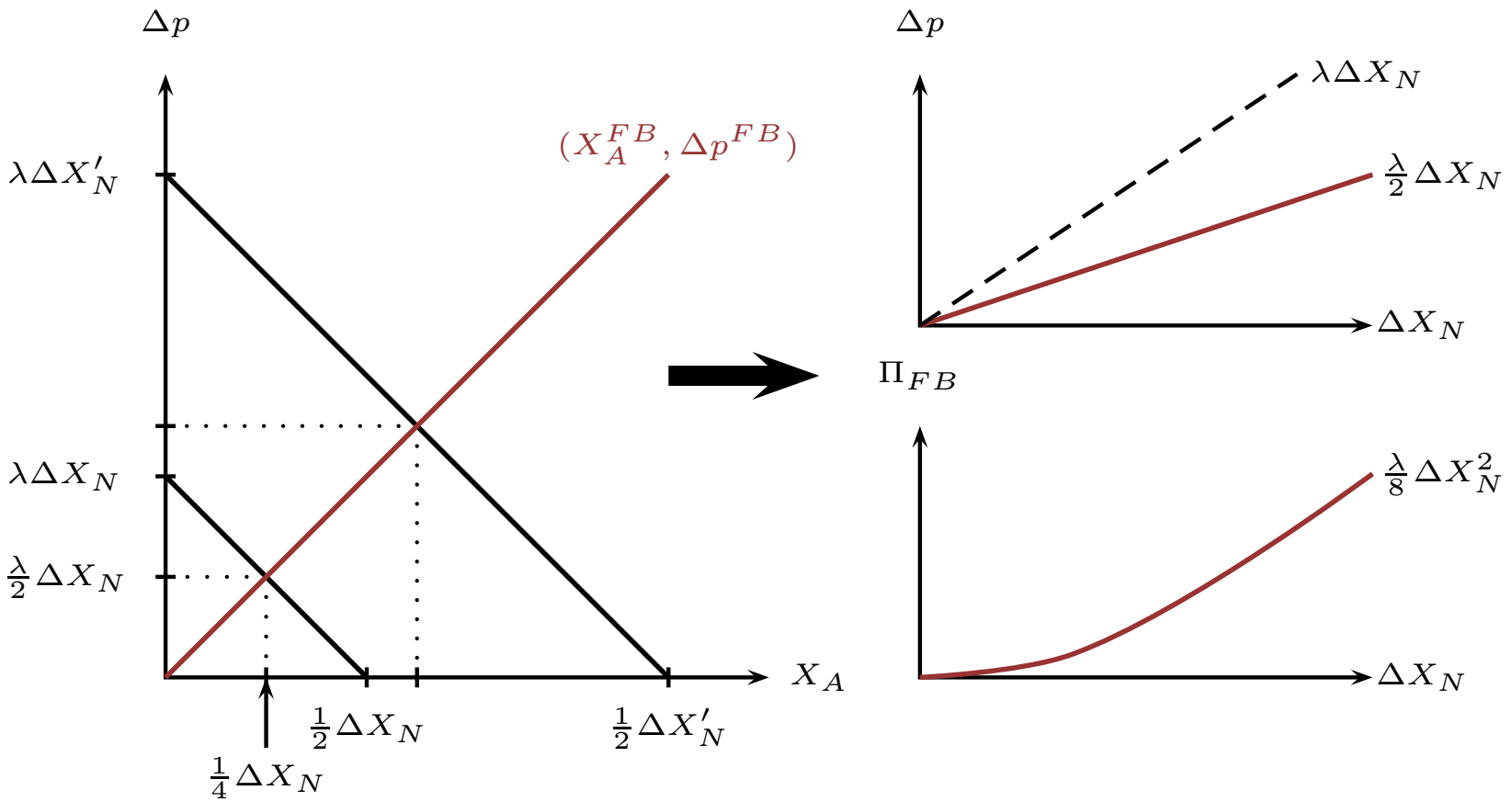
The Infinitely Repeated Game

- **New arbitrage opportunity cycle occurs every time period.**
  - ↪ Bank is long-lived and participates in each cycle.
    - ∗ Has discount factor δ.
    - ∗ **δ a Proxy for Many Things:** Impatience, frequency of new arbitrages, bank risk-aversion, internal agency problems, etc.
  - ↪ Arb only lives and plays for only one cycle.
    - ∗ New ideas are hard to come by.
  - ↪ If Bank chooses to hold-up an Arb, there is a probability p that word leaks out and all future Arbs know of this expropriation.
    - ∗ For simplicity, assume that p doesn't depend on the degree of expropriation.

Outline of Exposition

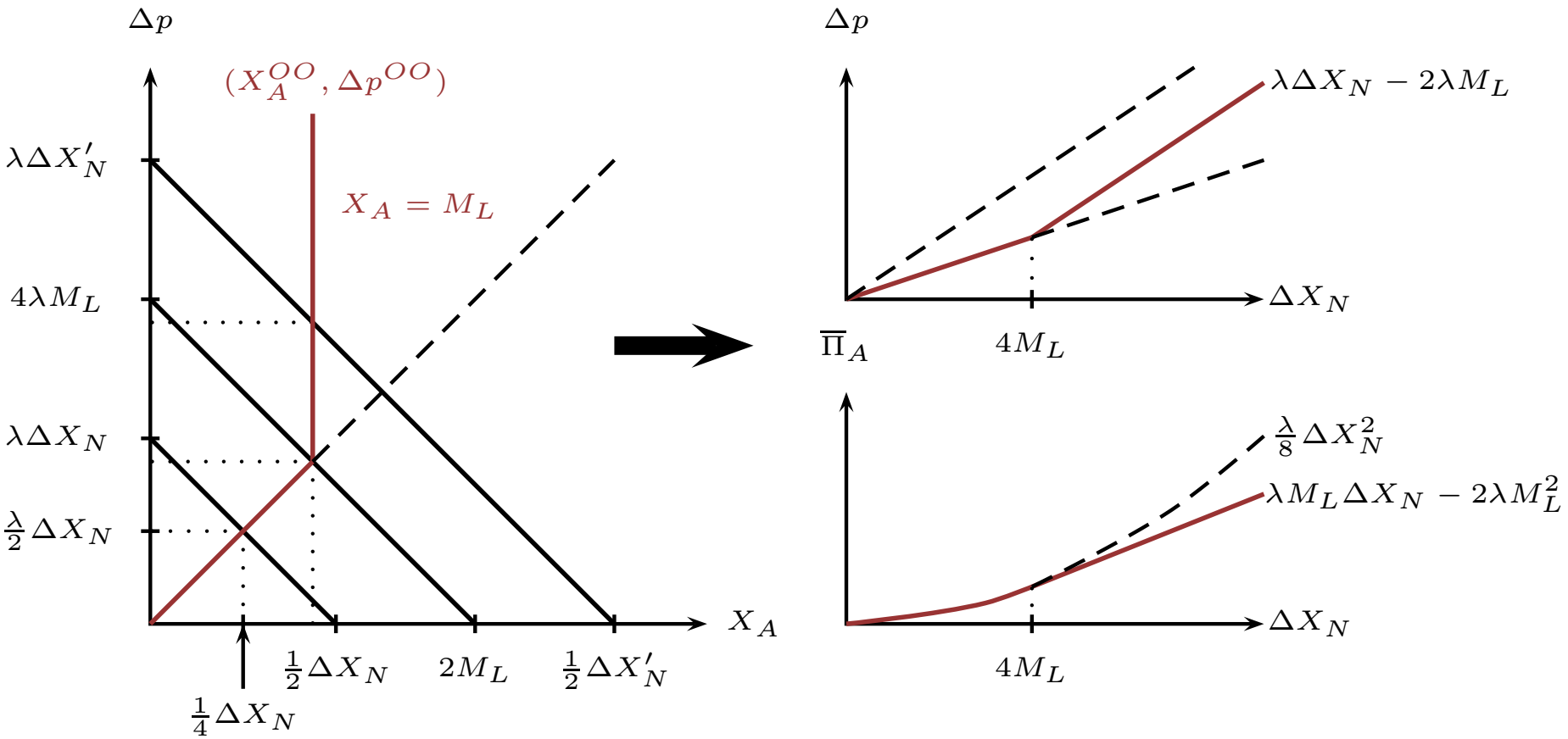
- **Benchmark Outcome:** Arbitrage with No Financial Constraint.
- **One-Shot Equilibrium:** Arbitrage with Financial Constraint and No Relationships.
- **Relational Equilibrium:** Arbitrage with Financial Constraint and Relationships.
- **Refinement:** Competition and Uniqueness.
- **Example:** Normal Distribution for $\Delta X_{N,t}$

Benchmark Outcome



- **Result:** Since Arbs are never constrained, they eliminate half of the mispricing.
  - ↪ Information monopoly leads to some residual mispricing but not because of FC. This is not what we focus on in this paper.

Outside Option: No Communication



- **Result:** Once Arb hits his FC, he only has a *fixed* impact on equilibrium mispricing.
  - ↪ FC binds starting at $\Delta X_N = 4M_L$.

One-Shot Equilibrium

Proposition (One-Shot Game)

The arbitrageur will always choose not to reveal his information. As a result, $X_A^* = X_A^{OO}$ *and* $\Delta p^* = \Delta p^{OO}$.

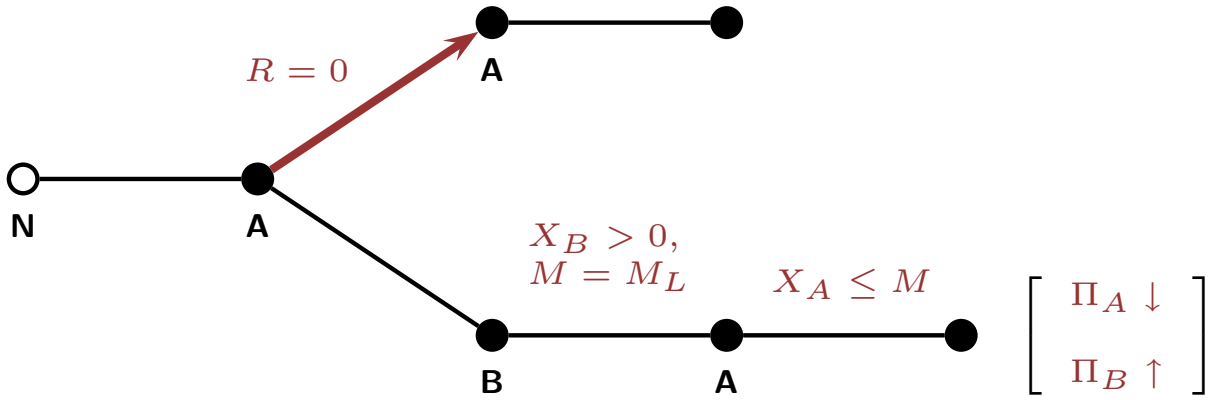
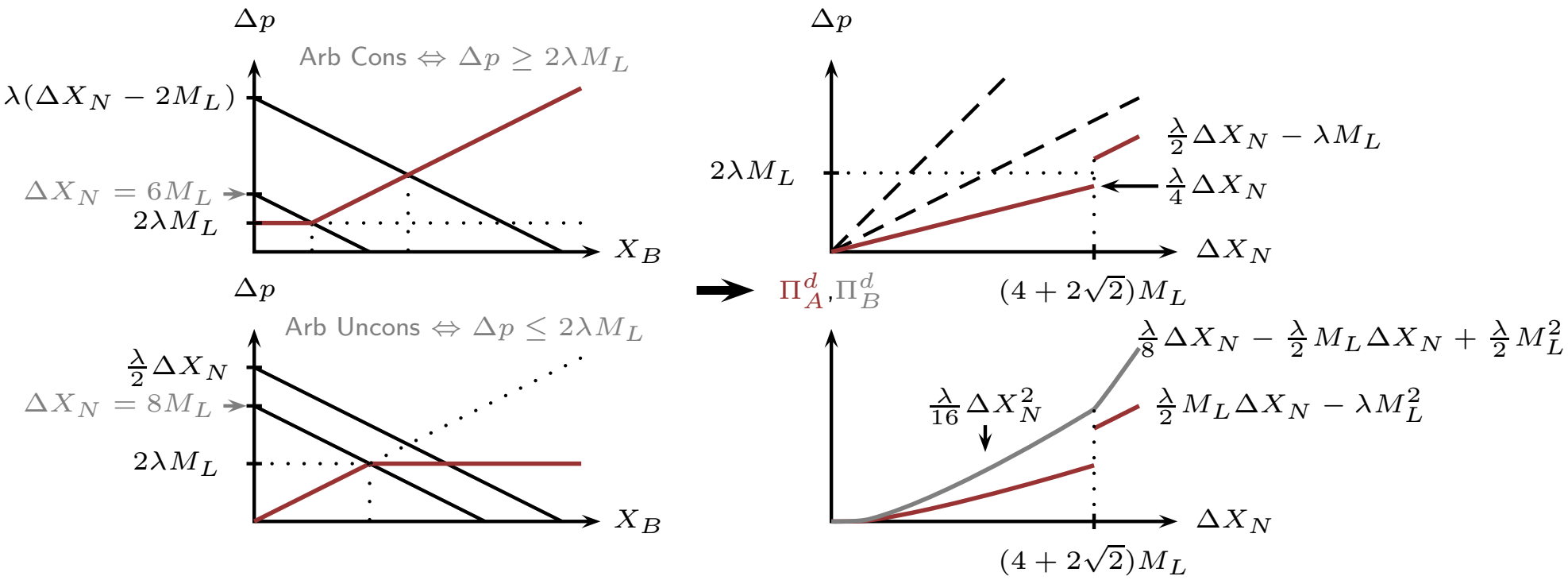


Figure 6: Intuition for Proposition.

- **Result:** Complete breakdown in the raising of informed capital.
- For $\Delta X_N > 4M_L \Rightarrow$ Arb is constrained $\Rightarrow$ Limited arbitrage.

Deviation Profit: Maximal Expropriation



- **Keep in Mind:** Object of interest here is Π_B^d.
  - ↪ Enters the Bank's IC constraint.

Relational Equilibrium

- Restrict attention to efficient equilibria...
- **Efficient Equilibria:** *An SPE of the repeated game with payoffs* $(\Pi^1_{A,t}, \Pi^1_{B,t})$ *is efficient if and only if there does not exist another SPE of the game with payoffs* $(\Pi^2_{A,t}, \Pi^2_{B,t})$ *such that: (i) for every* $\Delta X_{N,t}$, $\Pi^2_{A,t} \geq \Pi^1_{A,t}$, *and (ii)* $V^2_t \geq V^1_t$ *where* V_t *is the value of the relationship to the bank at* t*:*

$$V_t = E_t \left[\sum_{j=1}^{\infty} \delta^{t+j} \Pi_{B,t+j} \right].$$

- **Arb's IR:** $R^*_t = 1 \Leftrightarrow \Pi_{A,t}(R_t = 1) \geq \overline{\Pi}_{A,t}$.
- **WOLOG:** $(M^*_t, X^*_{B,t}) \in \{(M^c_t, X^c_{B,t}), (M_L, X^d_{B,t})\}$.
- **Bank's IC:** Cooperate $\Leftrightarrow \Pi_{B,t}(M^c_t, X^c_{B,t}) + V^*_t \geq \Pi^d_{B,t} + (1-p)V^*_t$.
- **Punishments:** Maximal punishments are *credible* because the arbitrageur can do business with another identical bank. Ignores switching costs though.
- **Fundamental Question:** What's the best *cooperation/communication* that can be supported?

Optimal Collusion Results

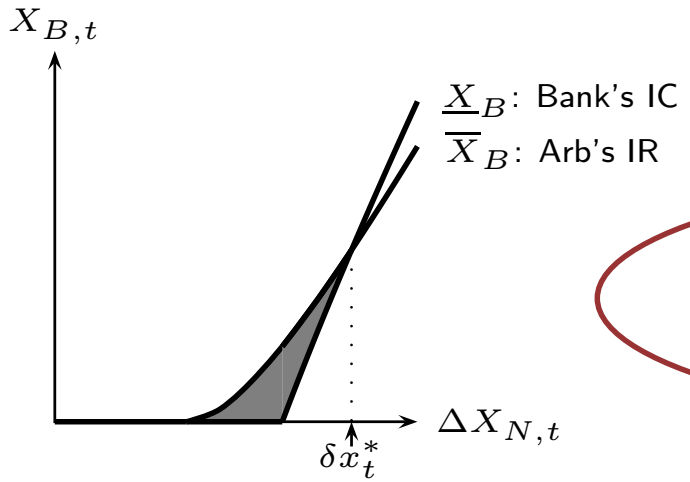
Lemma (Optimal Collusion 1)

If $R_t^* = 1$, then an efficient equilibrium satisfies:

$$X_{B,t}^* + M_t^* = \frac{1}{4}\Delta X_{N,t} \quad \textit{and} \quad X_{A,t}^* = M_t^*.$$

That is, the bank and the arbitrageur collude to achieve first-best profits.

- **Intuition:** If you're going to collude, do it right!
- **Corollary:** Using Arb's IC and Bank's IR constraints can get *bounds* on $X_{B,t}^*$...



Shaded area denotes feasible $X_{B,t}^*$. No communication feasible if $\Delta X_{N,t} \geq \delta x_t^*$. Expression for δx_t^* in the paper.

Optimal Collusion Results

- **Remark:** The last lemma shows that informable finance is *limited* in its capacity to improve the institution of arbitrage. Further, it fails when it is needed *most*.
  - ↪ **Novel Result:** Obtain opposite results if we consider a standard repeated game (constant $\Delta X_{N,t}$).

Lemma (Optimal Collusion 2)

All efficient equilibria satisfy:

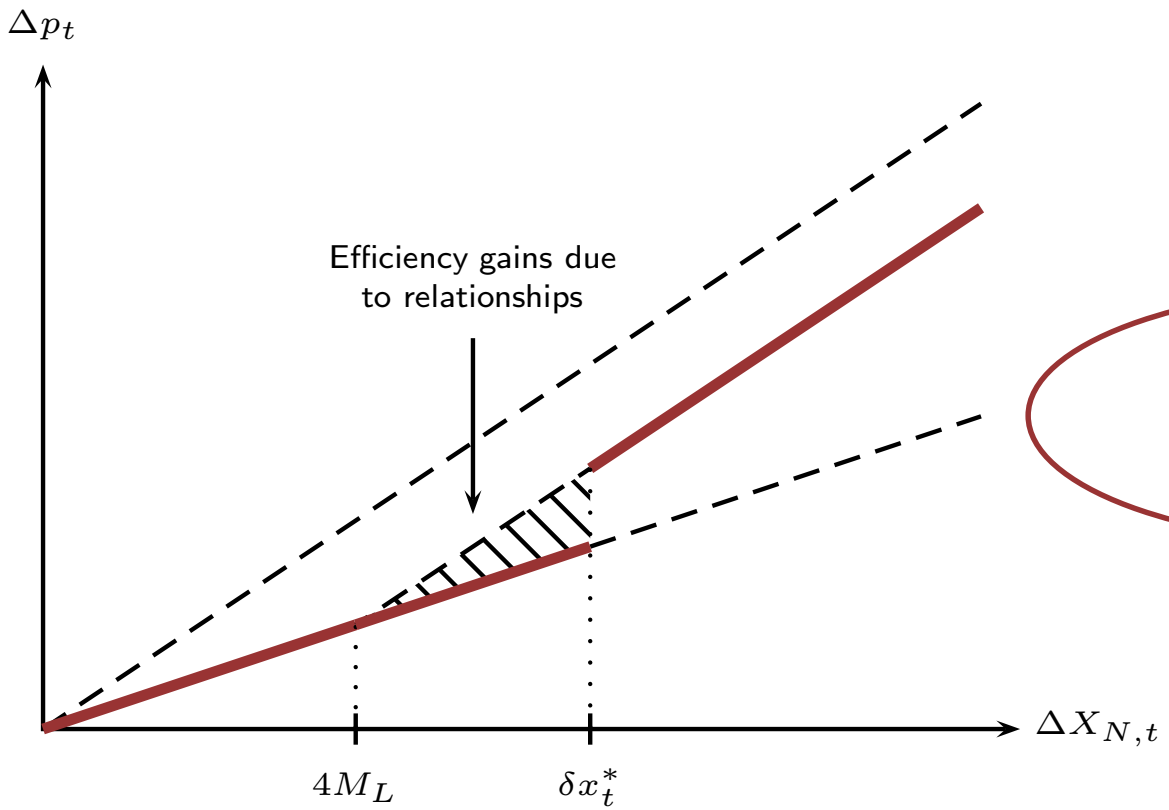
$$R_t^* = \begin{cases} 0 & \text{if } \Delta X_{N,t} \leq 4M_L \\ 1 & \text{if } 4M_L \leq \Delta X_{N,t} \leq \delta x_t^* \\ 0 & \text{otherwise} \end{cases} .$$

- **Intuition:** Collude whenever possible!

Equilibrium Price Spread

Proposition (Characterizing Equilibrium)

The optimal collusion results characterize the equilibria. In all equilibria, the equilibrium price spread is shown below.



Discontinuity at δx_t^* is due to breakdown of communication. Trading drops from $\delta_t^*/4$ to M_L.

First Comparative Static: Wealth

- We now make the assumption that the surplus allocation rule, α_t, is independent of M_L. It follows that:

Proposition (Wealth and the Limits to Arbitrage)

Let $\delta_t^*(X)$ denote the value of δ_t^* in an equilibrium with $M_L = X$. Assuming that:

- $0 < M_L' < M_L''$
- $\delta_t^*(M_L') > 0$,

it follows that $\delta_t^*(M_L') > \delta_t^*(M_L'')$. In words, arbitrage becomes more limited as Arb wealth increases.

- **Intuition:** Higher Arb wealth $\Rightarrow$ Arb's outside option is higher $\Rightarrow$ Bank keeps fewer profits $\Rightarrow$ V_t is lower $\Rightarrow$ less commitment capacity.

Competition and Uniqueness

- **Previous Characterization Allows for Tons of Equilibria:**
  - ↪ Narrow set by looking at how competition helps determine the surplus allocation rule.
- **Consider Two Simple Cases:**
  - ↪ **Perfect Competition:** Bank's bid away as much of the surplus as they possibly can commit to ($X^*_{B,t} = \underline{X}_{B,t}$).
  - ↪ **Monopoly:** Bank captures all the surplus and bind Arb to his outside option ($X^*_{B,t} = \overline{X}_{B,t}$).
- Solving for either equilibrium boils down to finding the *largest* solution to a fixed-point problem. This is unique.
- Existence of this problem is trivial ($V^* = 0$ is always a solution).
- Fixed surplus allocation rule + iid $\Delta X_{N,t} \Rightarrow$ Stationary equilibrium.

Normal Distribution Example with $p = 1$

$\langle \delta \mathbf{x}^* \rangle \equiv \frac{\delta \mathbf{x}^*}{\sigma}$	$\longleftarrow$	PC	$\longrightarrow$	$\longleftarrow$	M	$\longrightarrow$
freq. cons. $\setminus$ δ	0.75	0.85	0.95	0.75	0.85	0.95
80%	4.41	6.13	10.30	7.95	14.85	49.34
70%	2.52	3.80	6.52	4.32	7.91	25.86
62.5%	0	2.88	5.03	2.77	5.33	17.01
50%	0	0	3.53	0	2.71	8.92
40%	0	0	0	0	0	5.42
30%	0	0	0	0	0	3.16
no rel. below (%)	*69.35*	*59.87*	*41.63*	*61.63*	*49.65*	*29.04*
$\hookrightarrow$	*2.19*	*2.24*	*2.37*	*2.36*	*2.47*	*2.70*

Second Comparative Static: Competition

- The previous table highlights the second comparative static (which doesn't rely on particular distributional assumptions):

Proposition (Competition and the Limits to Arbitrage)

Comparing the efficient SPE from the monopoly and perfect competition cases, we have that:

$$V_M^* \geq V_{PC}^* \qquad \textit{and} \qquad \delta x_M^* \geq \delta x_{PC}^*.$$

- **Intuition:** More competition $\Rightarrow$ Bank bids away more surplus $\Rightarrow$ V_t is lower $\Rightarrow$ less commitment capacity.

Explicit Contracting: The Case of Labor Contracts

- **Contracting Environment:**
  - ↪ Banks can offer labor contracts specifying a *performance-sensitive wage*, $W(\Pi)$, and a *maximal trading budget*, $\overline{M}$.
  - ↪ **Limited Liability:** $W(\Pi) \geq 0$.
  - ↪ **Weak Monotonicity:** $W(\hat{\Pi}) \geq W(\Pi)$ for all $\hat{\Pi} \geq \Pi$.
  - ↪ Offer can be *renegotiated* following acceptance and prior to performance realization.
  - ↪ Employees do not punish firm when firm expropriates an independent Arb.
- **Hiring Environment:**
  - ↪ Proportion θ are Specs, $1-\theta$ are Arbs. Type is private information.
  - ↪ Specs have no information and earn a rent from risk-shifting because of limited liability.
  - ↪ **Outside Wage Offer:** $\overline{w} > 0$.
  - ↪ For simplicity, assume $\theta \to 1 \Rightarrow$ *Fly-by-night constraint.*
- We solve for the equilibrium in the case of a monopolistic bank.

Explicit Contracting (Continued)

Proposition (Labor Contract Offer)

Let Ω denote the set of arbitrageur types $\Delta X_{N,t}$ that are hired by the bank. We have that: (i) the optimal employment contract specifies a finite upper bound $\overline{M}$, (ii) $\Omega \subseteq [M_L/4, \overline{M}/4]$, (iii) the Specs' screening condition binds:

$$\overline{w} = \max_{X \leq \overline{M}} E^G[W(X \cdot \tilde{\Pi})],$$

(iv) the wage offer is given by:

$$W(\Pi) = \begin{cases} 0 & \text{if } \Pi \leq 2\lambda M_L^2 \\ 2M_L\sqrt{2\lambda\Pi} - 2\lambda M_L^2 & \text{if } 2\lambda M_L^2 \leq \Pi \leq 2\lambda\overline{M}^2 \text{ and } 2\sqrt{\frac{2\Pi}{\lambda}} \in \Omega \\ \sup_{\hat{\Pi}<\Pi} W(\hat{\Pi}) & \text{if } 2\lambda M_L^2 \leq \Pi \leq 2\lambda\overline{M}^2 \text{ and } 2\sqrt{\frac{2\Pi}{\lambda}} \notin \Omega \\ 4\lambda M_L\overline{M} - 2\lambda M_L^2 & \text{if } \Pi \geq 2\lambda\overline{M}^2 \end{cases},$$

and (v) Arbitrage is still limited.

Explicit Contracting (Continued)

Proposition (Contracting and the Limits to Arbitrage)

If $\delta x^*_M > 4\overline{M} > 4M_L$, then allowing explicit contracts leads to more severe limits to arbitrage.

- **Intuition:** The bank has *less* to lose from deviation because it keeps its profits from continuing to employ proprietary traders.
  - ↪ Bank needs a bigger share of profits to pre-commit $\Rightarrow$ δx^* becomes smaller.
  - ↪ Arbitrage is less limited only if explicit contracting *alone* dominates relational contracting.
- Similar intuition to Baker-Gibbons-Murphy (1994).

The Market Structure of Arbitrage

- **Market for Funding Arbs:** Also includes *fund-of-funds* and *seeders*. Both financiers provide capital to *multiple* Arbs at once.
- The emergence of these financiers is consistent with an *optimal institutional response* to our agency problem.
  - ↪ *Break link* between expertise and expropriation.
  - ↪ *But:* Can share one Arb's info with another fund in their portfolio.
  - ↪ Contracting is consistent with avoiding this (Cestone-White (2003)).
- **But Investment Banks Provide the Bulk of Financing... Why?**
  - ↪ Other market participants wealth constrained?
  - ↪ Bank advantage due to joint production?
- **Benefit and Cost of Chinese Walls:** An expertise-expropriation risk trade-off.

Financing Innovation

- **General Informable Finance:** Similar framework to our model is likely to be relevant in the market for financing innovation.
- **On Cisco's Reputation and Access to Investment Opportunities:** *"We are extremely conscious that corporations and entrepreneurial ventures can be in conflict... if [the entrepreneurs] don't know us or have never interacted with Cisco, there is an initial concern that needs to be overcome... We've overcome such concerns by building a track record. We have enough references within the venture capital community that we can say, 'Hey, why don't you talk to John Doerr or go to Don Valentine and ask them what they think about having Cisco as an investor.' Pretty unanimously we get over the hurdle."* (Mike Volpi, Head of Cisco Ventures, in Gupta p.32)
- **Implications of More General Model on the Market for Financing Innovation:**
  - ↪ Importance of geographical clustering and social networks.
  - ↪ Big innovations and innovation waves.
  - ↪ Insufficient and distorted incentives.
  - ↪ Discussed in more detail in Kondo-Papanikolaou (2006).

Conclusion

- **Limited Arbitrage** ⇒ Informable finance fails when it is needed *most*.
- Why and when Arbs are so secretive.
- Similar environments may exhibit very *different* levels of mispricing.
- Competition, higher Arb wealth, and explicit contracting can actually *worsen* the limits to arbitrage problem.
- Implications for the market structure of arbitrage ⇒ Possible to find smart financiers that can't expropriate?
- Economically broader applications of the model... **Informable Finance and the Limits of Reputation**.

  ↪ Please tell us if you have specific examples or anecdotes!