# Investigation of the Background $\gamma$ Radiation and Poisson Statistics

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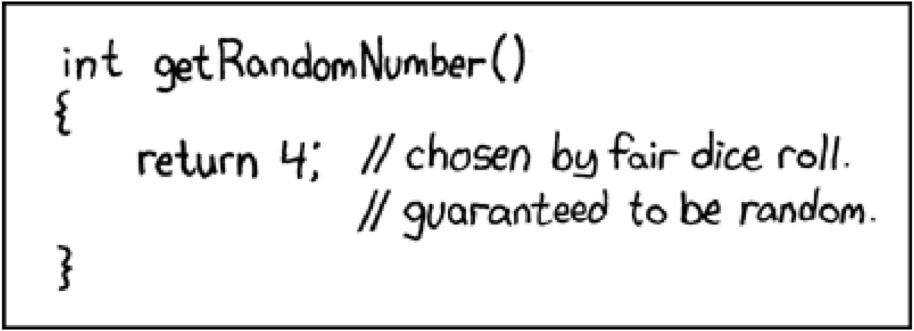
# Random Variables

Question: What does it mean for something to be "random"?

# Random Variables

## XKCD's Answer:



# Random Variables

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Wikipedia's Answer: "Let $(\Omega, \mathcal{F}, P)$ be a probability space and $(E, \mathcal{E})$ a measurable space. Then an $(E, \mathcal{E})$*-valued random variable* is a function $X : \Omega \to E$ which is $(\mathcal{F}, \mathcal{E})$-measurable.
"The expanded definition is following: a probability space is a triple consisting of:

- the sample space $\Omega$ — an arbitrary non-empty set,
- the $\sigma$-algebra $\mathcal{F} \subseteq 2^{\Omega}$ (also called $\sigma$-field) — a set of subsets of $\Omega$, called events, such that:
  - $\mathcal{F}$ contains the empty set: $\emptyset \in \mathcal{F}$,
  - $\mathcal{F}$ is closed under complements: if $A \in \mathcal{F}$, then also $(\Omega \setminus A) \in \mathcal{F}$,
  - $\mathcal{F}$ is closed under countable unions: if $A_i \in \mathcal{F}$ for $i = 1, 2, \ldots$, then also $(\bigcup_i A_i) \in \mathcal{F}$
- the probability measure $P : \mathcal{F} \to [0, 1]$ — a function on such that:
  - $P$ is countably additive: if $\{A_i\} \subseteq \mathcal{F}$ is a countable collection of pairwise disjoint sets, then $P(\bigsqcup A_i) = \sum P(A_i)$, where '$\bigsqcup$' denotes the disjoint union,
  - the measure of entire sample space is equal to one: $P(\Omega) = 1$."

# Random Variables

Yikes!

# Random Variables

Let's try again.

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Question: What does it mean for something to be “random”?

Answer:

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Question: What does it mean for something to be “random”?

Answer: Why do we care?

# Random Variables

Question: Why *do* we care?

# Random Variables

Question: Why *do* we care?

Answer: We want to be able to talk quantitatively about the relationship between measurements and theory.

# Random Variables

Question: What does it mean for something to be "random"?

# Random Variables

**Law of Large Numbers:** The average of a series of identical statistically independent random variables almost always converges to the true mean.

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# Random Variables

## Law of Large Numbers:

- $\lim_{n\to\infty} P\left(|\overline{X_n} - \mu| > \varepsilon\right) = 0$ (weak law)
- $P\left(\lim_{n\to\infty} \overline{X_n} = \mu\right) = 1$ (strong law)

# Random Variables

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Law of Large Numbers: The average of a series of identical statistically independent random variables almost always converges to the true mean.

# Random Variables

**Question:** Why is the law of large numbers enough?

**Answer:**

# Random Variables

Question: Why is the law of large numbers enough?

Answer: Because *everything* is a random variable!

# Binomial Distribution

Consider an event with two outcomes.

# Binomial Distribution

$$P(x; n, p) = \binom{n}{x} p^x q^{n-x}$$
$$= \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

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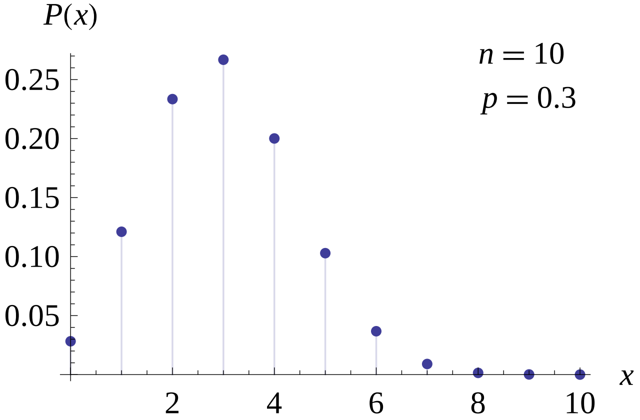
# Binomial Distribution

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x}$$



# Poisson Distribution

Consider the limit of $p \ll 1$, with $\mu = np$ fixed.

$$P(x;\mu) = \frac{\mu^x}{x!}e^{-\mu}$$

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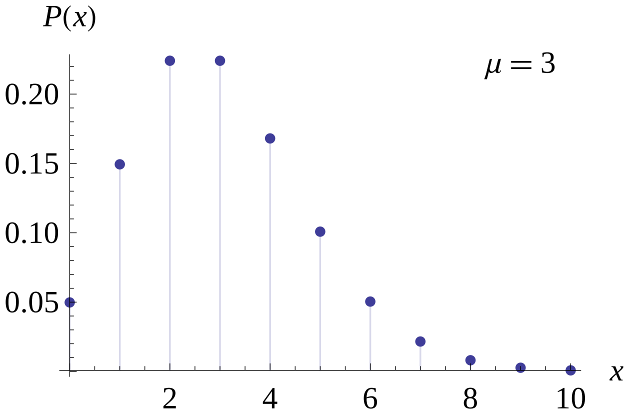

# Poisson Distribution

Question: Why do we care about this limit?

Answer:

# Poisson Distribution

Question: Why do we care about this limit?

Answer: It's everywhere! (when we count statistically independent events)

# Gaussian Distribution

Consider the limit for $\mu$ large.

$$P(x;\mu) = \frac{1}{\mu\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\mu}\right)^2}$$

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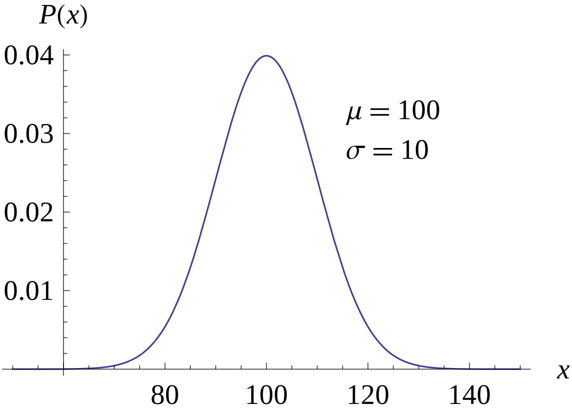
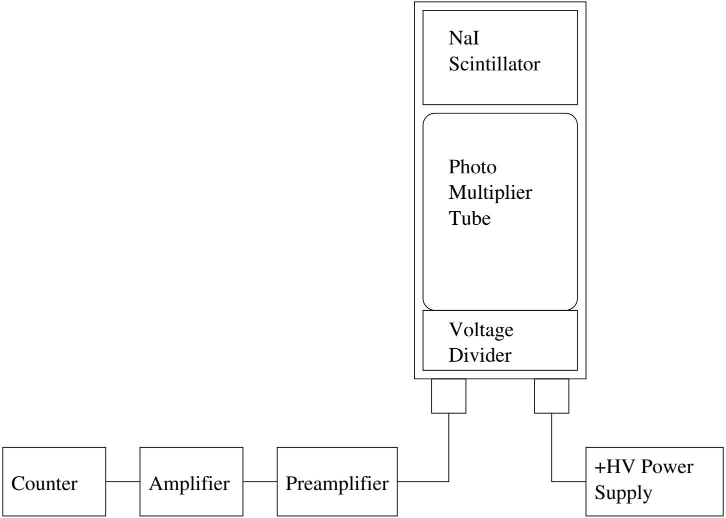
$$P(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



# Methodology



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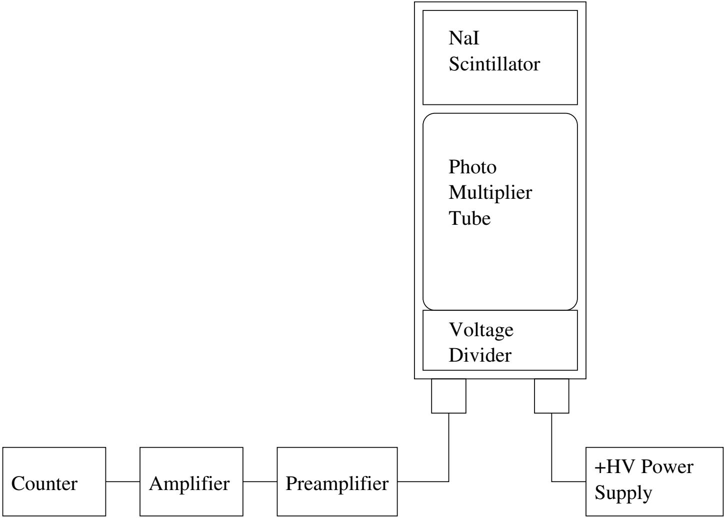

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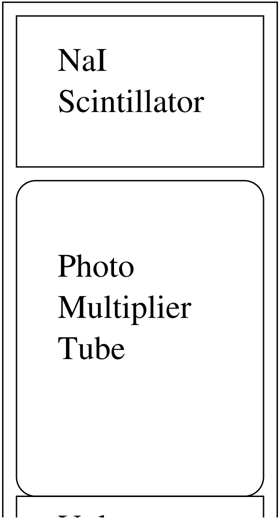

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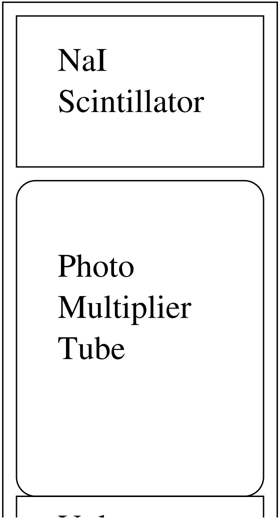
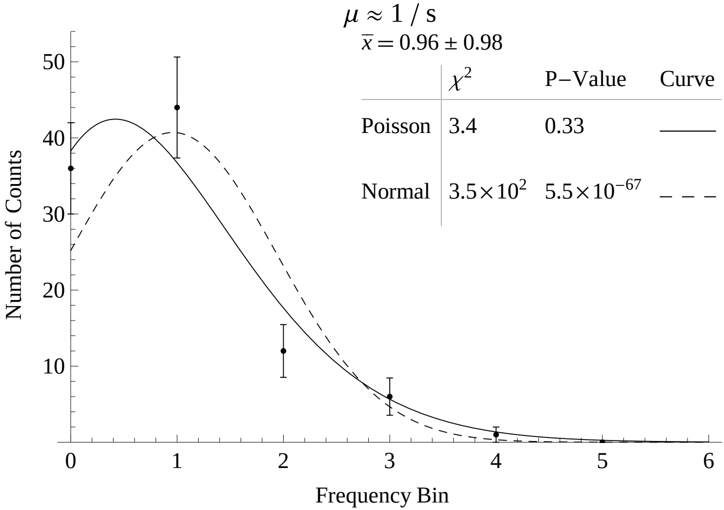
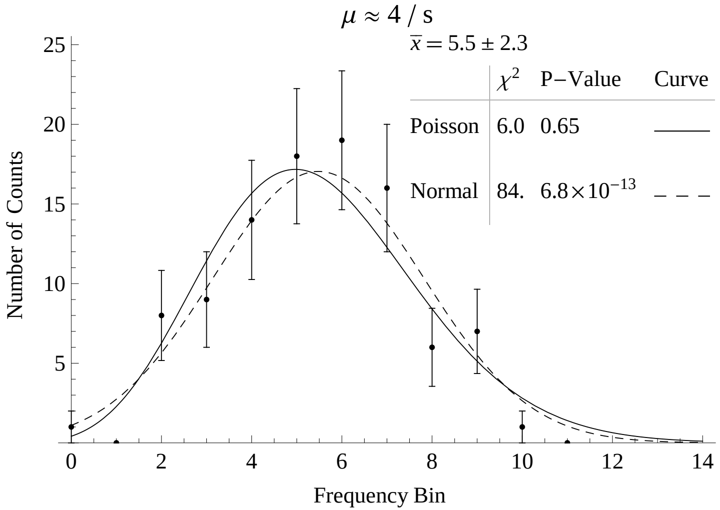


# Results

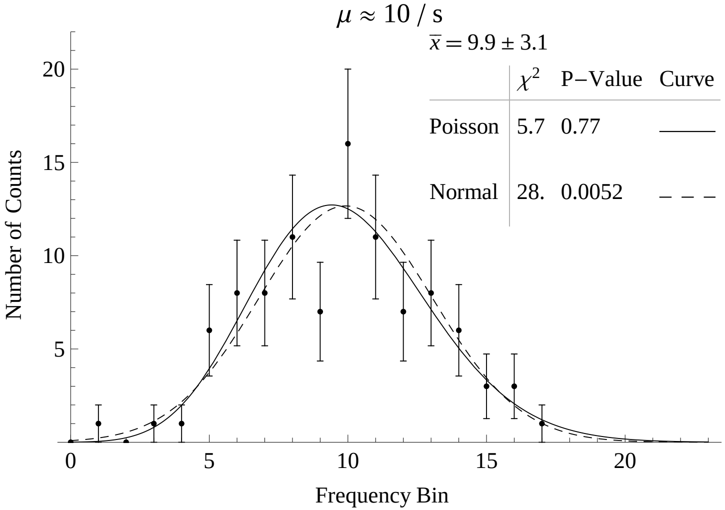

μ ≈ 1 / s
x̄ = 0.96 ± 0.98
χ2
P−Value
Curve
Poisson
3.4
0.33
Normal
3.5×10^2
5.5×10^−67
Number of Counts
Frequency Bin
0 1 2 3 4 5 6
10 20 30 40 50

# Results



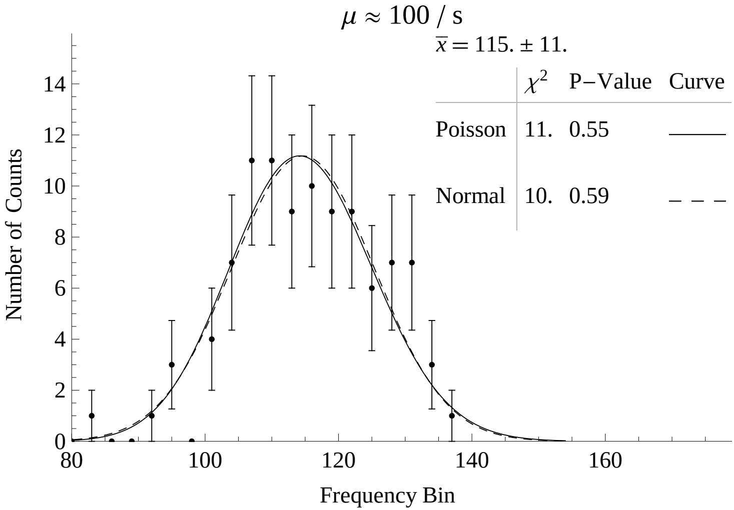
μ ≈ 4 / s
x̄ = 5.5 ± 2.3
χ² P−Value Curve
Poisson 6.0 0.65
Normal 84. 6.8×10⁻¹³
Number of Counts
Frequency Bin
0
2
4
6
8
10
12
14
5
10
15
20
25

# Results



$\mu \approx 10$ / s
$\bar{x} = 9.9 \pm 3.1$
$\chi^2$ P–Value Curve
Poisson 5.7 0.77
Normal 28. 0.0052
Number of Counts
Frequency Bin
0
5
10
15
20

# Results



$\mu \approx 100$ / s
$\overline{x} = 115. \pm 11.$
$\chi^2$ P−Value Curve
Poisson 11. 0.55
Normal 10. 0.59
Number of Counts
0
2
4
6
8
10
12
14
80
100
120
140
160
Frequency Bin

# Results

| $\mu$ | Poisson $\chi^2$ | Poisson P-value | Normal $\chi^2$ | Normal P-value |
|---|---|---|---|---|
| $\mu \approx 1$ | 3.4 | 0.32 | 350 | $5.5 \cdot 10^{-67}$ |
| $\mu \approx 4$ | 6.0 | 0.65 | 84 | $6.8 \cdot 10^{-13}$ |
| $\mu \approx 10$ | 5.7 | 0.77 | 28 | 0.0052 |
| $\mu \approx 100$ | 11 | 0.55 | 10. | 0.59 |

Thank You!

Any questions?