WAVE PROPAGATION
An Introduction to Engineering Analyses

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Wave Propagation
— An Introduction to Engineering Analyses —

Sophomores → (2.001 and 2.003) …
Style: Extended and Repetitive Expositions
200+ End-of-Chapter Problems

Chapter 1 Introduction to Wave Propagation

Vignette I

Chapter 2 The Classical Wave Equation

Vignette II and Vignette III

Chapter 3 Wave Propagation in Infinite Media

… ■ Examples … // … ♦ Examples …

Vignette IV

Chapter 4 Wave Propagation in Semi-Infinite Media

Vignette V and Vignette VI

Chapter 5 Wave Propagation in Finite Media
Classical Wave Equation

\[ \frac{\partial^2 z(x,t)}{\partial t^2} = c^2 \frac{\partial^2 z(x,t)}{\partial x^2} \]

Wave Functions

\[ z(x,t) = f(x - ct) + g(x + ct) \]

where \( f(x) \) and \( g(x) \) are sample functions.
Infinite Continua

Strings  Rods  Circular Shafts
Shear Beams  Electric Transmission Lines

Initial Conditions on Infinite Systems

Initial conditions: $\xi(x,0) = A(x)$ and $\dot{\xi}(x,0) = 0$, where

$$A(x) = \begin{cases} 
A_0(1 - x^2/a^2), & |x| \leq a \\
0, & |x| > a
\end{cases}$$

depicted as

Find: $\xi(x,t)$ [and $\dot{\xi}(x,t), F(x,t)$]
Displacements of uniform rod released from rest, shown at increasing times.
Domains 1 through 4 on $x$-$t$ plane.
Three-dimensional schematic of displacement wave propagation in rod.
• Domain of Dependence

• Time Lags

• Transmission of Energy by Arbitrary and Harmonic Waveforms
♦ Fourier Series

♦ Fourier Integral of Tone Burst Wave

\[ f(x) \]

♦ Ultrasonic Attenuation of Tone Burst Wave

♦ NDE of Impact-Damaged Fiber Composites

♦ NDE of Fatigued Fiber Composites
• Chapter 4  Wave Propagation in Semi-Infinite Media

Reflection and Transmission Coefficients at Junctions; and Reflection Coefficients at Boundaries

\[ \xi_i = f_i(x - c_1 t) \]

\[ \xi(x, t) \]

\[ x = 0 \]

Rod 1
\[ \rho_1, A_1, E_1 \]

Rod 2
\[ \rho_2, A_2, E_2 \]

• Vignettes

• Chapter 5  Wave Propagation in Finite Media

One-Dimensional Wave Fields in Finite Media
[Timewise Global and Point Variations]
Vignettes

I. Is There a Smallest Quantity of Energy?

\[ h = 6.62607015 \times 10^{-34} \text{ J} \cdot \text{s} \]

\[ \mathcal{E} = h \cdot \bar{f} \]

\[ J \cdot \text{s} = (\text{N} \cdot \text{m}) \cdot (\text{s}) = (\text{kg} \cdot \frac{\text{m}}{\text{s}^2}) \cdot \text{m} \cdot \text{s} = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1} \]

II. Gravitational Waves & Laser Interferometer Gravitational-Wave Observatory (LIGO)

III. NDE of Composite Materials and Structures

IV. Sound Waves and Sound Channels in the Ocean

V. Domino Waves

VI. Falling Slinky
II. Gravitational Waves & LIGO

Flat space-time

Curved space-time
IV. Sound Waves and Sound Channels in the Ocean
V. Domino Waves
Field and Point Timewise Motion

Free-free finite length rod.

Applied force $F(t)$.

Midpoint displacement $\xi(l/2, t)$ of elastic rod.
Video of Falling Slinky

https://tinyurl.com/y2psyp7y
(Deactivated via this Site)
VI. Falling Slinky

Statically held Slinky

Bottom end of Slinky remains motionless

(a) (b) (c)

Downward moving top creating collapsed region

\[ \hat{g} \]
Collision of Elastic Rods

Initial Displacement: $\xi(x, 0) = 0, \ 0 < x < 3l$

Initial Particle Velocity:

$$\dot{\xi}(x, 0) = \begin{cases} 
 v_0, & 0 < x < l \\
0, & l < x < 3l 
\end{cases}$$
Collision of Elastic Rods

Initial Displacement: $\xi(x, 0) = 0, \ 0 < x < 3l$

Initial Particle Velocity:

$$\dot{\xi}(x, 0) = \begin{cases} v_0, & 0 < x < l \\ 0, & l < x < 3l \end{cases}$$

$$= \frac{v_0}{3} + \begin{cases} \frac{2v_0}{3}, & 0 < x < l \\ -\frac{v_0}{3}, & l < x < 3l \end{cases}$$

*Rigid-body and elastic* initial particle velocities.
Displacement waves, rigid-body displacement, and their sums for elastic rod system after collision.
Displacement waves, rigid-body displacement, and their sums for elastic rod system after collision (continued).
Chapter 5

First reflection of initial left-going wave

$F(x, 0)$

$\frac{EA v_0}{2c_p}$

$\frac{EA v_0}{4c_p}$

$R_F = -1$

$x = 0$

$\sum$

$l$

$2l$

$x = 3l$

(a) $t = 0$.

$F(x, t_1)$

$\frac{EA v_0}{2c_p}$

$\frac{EA v_0}{4c_p}$

$-\frac{EA v_0}{2c_p}$

$-\frac{EA v_0}{4c_p}$

$R_F = -1$

$x = 0$

$\sum$

$l$

$2l$

$x = 3l$

First reflection of initial left-going wave

(b) $t = t_1 = l/2c_p$.

Force waves in elastic rod system after collision.
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Dedication

To A. Neil (1964) and Jane Pappalardo, by measure of profound and indelible devotion and love, MIT Royalty.