Transition challenges for alternative fuel vehicles: Consumer acceptance and sustained adoption

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Technical appendix

1 Introduction

The model described in this Essay is designed to capture the diffusion of and competition among multiple types of alternative vehicles, along with the evolution of the ICE fleet. For example, the model can be configured to represent ICE and alternatives such as ICE-electric hybrid, CNG, HFCV, biodiesel, E85 flexfuel, and electric vehicles. However, the Essay focuses on intuition about the basic dynamics around the diffusion of alternatives to ICE by considering two platforms, ICE and an alternative vehicle, and makes a number of other simplifying assumptions that allow us to explore the global dynamics of the system. In this appendix I discuss additional components of the full model, highlighting those structures required to capture the competition among multiple alternative platforms.

The appendix is divided into three sections. The first section provides elaborations on the model. The second section provides connections to, and differences from the original Bass structure as discussed in the Essay. The model and analyses can be replicated from the information provided in the Essay. The last section provides a link to the full model and analysis documentation.

2 Elaborations on the model

This section elaborates segments of the model that were highlighted in the paper but not fully expanded due to space limitations.

a) Vehicle fleet aging chain

For simplicity, the age structure of the fleet is not treated in the paper. Below we lay out how this is incorporated in the full model.

The total number of vehicles for each platform *j, j={1,..., J}*, of each age cohort *m,* V_{j_m}, accumulates net vehicle replacements and aging (see Figure 1):

$$\frac{dV_{j_m}}{dt} = v^r_{j_m} + v^a_{j_m} \quad \text{(A1)}$$

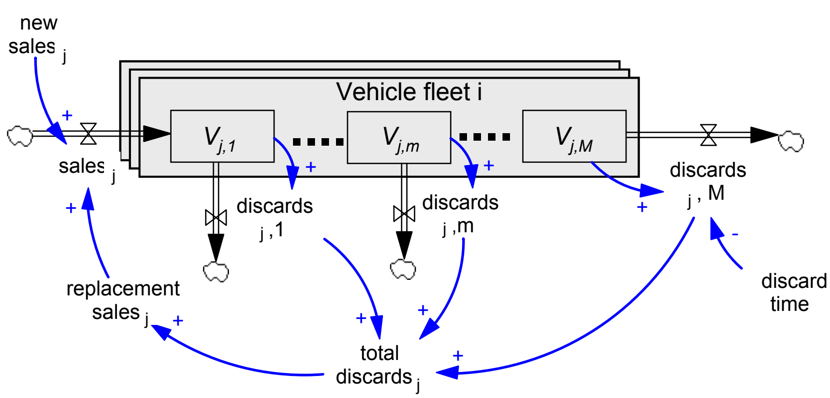

Figure 1 Vehicle replacement with aging chain.

Aging captures vehicles coming from a younger cohort less those aging into the next cohort:

$$v^{a}_{j_m} = v^{a+}_{j_m} - v^{a-}_{j_m} \tag{A2}$$

with

$$v^{a+}_{j_m} = \begin{cases} 0 & m=1 \\ v_{i_{m-1,m}} & m>1 \end{cases} \qquad v^{a-}_{j_m} = \begin{cases} v_{i_{m,m+1}} & m \le M \\ 0 & m=M \end{cases} \tag{A3}$$

while

$$v_{i_{m,m+1}} = f^{s}_{i_m} V_{i_m} \Big/ \tau^{c} \tag{A4}$$

Where $f^{s}_{i_m}$ is the survival fraction for each cohort.[1,2]

Net vehicle replacements are new vehicle sales, s_{j_m}, less age dependent discards, d_{j_m}:

$$v^{r}_{j_m} = s_{j_m} - d_{j_m} \tag{A5}$$

We do not consider the used car market here. New vehicle sales enter the first age cohort, thus:

$$s_{j_m} = \begin{cases} s_j & m=1 \\ 0 & m>1 \end{cases} \tag{A6}$$

Total sales for platform j, s_j, consist of initial and replacement purchases:

$$s_j = s^{n}_{j} + s^{r}_{j} \tag{A7}$$

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[1] Annual survival (and/or scrappage) rates by model year can be derived from registration data (e.g. by L. Polk &Co, AAMA).

[2] In equilibrium average vehicle life λ^{v} is found by: $\lambda_j = \sum_{m=1}^{M-1} \left(\prod_{m'=1}^{m-1} f^{r}_{j_{m'}} \right) \lambda^{c} + \prod_{m'=1}^{M-1} f^{r}_{j_{m'}} \lambda^{c_M}$

The full model allows for growth in the fleet as population and the number of vehicles per person grow. In the paper population and the number of vehicles per person are assumed constant, implying the total fleet is in equilibrium and initial purchases are zero. Vehicles sales for platform *j* arise from the replacement of discards from any platform *i* and cohort *m*, $d^r_{i_{m'}}$:

$$\sum s^r_j = \sum_{i,m'} \sigma_{i_{m'}j} d^r_{i_{m'}} \tag{A8}$$

where $\sigma_{i_n j}$ is the share of drivers of platform *i* cohort *n* replacing their vehicle with a new vehicle of platform *j*. The share switching from *i* to *j* depends on the expected utility of platform *j* as judged by the driver of vehicle *i*, cohort *n*, $u^e_{i_n j}$, relative to that of all options $u^e_{i_n j'}$.

Thus:

$$\sigma_{i_m j} = \frac{u^e_{i_m j}}{\sum_{j'} u^e_{i_m j'}} . \tag{A9}$$

To capture a driver's consideration set we introduce the concept of familiarity among drivers of vehicle *i* with platform *j*. The model can be elaborated to include cohort-specific levels of familiarity, recognizing that drivers of, say, a 10 year old ICE vehicle have a different (presumably lower) familiarity with new ICE vehicles than the driver of a 1 year old vehicle. Such distinctions may matter when vehicle attributes change rapidly, as is likely for early AFVs as experience and technology rapidly improve. (Further disaggregation would eventually lead to an agent-based representation where each driver has an individual-specific level of familiarity with different platforms).

These issues will be treated in future work. For simplicity I assume here that familiarity is equal across all cohorts of a given platform and remains F_{ij} , thus expected utility is:

$$u^e_{i_m j} = F_{ij} * u_{i_m j} . \tag{A10}$$

b) Initial purchases and fleet growth

New car sales for fleet *j* are:

$$s^n_j = \sigma^n_j s^n \tag{A11}$$

where the share σ_j is equal to the share of replacement sales: $\sigma^n_j = s^r_j \Big/ \sum_i s^r_i$.

Total new car sales allow the total fleet $V = \sum_{j,m} V_{j_m}$ to adjust to its indicated level V*:

$$s^n = \frac{\max\left[0, \left(V^* - V\right)\right]}{\tau^v} \tag{A12}$$

where total desired vehicles $V^* = \rho^v * H$ is product of the target or desired number of vehicles per household ρ and total households H, and τ^v is the fleet adjustment time. The max function ensures sales remain nonnegative in the case where V^* falls below V (a possibility if there is a large unfavorable shift in the utility of AFVs when the installed base is small).

Discards, d_{j_m} are found by:

$$d_{j_m} = \begin{cases} (1 - f^s_{j_m}) V_{j_m} \Big/ \lambda^c & m < M \\ V_{j_m} \Big/ \lambda^{cM} & m = M \end{cases} \tag{A13}$$

where λ^c is the cohort residence time; λ^{cM} is the residence time of the last cohort.

The number of discards people choose to replace is give by:

$$d^r_{j_m} = f^r d_{j_m} \tag{A14}$$

where f^r is the nonnegative part of the difference between total discards and the indicated contraction rate as a fraction of the total discard rate:

$$f^r = \frac{\max\left[0, d - v^{c*}\right]}{d} \tag{A15}$$

Here $d = \sum_{i,m} d_{i_m}$ is total discards, and $v^{c*} = \frac{\max\left[0, V - V^*\right]}{\tau^v}$ is the indicated fleet contraction rate. The fleet of a particular platform can contract when, for example, the perceived utility of that platform suddenly falls (say, due to unfavorable shifts in fuel costs or perceived safety, reliability, or costs) and if the existing installed base is small enough and young enough so that discards from normal aging are small.

c) Co-flows

The model accounts for transfer of familiarity and perceived performance associated with those drivers who switch platforms. I will capture this through the co-flow structure (Sterman 2000). The formal structure is identical for both and I will discuss the familiarity co-flow as an example. The familiarity of drivers of platform *i* with platform *j* is updated through social exposure, as discussed in the paper. When a driver switches from platform *i* to *k*, their familiarity with platform *j* is transferred from F_{ij} to F_{kj}. For example, consider a model in which three platforms are portrayed, say, ICE, hybrids, and HFCVs (denoted platforms 1, 2, and 3, respectively). When an ICE driver switches to a hybrid, the familiarity of that driver with HFCVs, previously denoted F_{13}, now becomes

F_{23}. In the two platform simulations considered in the paper these dynamics do not matter since all drivers are assumed to be fully familiar with ICE, and AFV drivers are assumed fully familiar with AFVs, so the only dynamic relates to the growth of familiarity of ICE drivers with AFVs (F_{12}).

To model the transfer of familiarity as drivers switch platforms, it is convenient to consider the evolution of familiarity at the population level:

$$\frac{d(F_{ij}V_j)}{dt} = V_i \frac{dF_{ij}}{dt} + F_{ij}\frac{dV_i}{dt} = f_{ij}^u + f_{ij}^t \quad \text{(A16)}$$

where the first term, which we call f_{ij}^u, captures updating of familiarity with platform *j* by drivers of platform *i*, as discussed in the paper. The second term, denoted f_{ij}^t, captures the transfer of familiarity arising from drivers who switch platforms. When familiarity is updated much faster than fleet turnover (and therefore switching), the second term has limited impact on the dynamics of familiarity. On the other hand, when fleet turnover is very fast, the transfer of familiarity as drivers switch platforms can be important.

Familiarity updating is formulated as described in the paper: updating of total familiarity is the average update from social exposure, including familiarity decay (equation 5 of the paper), over the total number of drivers V_i:

$$f_{ij}^u = \left[\eta_{ij}\left(1 - F_{ij}\right) - \phi_{ij}F_{ij}\right]V_i \quad \text{(A17)}$$

where η_{ij} is the total impact of total social exposure to platform *j* on the increase in familiarity for drivers of platform *i*, and ϕ_{ij} is the fractional loss of familiarity about platform *j*.

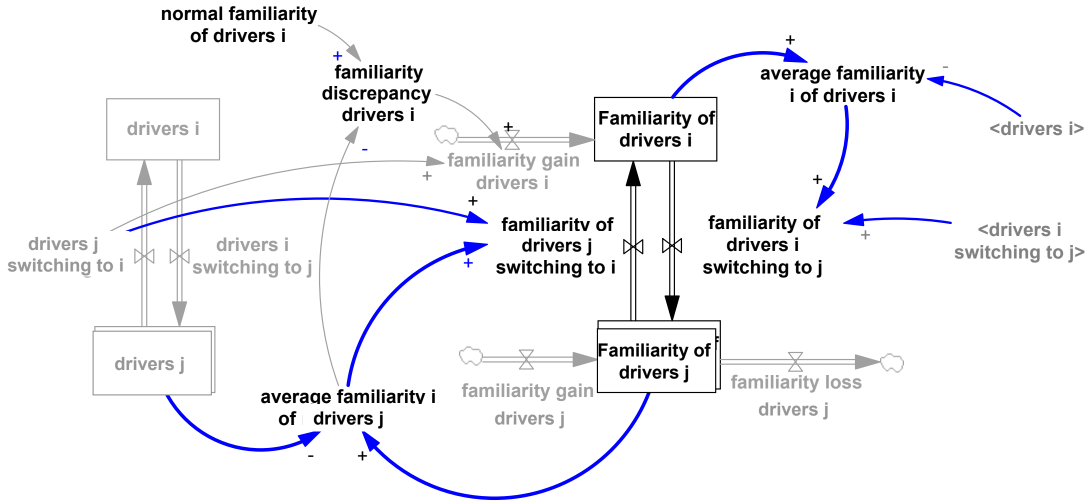

Figure A2 Familiarity change for drivers that switch between platforms

The transfer term captures two that track the movement of the familiarity of a driver of platform *i* with platform *j*, one arising from vehicle sales and one arising from discards:

$$f_{ij}^{t} = f_{ij}^{s} - f_{ij}^{d} \,. \tag{A18}$$

The first term, f_{ij}^{s} , captures the transfer of familiarity through sales:

$$f_{ij}^{s} = s_i^n F_{ij} + \begin{cases} \sum_k s_{ki}^r F_{kj} & i \neq j \\ \sum_{k \neq j} s_{ki}^r & i = j \end{cases} \tag{A19}$$

This term contains the flow of new drivers purchasing platform *i*, and their average familiarity with platform *j*, assumed to equal the familiarity of current drivers of *i* with platform *j*. The second term is the transfer of familiarity associated with the flow of drivers of platform *k* replacing their vehicles with one of platform *i*. The average familiarity of these drivers with platform *j* is transferred as they switch. We assume drivers become fully familiar with the platform they are driving, so those who purchase a vehicle of platform *j* (the case *i*=*j*) achieve full familiarity with platform *j* (in a time much shorter than the other time constants).

The second term in equation (A18) captures the transfer of familiarity with platform *j* associated with drivers of platform *i* through discards:

$$f_{ij}^{d} = d_i F_{ij} \tag{A20}$$

where $d_i \equiv \sum_m d_{i_m}$ is total discards .

The transfer term f_{ij}^{t} was used in the simulations of the paper, for the relevant cases. The transfer of familiarity as drivers switch platforms has a small but significant contribution to the dynamics: early alternative fuel adopters who switch back from the alternative to ICE have full familiarity with the AFV, and contribute strongly to word of mouth. Technically, a balancing loop is generated, in similar fashion as marketing effectiveness, with strength $\left[1-u_{jj}/\sum_i u_{ji}\right]/\lambda^{v}$. However, a more complicated result emerges when learning about performance through social exposure is involved, as early adopters might learn about mediocre performance. Hence, their word of mouth results in lower perceived attractiveness of alternatives among others.

Other co-flows that follow the same logic are those guide an adjustment of installed base performance, P_j^{f} to the new vehicle performance P_j^{n} , and those that allow the perceived performance P_{ij}^{e} to be updated when drivers switch platforms. The first one is a simple co-flow that only changes with sales and discards. The perceived performance updated implies taking all influences of Equation 17 into account. This is done by separately capturing drivers of a platform $j\,P_{jj}^{e}$, and non-drivers of platform $j, P_{ij}^{e}, i \neq j$.

3 Stipulations

a) Equivalence to Bass

Here we recover the Bass equation from the familiarity model in this paper, for durable goods, with validity for low familiarity. The formulation differs from those of the standard Bass models through the decoupling of exposure, familiarity and the adoption decision, the word of mouth through non-users and the discrete choice replacement, for durable goods.

The original Bass model is describes diffusion of isolated technologies and is specified as follows:

$$\frac{dV^{B}}{dt} = \left(\alpha^{B} + c^{B}\left(V^{B}/N\right)\right)\left(N - V^{B}\right) \qquad \text{(A21)}$$

Where the marketing effectiveness α^{B} and contact rate c^{B} have the same interpretation as in the familiarity model. The functional form is a the logistic growth and the associated dynamics yield an S-shape curve.

To recover the Bass model, we ignore population change, which is sensible with the shorter time horizons of product replacements in usual Bass settings), aging chains (the arguments below can be easily expanded), heterogeneity in contact effectiveness, and word of mouth through non users, and denote this simplified version '*1*':

$$\frac{dF_{ij}^1}{dt} = \eta_{ij}^1\left(1-F_{ij}^1\right) - \phi\left(\eta_{ij}^1\right)F_{ij}^1 \qquad \text{(A22)}$$

We ignored here the higher order terms that involve transfer of familiarity through sales and discards. Simplifying further, setting contact effectiveness between drivers of platforms *j* with non-drivers equal for all *j* and *i≠j*, with $c^d = c_{ijj}$,for all *i≠j*:

$$\eta_{ij}^1 = \alpha_j + c^1\left(V_j^1/N\right) \qquad \text{(A23)}$$

Now we specify the sales rate for a platform *j*, which is identical to the actual familiarity model $s_j = \sum_{i\neq j}\sigma_{ij}V_i/\lambda$, with λ being the vehicle life.

We further assume perceived utility to equal actual utility and derive the Bass equation for durable goods, with validity for low familiarity. When the product of familiarity and relative attractiveness is low, we can make a first order approximation for the share going to i from j:

$$\sigma_{ij} = \begin{cases} \dfrac{F_{ij}u_j}{u^o + u_i + \sum_{j'\neq i}F_{ij'}u_{j'}} \approx \tilde{u}_{-j}F_{ij} & i\neq j; \tilde{u}_{-j} \equiv \dfrac{u_j}{u^o+u_i} \\ \dfrac{u_j}{u^o+u_j+\sum_{j'}F_{jj'}u_{j'}} \approx \tilde{u}_j & i=j; \tilde{u}_j \equiv \dfrac{u_j}{u^o+u_j} \end{cases} \qquad \text{(A24)}$$

Then, letting all alternatives to *j* yield the same utility, the net sales rate equals the new vehicle sales minus the discards that are not replaced:

$$
\begin{aligned}
\frac{dV_j}{dt} &= s_j - d_j \approx \left[\sum\nolimits_{k\neq j} \tilde{u}_{-j} F_{kj}\right]\frac{1}{\lambda} - \left(1-\tilde{u}_j\right)\frac{V_j}{\lambda} \\
&= \frac{1}{\lambda}\left(\tilde{u}_{-j} F_j \left(N - V_j\right) + \left(1-\tilde{u}_j\right) V_j\right)
\end{aligned}
\tag{A25}
$$

Further, with adoption dynamics slow relative to the familiarity dynamics, which is justified for durable goods, we use the steady state familiarity as a function of the number of adopters. Ignoring the word of mouth through non-drivers, and the higher order terms that include that include transfer of familiarity through discards:

$$
\frac{dF_{ij}^1}{dt} = \eta_j^1\left(1-F_{ij}^1\right) - \phi\left(\eta_j^1\right) F_{ij}^1 = 0 \tag{A26}
$$

Using a piecewise linear expression for equation (7):[3]

$$
\phi\left(\eta_j\right) = \begin{cases} 0 & \eta_j < \eta_0 - \dfrac{0.5}{\varepsilon} \\ \phi_0\left[0.5 + \varepsilon\left(\eta_0 - \eta_j\right)\right] F_{ij} & \textit{otherwise} \\ \phi_0 & \eta_j > \eta_0 + \dfrac{0.5}{\varepsilon} \end{cases}
$$

We get for the equilibrium familiarity:

$$
F_{ij}^{1*} = \begin{cases} \dfrac{\eta_j^1}{\eta_j^1 + \phi_0\left(0.5 + \varepsilon\left(\eta_0 - \eta_j^1\right)\right)} & \eta_j^1 \le \eta_0 + \dfrac{0.5}{\varepsilon} \\ 1 & \eta_j^1 > \eta_0 + \dfrac{0.5}{\varepsilon} \end{cases}
$$

Then, with η_j^1 small compared to ϕ_0, and, and by definition of the interesting case where familiarity is not saturated yet, $\eta_j^1 << \eta_0 + 0.5/\varepsilon$, and thus:

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[3] This functional form for forgetting leads to results that are indistinguishable from the non-linear form used in the paper.

$$F_{ij}^{1*} \approx \eta_j^1 / \phi = \left(\alpha_j + c^1\left(V_j^1/N\right)\right) / \phi_0 \quad \text{(A27)}$$

and thus, combining with equation (A25):

$$\frac{dV_j^1}{dt} \approx \left[\frac{\tilde{u}_{-j}}{\phi_0 \lambda}\left(\alpha_j + c^1\left(V_j^1/N\right)\right)\left(N - V_j^1\right) - \frac{\left(1-\tilde{u}_j\right)}{\lambda} V_j^1\right]$$

Equilibrium conditions are found by including the exit rate from discards. This is discussed elsewhere. Here we are interested to convert to Bass. Which can be rewritten as:

$$\frac{dV_j^{1*}}{dt} = \kappa_j\left[\alpha_j^{'} + c\left(V_j/N\right)\right]\left(N - V_j\right) - y_j \quad \text{(A28)}$$

Where, $\kappa_j \equiv \tilde{u}_{-j} / \phi_0 \lambda$ is the conversion parameter between Bass and the familiarity model that captures the relative attractiveness, replacement rate, and forgetting rate are convoluted in the Bass model, but explicit in the familiarity model. Further, $\alpha_j^{'} \equiv \alpha_j + \left(1-\tilde{u}_j\right) / \tilde{u}_{-j}\, \phi_0$ is the adjusted marketing effect, of which the second term, in multiplication with the conversion parameter captures the "free marketing" exposure that derives from drivers who discard their vehicles and become non-drivers (which are not included in the original Bass model). Finally, $y_j = \left(1-\tilde{u}_j\right) N/\lambda$ is a constant adjustment that accounts for discards, offsetting any adoption. Note that when drivers are zero, the last two effects cancel out, naturally preserving non-negativity.

With equation (A28) we have derived at the original Bass model, except for a correction term. Note that the connection implies structural equivalence, this only held under specified conditions, for instance assuming equilibrium familiarity and ignoring the role of non-drivers. Because of the equilibrium assumption, the complex dynamics have been filtered out. This derivation illustrates the connection of the parameters of the two models, as well as an interpretation of the Bass parameters in the context of competitive platforms. This interpretation will also be used in the analysis section of the Essay.

b) Platform competition: The familiarity model compared to Bass

The Essay illustrates the strong tipping dynamics that the familiarity model reveals for competing entrants, as a function of their respective marketing programs (Figure 7b and 7c).Here we compare the dynamics of platform competition to that what can be generated by Bass models. We proxy the Bass model with platform competition, by deriving the equilibrium familiarity and absence of word-of-mouth from non-drivers (see Appendix 2a), and combine this with the multiplatform logit decision structure. Figure A3 shows the results, using exactly the same scenario as in Figure 7c and 7d of the Essay.

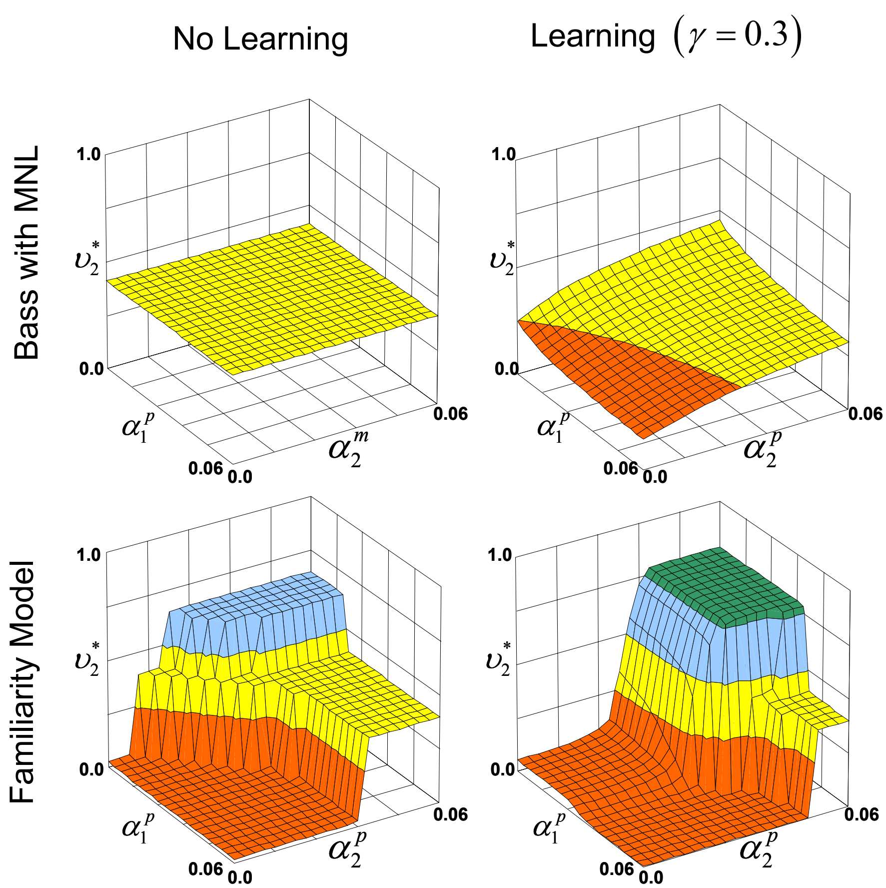

Figure A3 Multiplatform competition: comparing the Familiarity model (Bottom – identical to Figure 7b and c of the paper) with the Bass representation the Bassrepresentation, combined with the MNL decision structure (Top).

We see that the dynamics depart considerably. In absence of learning, in the Bass model there is always convergence to the equal equilibrium share (as the background marketing is nonzero, equaling 0.01). When we include learning, we see that some path-dependency is created in the Bass representation, albeit very smoothly. The results from the Familiarity model contrast greatly to this (Bottom).

4 Model and analysis documentation

The model and analyses can be replicated from the information provided in the Essay.

In addition model and analysis documentation can be downloaded from

http://web.mit.edu/jjrs/www/Thesis Documentation.htm

5 References

Sterman, J. (2000). *Business Dynamics: systems thinking and modeling for a complex world.* Boston, Irwin/McGraw-Hill.