Identifying challenges for sustained adoption of alternative fuel vehicles and infrastructure

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Technical appendix

1 Introduction

The model described in this Essay is designed to capture the diffusion of and competition among multiple types of alternative vehicles and their fueling infrastructure. In the appendix I discuss additional components of the full model, highlighting those structures required to capture the full model. Further, this appendix provides additional information to accompany the model and the analysis of Essay 2. Each subsection is pointed to from a paragraph within the Essay. Following this introduction, subsequent sections group issues by:

2 **Elaborations on the model** that provide details on expressions that were not fully expanded due to space limitations (in particular we discuss functional forms).

3 **Derivations,** which discuss expressions that be derived through closed form derivations. These were highlighted in the paper but not fully expanded due to space limitations.

4 **Notes on simulations**, providing completions or complementary information to analysis in the paper.

5 **Stipulations:** Additional notes that provide insight in the model or analysis

6 **Model analysis and documentation:** Essay 2, in combination with the first two Appendix sections allows replicating the model. The third section allows the reader to replicate those analyses that did not provide sufficient information in the Essay to do so. Here I point to additional supporting documentation to do so.

7 References

2 Elaborations on the model

This section elaborates segments of the model that were highlighted in the paper but not fully expanded due to space limitations. These elaborations include in particular selected functional forms for functions that were provided in general form in the model.

a) Refueling sensitivity parameter

The refueling sensitivity parameter captures the propensity of drivers to refuel outside the location where they reach their normal refueling buffer r_{iz}^{b} (Equation 7 of the Essay).

The two functions *g* and *h* determine how this propensity depends on the effective buffer and driving range, relative to the trip length. The functions should increase in both inputs, but are bounded, as the relevant area of sensitivity is at the order of the trip length (if one has a top-off buffer of 100 miles and the trip is 20 miles, we can typically refuel anywhere we like along the way). We use:

$$g\left(r_{iz}^{b}/r_{i\omega_{zz'}}^{t}\right)=\min\left[1,r_{iz}^{b}/r_{i\omega_{zz'}}^{t}\right]^{\eta^{b}}$$
$$h\left(\left[r_{iz}^{f}-r_{iz}^{b}\right]/r_{i\omega_{zz'}}^{t}\right)=\min\left[1,\left[r_{iz}^{f}-r_{iz}^{b}\right]/r_{i\omega_{zz'}}^{t}\right]^{\eta^{f}}$$

The default setting in the model of both η^{b} and η^{f} are set equal 1.

The reference sensitivity β_{ref}^{f} determines how the refueling location sensitivity is constraint by a combination of behavioral and physical constraints of refueling on a different location. Formally, it gives the elasticity of refueling shares to a change in the utility of a refueling at a location, when the refueling buffer equals the trip length determined by the physical and behavioral factors. Here we assume this to be fixed (see

Table 3). Note that if drivers always intend to top-off at their topping-off buffer, this parameter is zero.

b) Scale economies for fuel stations

Fixed costs are defined in Equation 23 of the Essay as:

$$c_{vs}^{k} = c_{vs}^{k,ref} f^{k}\left(y_{vs}/y^{ref}\right); f\left(0\right) > 0; f\left(1\right) = 1; f' > 0; f'' < 0$$

Economies of scale in fuel station cost follow the standard diminishing returns to scale function:

$$f^{k}\left(y_{vs}/y^{ref}\right) = \left(y_{vs}/y^{ref}\right)^{\eta^{k}}$$

Further, station cost may, labor, and land may differ per region. In particular fuel stations in urban areas have a totally different costs than those in rural areas. Higher cost in urban areas wil suppress expansion and entrance. Population is a good proxy for consistent variation between them. Thus, I include a population dependenent factor:

$$c_{vs}^{k,ref} = c_{vs}^{k,o} + c_{vs}^{k,h}\left(h_{s}/h^{avg}\right)^{\eta^{h}}$$

In the simulation I use the following parameters: $\eta^{k} = 0.25$, and

$\eta^{h} = 0.25; c_{vz}^{k,o} = 250,000; c_{vz}^{k,h} = 250,000$

Note that these parameter settings disvafovor adoption in urban areas relative to rural areas. Effects of excluding this have limited impact on the dynamics.

A note on explicit representation of multi-fuel stations:

Assuming that scale economies do not hold across technologies, it is reasonable to exclude the role of multi-fuel stations. Under such conditions, we can see a multifuel station as two neighbouring monofuel stations. This is assumption is reasonable when involving entirely different fuels such as natural gas and gasoline. Flexfuels are more likely to be substitutes from the station's perspective and require some more complicated scaling. In that case multi-fuels might offer lower barriers than specialized stations. In the current simulations I exclude the explicit representation of hybrid fuel stations.

c) Entry and exit sensitivity to profits

Equation 30 in the Essay defines the industry growth rate as

$$K_v^n = g_v^k K_v$$
$$g_v^k = g_v^{k0} f^g\left(\overline{\pi}_v^e\right); f\left(\ll 0\right) = 0; f\left(0\right) = 1; f' \geq 0; f'\left(\gg 1\right) = 0;$$

The constraints imply, first, that the growth rate equals g_v^{k0} when perceived returns on investment equal desired returns; second, that the growth rate increases with return on investment, which could differ by fuel, because of potential variation in constraints. Further, the shape is bounded, at zero, for extremely negative profits, and, at some finite value, for extremely high returns. The most general shape that satisfies these conditions is an S-shape. The logistic curve is used here, with the following parameter settings:

$$f^e\left(\overline{\pi}_v\right) = f^{LG}\left(\overline{\pi}_v; \pi_v^0; 1; 0; 10; \alpha^{\max}\right).$$

See this Appendix, section 2f for a detailed specification of the functional form and interpretation of the parameter entries, but in short, the elasticity of industry growth to market profits equals 1 at the normal profits, and the growth entrance rate is smoothly bounded by 0 and 10 times the normal growth rate.

Exits, specified in Equation 32 of the Essay also follow an S shape, but have a negative elasticity of 1:

$$f^{x}\left(\pi_{vz}^{x}\right)=f^{LG}\left(\pi_{vz}^{x};\pi_{v}^{0};-1;0;10;\alpha^{\max}\right)$$

See this Appendix, section 2f for a detailed specification of the functional form and interpretation of the parameter entries, but in short, the elastivity of exits to profits equals -1 at the normal profits, and the growth entrance rate is smoothly bounded by 10 10 times the normal exit rate (at large losses) and 0 (large profits) .

d) Expected return on investment

Entrepreneurs derive the perceived net present value of operation over planning horizon τ^{p} and continuous time discount rate β . Expected retuns on investment $\pi_{vz\beta}^{e}$ are determined by the net present value of expectected revenues, minus net present value of costs, divided by net present value of cost:

$$\pi_{vz\beta}^{e}=\left(r_{vz\beta}^{e}-c_{vz\beta}^{e}\right)\Big/c_{vz\beta}^{e}$$

The net present value of a constant stream s (income or expense) is represented by:

$$s_{\beta}=\int_{0}^{\tau^{p}}se^{-\beta t}=s/\beta\left[1-e^{-\beta\tau^{p}}\right]=\nu_{\beta}s \qquad \text{(A1)}$$

This formulation is a good representation for expected net present value of, say, cost of capacity, or price. However, other values, in particular sales, adjust gradually over time. For, instance, the expiration of subsidies can be anticipated, which results in a gradual reduction of entrance in the last years of such a program. Similarly, placement of 5 stations in a periphery around, say, Sacramento can considerably increase the

attractiveness of AFVs, driving up sales of vehicles of that platform, followed by increased fuel consumption, but the impact on the return on investment depends strongly on the adjustment time (in relation to discount rate). The more general representation of an expected net present value of a variable stream s_β that adjusts with adjustment rate λ to its indicated value s^* equals:

$$s_\beta^e = \int_0^{\tau^p} \left[s^* - \left(s^* - s \right) e^{-\lambda t} \right] e^{-\beta t} = s_\beta + \triangle s_\beta - \triangle s_{\beta'} \qquad \text{(A2)}$$

Where $\triangle s_\beta = s_\beta^* - s_\beta$ is the net present value of the goal (structurally defined in equation (A1)) and, with and $\beta' = \beta + \lambda$, $\Delta s_{\beta'}$ is the correction for the time needed to adjust to it. Note that if $\lambda \to \infty$, the third term drops out, and net present value equals that of a constant s^* stream. Further, the net present value of the sum of two variables is additive, while the net present value of the product of two variables is found through additivity in the adjustment rate and we can also write for therefore we can also write for equation (A2): $s_\beta^e = s_\beta + \triangle s_\beta^e$ and:

$$\triangle s_\beta^e = \triangle s_\beta - \triangle s_{\beta'} = \left(\nu_\beta - \nu_{\beta'} \right) \triangle s$$

The main challenge for stations is estimation of future sales $s_{vz\beta}^{fe}$ at entrance, which feeds into revenues $\left(r_{vz\beta}^e = p_{vz} s_{vz\beta}^e \right)$ and variable cost $\left(c_{vz\beta}^a = c_{vz\beta}^v + c_{vz\beta}^k ; c_{vz\beta}^v = c_{vz}^v s_{vz\beta}^e \right)$. We will discuss this here. Expected present value of revenues $s_{vz\beta}^e$ are those of current fuel sales plus the adjustment for growth $\triangle s_{vz\beta}^e$ induced by entrance, but can not exceed one's planned capacity:

$$s^{e}_{vz\beta} = \min\left[k^{*}_{vz}, \frac{F_{vz}}{\left(F_{vz}+1\right)} s_{vz\beta} + \frac{1}{\left(F_{vz}+1\right)} \triangle S^{e}_{vz\beta} \right]$$

The first term on the right hand side of the min function equals current demand patterns at stations, adjusted for sharing of sales by an increased base of fuel stations. The second term captures the share of (net present value of) an anticipated increase of sales due to increased coverage, going to a new station. This increase in sales comprises four components: i) closing the gap between demand and sales, in the case of full utilization ii) increased share of current driver's refuelings in that area, iii) an increase of trips by adopters iv) an increase in adoption.

In an earlier version this has been derived and implemented, using the actual demand elasticities. An alternative, simpler, approach that is used in this and is now discussed. Potential entrants expect that demand in a zone can grow more the wider the gap is between perceived potential demand and perceived actual demand. Perceived potential demand s^{m}_{vs}, equals the total current demand in that region, of which the potential for the entrant is corrected by α_{vs}, that captures fuel specific factors (e.g. higher fuel efficiencies result in less potential demand), and contextual factors (the aggregate of factors discussed above):

$$\Delta s^{*}_{vs} = f^{*}_{vs} \max\left[0, \left(\alpha_{vs} s^{m}_{vs} - s_{vs}\right)\right]$$

The effectiveness to attract more demand, f^{*}_{vs}, depends on how the infrastructure coverage is changed as a function of entrance. The heuristic follows the one that let us draw the demand curve in Figure 2 of the Essay. At zero existing stations, the responsiveness will be very low, similarly when infrastructure is already very abundant.

However, when stations are reasonably spars, say halfway the normal demand, entrance responsiveness is expected to be high:

$$f_{vs}^{*} = f_{vs}^{0} g\left(s_{vs}/s^{ref}\right) h\left(s_{vs}/s^{ref}\right) \begin{cases} g(0)=0; g(1)=1; g' \geq 0; g''(0) > 0; \\ h(0)=1; h(1)=0; h' \leq 0; h''(0) < 0; \end{cases}$$

We use two standard symmetric, bounded at 0 and 1 logistic curve functions (see Appendix 2f), with sensitivity parameters of respectively *g* and *h* being 2 and -2.

e) Exits: weight of expected profits for mature stations

Stations enter based on expected return on investment, and during a honeymoon period, losses may well be anticipated. Equation 34 in the paper captures the different behavior for mature and new to industry stations, through the weight of the relevant expected profits. As we study early transition dynamics, it is important to capture the reality that new stations can stay in business, holding on to their business case, eventhough no profits are being made. New stations therefore base their exit rate on adjusted expected profits.

For the weight function given to recent profit streams increases with the average maturity of the stations we use the logistic curve:

$$f^{L}\left(L_{vz}\right) = f^{LG}\left(L_{vz}; 4; 0.25; 0; 1; 1\right)$$

See this Appendix, section 2f for a detailed specification of the functional form and interpretation of the parameter entries, but in short, the elasticity of growth to profits equals 4 at the normal profits, and the growth entrance rate is smoothly bounded by 0 and 1.

f) General forms, the logistic curve

In the Essay several functional forms were specified in general terms, including boundary constraints as normal values, extreme conditions, first and second derivatives. For several of them a general S-shape curve is a natural form. For those we specify here the exact expressions used in the simulation. While many forms are available, I use the Logistic Curve. I will present this here in a more general form, and specify parameters where applied:

$$f^{LG}\left(x;x_0;\beta;\max;\min;\alpha\right)\equiv\min+\frac{\alpha\left(\max-\min\right)}{\alpha+\exp\left[-\beta'\left(\frac{x-x_0}{x_{ref}}\right)\right]} \tag{A3}$$

With min and max as specfified, output at x=0 equal to:

The inflection point at x0 has value:

$$f^{LG}\left(x_0\right)=\frac{\alpha\max+\min}{\alpha+1}\Rightarrow$$
$$\alpha=\frac{f^{LG}\left(x_0\right)-\min}{\max-f^{LG}\left(x_0\right)}=\frac{y^{LG}\left(\max-\min\right)+\min-\min}{\max-\left(y^{LG}\left(\max-\min\right)+\min\right)}=\frac{y^{LG}}{\left(1-y^{LG}\right)}$$

Where y^{LG} is the locus of the inflection point as fraction between the max and the min. If we want to set $f^{LG}\left(x_0\right)$ to 1, provided min<1:

$$y^{LG}=\frac{1-\min}{\Delta};\Delta=\left(\max-\min\right)$$

Next, if

$$\beta'=\frac{\min+y^{LG}\Delta}{f_0^x y^{LG}\left(1-y^{LG}\right)\Delta}\beta;f_0^x=\frac{x_0}{x_{ref}}$$

Then the elasticity of output to the input at the inflection point equals:

$$\varepsilon_{LG/x} = \frac{x}{f^{LG}} \frac{df^{LG}}{dx} \Rightarrow \varepsilon_{LG/x}\Big|_{x_0} = \beta$$

Note further that the symmetric configuration, $\alpha = 1$, renders the standard logistic curve.

However, with minimum at min, maximum at max, elasticity at the inflection point specificed and the output equal to 1 at x_0, we have:

$\alpha = 1 \big/ (\max - 1) \equiv \alpha^{\max}$. Note that with max to infinity we get the exponential function:

$$f^{LG}\left(x; x_0; \beta; \infty; 0; 0\right) = \exp\left[\beta\left(\frac{x - x_0}{x_0}\right)\right]$$

3 Derivations

In this section I derive analytical expressions, including the average route effort (discussed with Figure 5), refills per trip, and the trip effort inputs average refueling distance, out of fuel risk and service time.

a) Notes on derivation of trip effort components

In the following treatment we assume that all searches for fuel occur within the zone (used interchangeably with "patch") *s* in which refueling is desired. This is justified as in the current analysis the zones are naturally chosen large enough such that in search for fuel within a zone, and small enough to allow capturing the effects of heterogeneous population concentrations. For deriving the average risk of running out of fuel, the average refueling effort, I use a discrete grid, with patches defined at a much smaller

scale than that of the patches *s* or *z*. Where preferred I will resort to polar coordinates, using (l,θ).

b) Route effort and probability of a refill

Here I explain how I derive at the route effort expression in Figure 5 (row 3 of the aggregate utility and effort column). For the average trip effort I use an approximation of the expected trip effort, which aggregates over the probability of refueling n times $p_{i\omega_{zz'}n}$, multiplied with the corresponding effort $a^{t}_{i\omega_{zz'}n}$. Assuming that various refueling events are uncorrelated, which holds true when averaging over a large population, this equals the effort of not having to refill, plus the summation (to infinity) over the number of rfill events n , of n multiplied with the refueling probability and the net effort of refueling 1 time, $a^{f1}_{i\omega_{zz'}} - a^{f0}_{i\omega_{zz'}}$

$$a^{t}_{i\omega_{zz'}} = \sum_{n} p_{i\omega_{zz'}n} a^{t}_{i\omega_{zz'}n} \approx a^{t0}_{\omega_{zz'}} + \sum_{n} np_{i\omega_{zz'}n}\left(a^{f1}_{i\omega_{zz'}} - a^{f0}_{i\omega_{zz'}}\right)$$

The product $np_{i\omega_{zz'}n}$ is the only part that is a function of n. This summation equals the expectd refills per trip, $\phi_{i\omega_{zz'}}$, and the previous expression can be further simplified to

$$a^{t}_{i\omega_{zz'}} = a^{t0}_{\omega_{zz'}} + \phi_{i\omega_{zz'}} a^{f}_{i\omega_{zz'}}$$

Where $a^{f}_{i\omega_{zz'}} \equiv \left(a^{f1}_{i\omega_{zz'}} - a^{f0}_{i\omega_{zz'}}\right)$. For each individual refueling location this equals the average effort of refueling $a^{f}_{i\omega_{zz'}}$.

c) Refills per trip

The refills per trip can be found by solving from Equations (6) and (8), using that the refills per effective tank range r_{iz}^{f} equals 1. Then:

$$\phi_{i\omega_{zz'}s}^{*} = r_{\omega_{zz'}}^{t} \Big/ \left(r_{iz}^{f} - \sum_{s\in\omega_{zz'}} \sigma_{i\omega_{zz'}s}^{f} r_{s}^{f} \right).$$

The denominator provides a corrected effective tank range that is reduced because of the search for fuel. We see that if the expected distance to obtain fuel approaches the actual tank range, this term diverges. This is the situation corresponds with the situation that there is not enough fuel to be found along the whole trip, to bring us home. The divergence is physically sound. Note for instance that at this point the utility for making the trip approaches zero (see Figure 5). However, the negative constraint is not. To deal with this in a consistent manner, I assume that the range to find fuel in each location is bounded by the actual tank range, representing an option to call a service to fill you up. However, the cost in time and money is very large, thus at this time the effect of refueling effort on utility is at this point already reduce it to zero, consistent with this, thus:

$$\phi_{i\omega_{zz'}} = r_{\omega_{zz'}}^{t} \Big/ \left(\max\left[0_{+}, r_{iz}^{f} - \sum_{s\in\omega_{zz'}} \sigma_{i\omega_{zz'}s}^{f} r_{s}^{f} \right] \right)$$

d) Average refueling distance

The average distance of a refueling point to the desired refueling location, $\left\langle r_{is}^{d} \right\rangle$ is found by summing over the probability that the nearest station is at a distance r_{l} from the desired refueling point in s, p_{isl}^{*} multiplied by the distance:

$$\left\langle r_{is}^{d} \right\rangle = \sum_{l \geq 0} \frac{n!}{l!(n-l)!} r_l p_{isl}^{*}$$

which equals the probability that at least one station exists at a ring with radius r_l and with dl, minus the probability that a station within that ring within r_l of s, p_{isl-1}:

$$p_{isl}^{*} = p_{isl} - p_{isl-1}$$

The probability of finding a station until l, equals 1 minus the probability of finding no station:

$$p_{isl} = 1 - p_{isl}^{0}$$

Given the poisson characteristics, p_{isl}^{0} this equals (e.g. Pielou 1977):

$$p_{isl}^{0} = \exp\left(-F_s A_l / A_s\right) \tag{A4}$$

Where $A_l = 2\pi r_l dl$.

Below I plot, for reference the relative effective trip duration, as a function of the effective tank range (wich determines the trip frequency), and thus the effective search time, and the trip length.

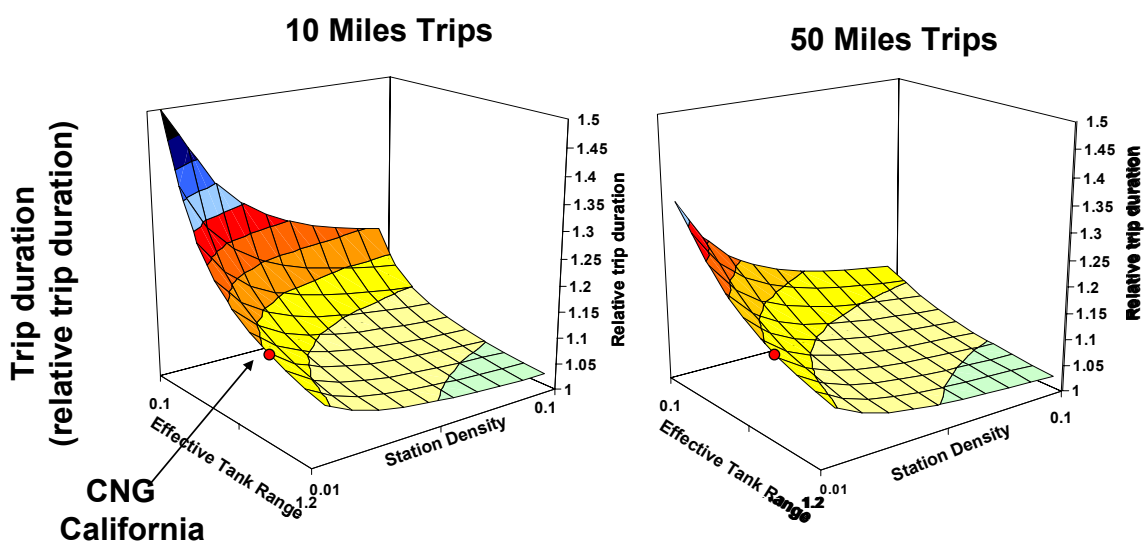

Figure A1 – trip duration

No parameters are required to derive this function. Trip duration is especially sensitive to station density for short trips.

e) Out of fuel risk

The expected risk of running out of fuel is derived by summing over probabilities of running out of fuel at distance r_l from the desired refueling location, given topping-off buffer r_{iz}^b. Such a probability requires not having encountered a station within one's driving radius r_l, p_{ls}^{0-}, times the probability of running out of fuel in location p_{izsl}^o, conditional upon not having run out of fuel before:

$$\left\langle o_{izs} \right\rangle = \sum_{l>1} p_{izsl}^{o1} p_{izsl}^{o0-}$$

Where an out of fuel in location *l* equals to the probability of getting out of fuel at a distance r_l from the desired refueling point p_{izsl}^o, conditional upon not having been out of fuel before:

$$p_{izsl}^{o1} = p_{izsl}^o c_{izsl-1}^{-o} \tag{A5}$$

The cumulative out of fuel probability is a function of the tank range r_i^f and the buffer r_{iz}^b:

$$c_{izl}^o \quad = f\left(r_l \,/ r_i^f\right); f\left(0\right) = 0; f\left(r_{iz}^b / r_i^f\right) = 0.5; f\left(1\right) = 1;; f' \geq 0$$

The probability of not having been out of fuel can be derived from this expression, when using simple exponential expressions for p_{izl}^o, otherwise it can be approximated through:

$$c_{izl-1}^{-o} = \left(1 - c_{izl-1}^o\right) \tag{A6}$$

While

$$p^{o}_{izl} = \left(c^{o}_{izl} - c^{o}_{izl-1}\right) \tag{A7}$$

Figure A2 shows a graphical representation for relatively short (10 miles) and longer trips (50 miles).

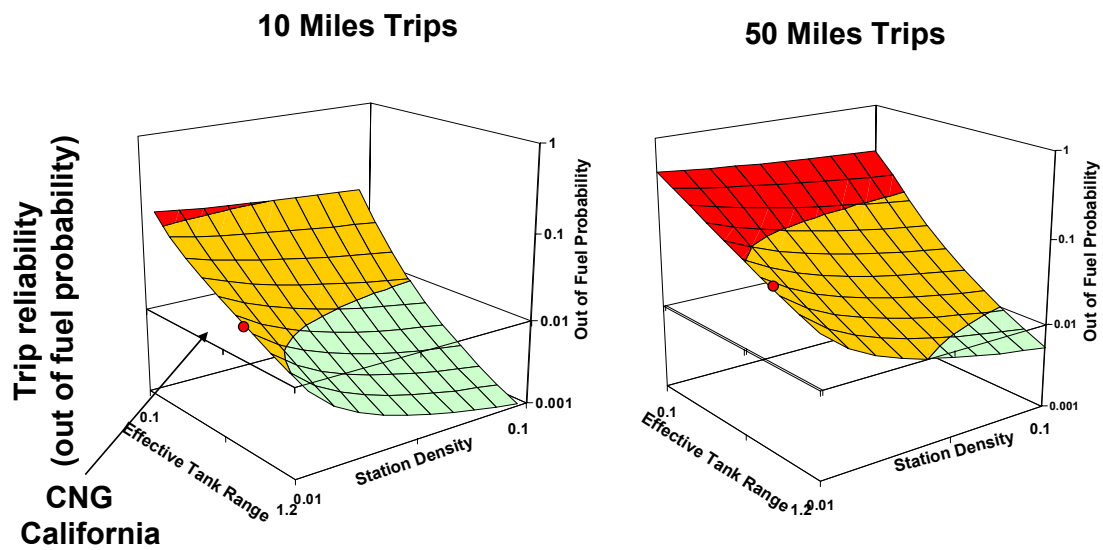

Figure A2– fuel reliability

Comparing this with the general results for search efforts, we note that the driving effort component is most easily affected during short trips, while the out of fuel risk grows faster in larger trips.

f) Mean waiting time for service

Disequilbirum between supply and demand are very critically felt at the pump. In Argentina and New Zealand that have experienced a take-off of CNG, waiting times have

been found to be in the order of 2 hours.[1] The mean waiting time at a station is derived through stationary solutions of basic queuing theory concepts. This provides insights on the average wait time as a function of average utilization, number of pumps and pump capacity. The assumptions we make are simplified, but provide excellent insights on the strong non-linearities involved. We assume customer arrival rate at stations in *s*, for drivers of platform *i* to be uncorrelated and Poisson distributed. Assuming more complex demand patterns, such as peak behavior would yield average wait times that are even larger. The average arrival rate per station is the sum over arrival rates from all regions $z, \lambda_{is}^{s} = \left(\sum_{z} \lambda_{vzs} \right) \Big/ F_{vs}$. The arrival rate for refills in region *s* for platform *v* from region s, λ_{vzs} ,is given by the average refills during trips between *z* and z', the actual trip distribution between *zz'* and the number of adopters in *z:*

$$\lambda_{vzs}^{s} = \sum_{i \in v, z'} \phi_{izz's} T_{izz'} V_{iz} \tag{A8}$$

With refills per trip from location *z* to *z'*, with underway *s*:

$$\phi_{izz's} = \sum_{w} \sigma_{izz'sw}^{f} \sigma_{izz'w}^{t} \phi_{izz'w} \tag{A9}$$

A second requirement for the (basic) queuing concepts is to assume the servicing time at the pump to be exponential. The strong assumption can be easily relaxed, for instance by assuming more sophisticated Erlang distributions, but this is sufficient to surface the strong non-linearity and analytically convenient. Other second order effects derive from, for instance, the number of stations in an area.

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[1] Jeffrey Seissler, Executive Director of the European Natural Gas Vehicle Association (ENGVA) – personal communication July 2006.

The average duration τ_{is}^{sf}, is found by averaging over the service duration from all customers:

$$\tau_{is}^{sf} = \sum_z \lambda_{izs}^{s} \tau_{izs}^{sf} \Big/ \sum_z \lambda_{izs}^{s} \tag{A10}$$

The steady state wait time, that is, the waiting time for $t \to \infty$ is derived from the constraint that the sum of over all probabilities of finding k customers in the system should be equal to 1. The derivations are done for total demand being smaller than supply (the interesting area). First we define the average station load factor, $\rho_{is} = \lambda_{is}^{s} \tau_{is}^{sf}$, which is by the foregoing assumption smaller than the number of stations, and the average stations available $\alpha_{is} = \left(y_{is} - \rho_{is} \right)$. Then, the probability that k customers demand fuel, P_k, expressed in the probability that no customers demand fuel P^0 (for derivation see e.g. Gnedenko and Kovalenko (1989)), omitting subscripts for clarity:

$$P_k = \begin{cases} \dfrac{\rho^k}{k!} P^0 & 0 \le k \le y \\ \left(\dfrac{\rho}{y} \right)^{k-y} P_y & k > y \end{cases}$$

Then, with $\left(y - \rho \right) / y \sum_{k>y}^{\infty} \left(\rho / y \right)^k = \left(\rho / y \right)^{y+1} \Rightarrow y^y \sum_{k>y}^{\infty} \left(\rho / y \right)^k = \rho^{y+1} \Big/ \left(y \left(1 - \upsilon \right) \right)$ the probability of having no customers waiting equals:

$$P^0 = \left[\sum_{k=0}^{y} \frac{\rho^k}{k!} + \frac{1}{\left(1 - \upsilon \right)} \frac{\rho^{y+1}}{y+1!} \right]^{-1} \tag{A11}$$

And the probability that all pumps are busy equals

$$P^q = \sum_{k=y}^{\infty} P_k = \frac{1}{\left(1 - \upsilon \right)} P_y = \frac{1}{\left(1 - \upsilon \right)} \frac{\left(y\upsilon \right)^y}{y!} P^0 \tag{A12}$$

In the case of one pump per station, this equals the average utilization of a station υ, according to intuition.

This intermediate outcome is important: it shows that when the average station utilization becomes high $(\upsilon \to 1)$, the probability that someone finds all pumps increases dramatically. Further, this probability is also highly dependent on the number of busy pumps, even when the total utilization is constant. Finally the mean waiting time can be shown to be:

$$\left\langle \tau_{is}^{sw} \right\rangle = \frac{P_{is}^{q}}{y_{is}\left(1-\upsilon_{is}\right)} \tau_{is}^{sf} \tag{A13}$$

Figure 3 shows, for one set of parameters, technological parity with gasoline stations and ICE vehicles (e.g. pump capacity, vehicle driving range), utilization and relative service time for increasing demand supply imbalance and increasing number of pumps per station. I use an estimated 8 pumps per station. Of interest is the steep non-linearity of service time for low utilization, especially for fewer pumps per station).

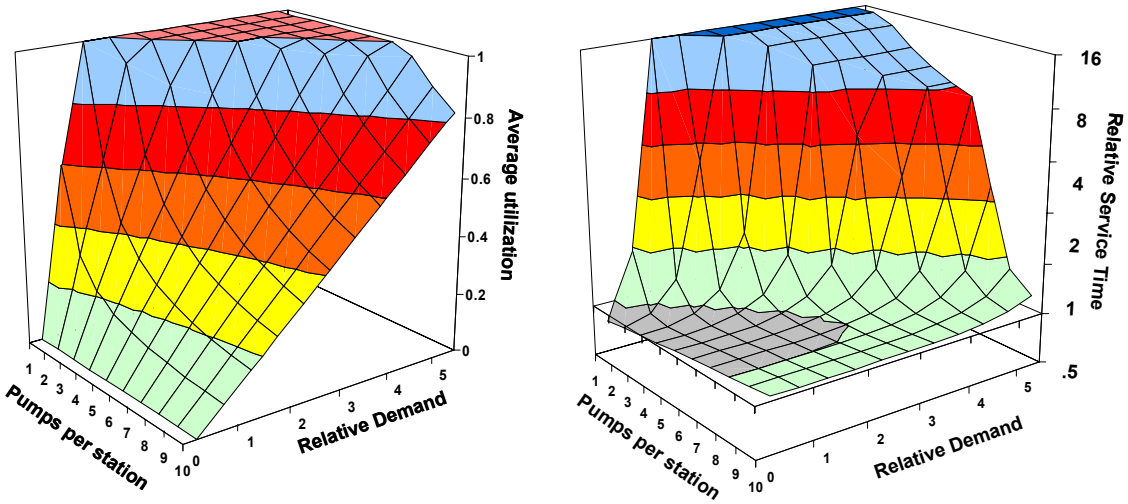

Figure A3– Utilization and Relative Service Time for increasing pumps per station and relative demand (Technological Parity with Gasoline stations and Drive Range).

4 Notes on simulations

a) Trip generation and trip relevance

For the simulation we derived the aggregate for each driver d from a two-parameter lognormal distribution:

$$f_n^t = f^{TOT'}\left(r^{t,mean}/r^t\right)\Big/\left(\sigma^t\sqrt{2\pi}\right)\exp\left[-\left(\frac{\sigma_t}{2\sqrt{2}}+\frac{\ln\left(r^t/r^{t,mean}\right)}{\sqrt{2}\sigma^t}\right)^2+\frac{\sigma_t^{\,2}}{2}\right]$$

with σ_t the standard deviation and mean distance, $\mathrm{r}^{\mathrm{t,mean}} \equiv \mathrm{r}^{\mathrm{t,ref}}\mathrm{e}^{0.5*\sigma_t^2}$; $f^{TOT'}$ equals the total annual trip frequency $f^{TOT'}$, times the cumulative distribution until a maximum range $r^{\max}$. Specific data can be derived from trip-tables (e.g Domencich et al. 1975), but here we assumed identical average trip behavior across the regions:

$r^{t,mean} = 27, r^{\max} = 120$ miles per trip and $\sigma^t = 0.5$, yielding the total f^{TOT} of 300 trips per

year (f^{TOT} derived through integration over the populations corresponds with average annual miles $m = h \int_0^{r^{\max}} \mathrm{r}^{\mathrm{t}} f_n^t$).

The total vehicle miles for a drivers of platform i equal:

$$m^{v,\max} = \sum_{z'} r_{zz'} T_{zz'}^{\max}$$

And are set to 15,000 miles per person per year. Subsequently, $T_{zz'}^{\max}$ was derived by dividing of trips between region z and z' assuming uniform distribution in radius .

Trips between regions are weighted by desired frequency and distance, thus, with $m^{v,\max} = \sum_{z'} r_{zz'}^{\eta^w} T_{zz'}^{\max}$, we have $w_{zz'} = m_{zz'}^{v,\max} \Big/ \sum_i m_{zz'}^{v,\max}$.

In the simulations $\eta^w = 2$. The combination of trip weight and frequency render the following distributions:

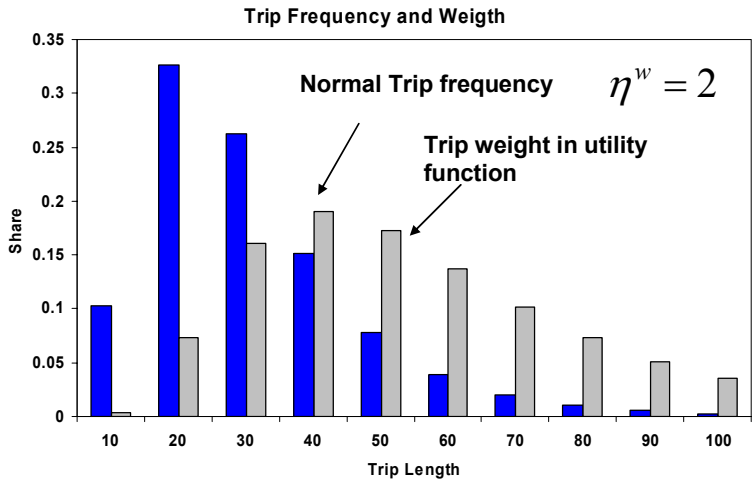

Figure A4 Normal Trip Frequency, and trip weight for the deteriming average trip attractiveness (see Figure5 in the Essay). To speed up the computation, throughout the analysis, drivers only select the direct route $\beta^t \to \infty$ (see Table 3).

b) Figure 12a – tipping point for a one patch model

Figure 12a in the Essay discusses the equilibrium dynamics as a function of the patch length. Under the assumption of one patch, the model structure corresponds with assumptions of uniform population distribution. This assumption does not bring out strategic location incentives on the supply side, and, even under rich behavioral assumptions we can plot a unique adoption curve as a function of the number of fuel stations, that will yield demand /supply responses that correspond with the qualitative sketch in **Figure 2.** This graph shows the equilibrium adoption fraction under the default

simulation assumptions:

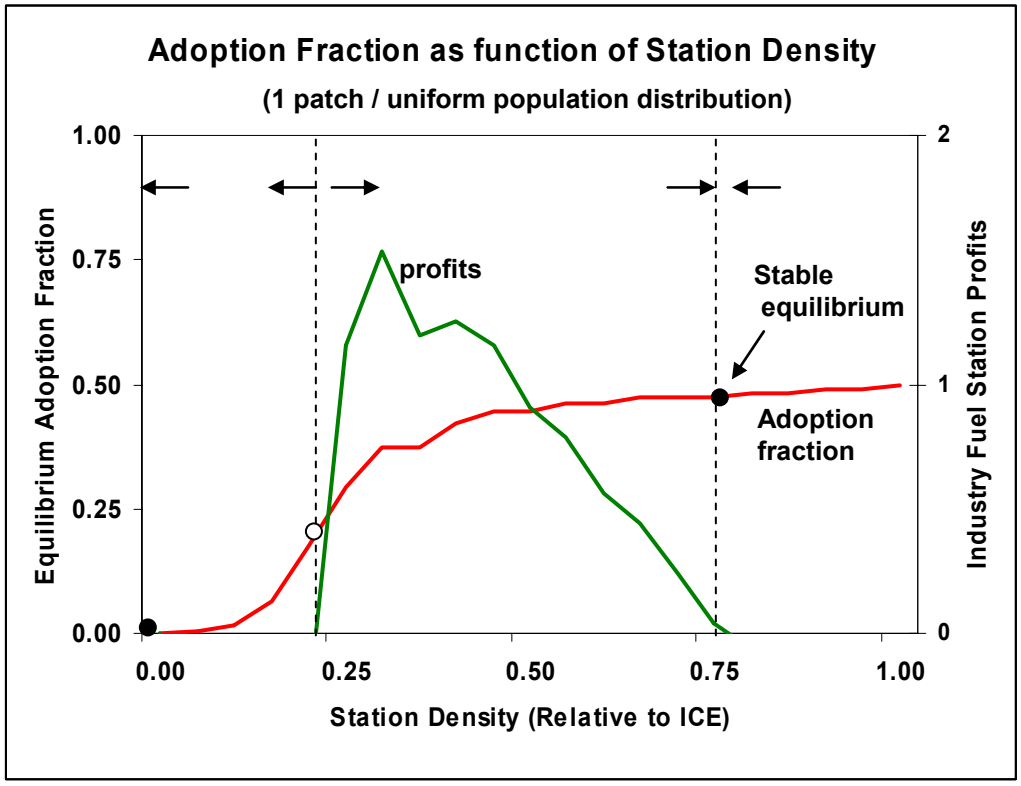

Figure A5 Adoption fraction and fuel station profits for a 1 patch simulation

Where profits equal zero (or for 0 fuel stations), we can expect an equilibrium. We see that under these assumptions 2 platforms can be supported. But getting towards that requires significant investment. While useful as a starting point, the assumptions for the uniform distribution ignore many feedbacks that involve critical dynamics.

c) Figure 12b – directed trip simulation

The full Los Angeles region is included, including San Diego. Further, to the north, Fresno, San Jose, and Sacramento. Figure A6 right provides summary statistics of population distribution for each landuse, as well as the average trip frequency and miles

(short and long trips). Minimum pop density indicates the selection criterium for each landuse type and is measured against the average population of the region. The region contains 83 % of the California populion and 58% of the land area. (Figure A6).

Region Parameters	Urban	SubUrban	Rural	Total
Minium Pop Density	10	0.75	0	
Area Fraction	0.09	0.29	0.62	1.00
Population Fraction	0.70	0.27	0.01	
Population (M)	19.8	7.6	0.33	28
Short Trip Frequency	97	89	86	94
Long Trip Frequency	24	11	7	20
Total Trip Frequency	122	100	93	114
Short Trip Distance	7,453	5,380	5,155	6,767
Long Trip Distance	6,131	2,870	1,992	5,120
Total Trip Distance	13,584	8,250	7,147	11,887

California Parameters	Urban	SubUrban	Rural	Region / California
Fraction of Total Area	0.05	0.19	0.75	0.58
Fraction of Population	0.63	0.32	0.05	0.83

.

Figure A6 Selected region for directed trip simulation (left) and summary statistics of geographical and the generated desired driving behavior (right).

Short-distance trips follow the same distribution as those in the base model (with random direction). However, to conserve computation time, trips of that category were cutoff beyond a 50 miles radius for urban and suburban population and a 30 miles radius for suburban and rural population. Long-distance trip destinations drawn from each location (23 for urban, 10 for suburban and 8 for rural), to a limited set of destinations as well,

weighted by population and distance from home to final destination: Population weights increased linearly with density: $w^p = \left(h_{z'}/h_{ref}\right)$ if $h_{z'} \geq h_{ref}$, 0 otherwise; distance between zones, with $w^d = \max\left[1, \left(d_{ref}/d_{zz'}\right)\right]$, with $d_{ref} = 100\,\text{miles}$. Thus, a long distance destination 200 miles away from one's home location was twice as likely to be drawn than a trip 400 miles away from *z*. Finally 6 hot-spot areas were handpicked: Las Vegas, Lake Tahoo, Crescent City, Alturas, Mamoth Lakes, with weight being set equal to a region with 10-20 times the average population. If destination fell outside the boundary of the region, the nearest point to the region was selected. (. Figure A7 shows the demand profile that is generated by total population and its trip profile in Mgallons/year/ zone (top). Also in Figure A7 (bottom) the actual gasoline stations. Average demand per station is 1.1 Mgallons/year. Therefore, the demand/ gasoline ratio in each region is a good indicator for proficiency of the generated trips.

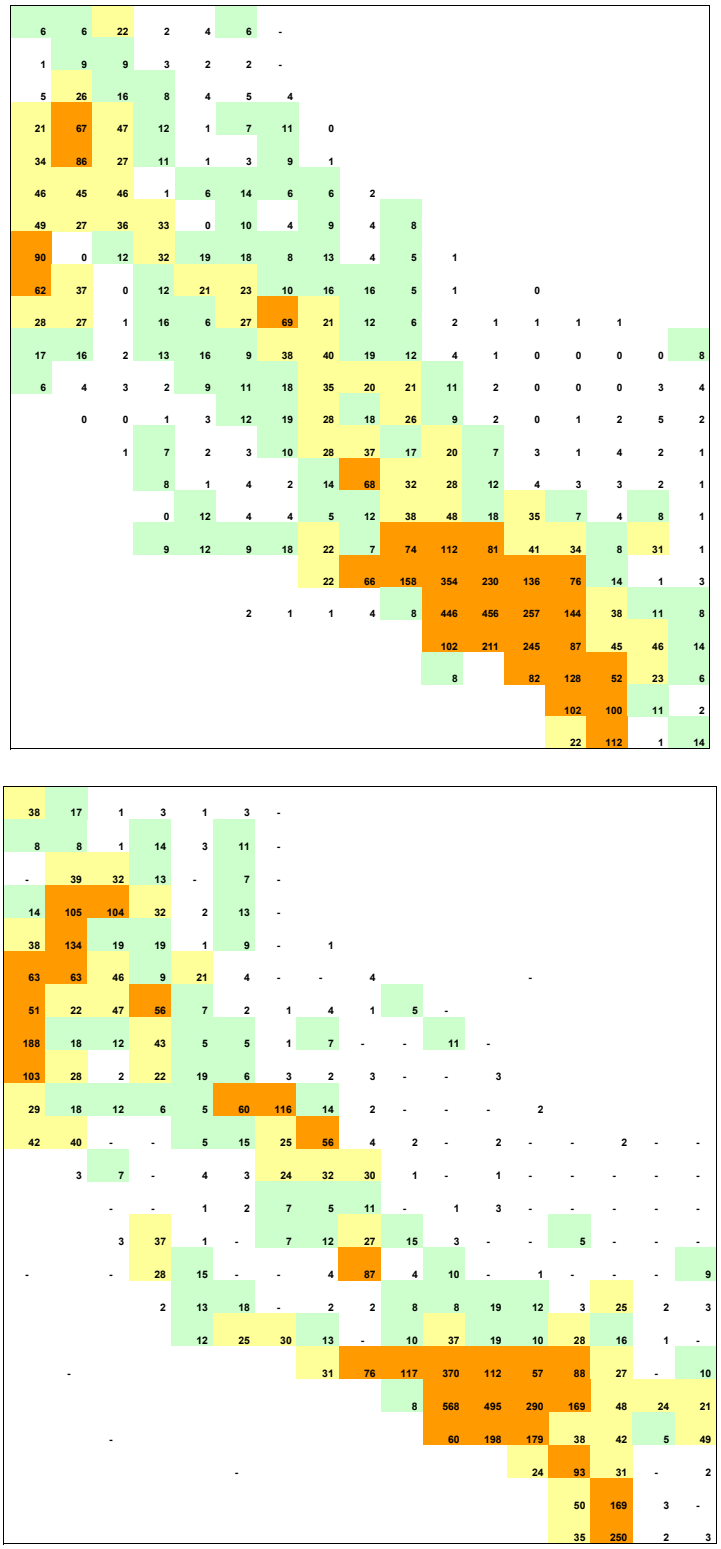

Figure A7 Comparing generated demand with actual supply with in selected region: top shows generated gasoline demand (Million Gallons per year per zone; D=6,751e6);

bottom shows the distribution of gasoline stations (Top, N=6,499). White areas <5 units; green <20; yellow < 50; orange ≥50.

d) Figure 13 – endogenous topping off

Figure 13 in the essay simulates the endogenous topping-off buffer. The general functional form was provided to be:

$$r_{iz}^{b} = f\left(u_{iz}^{t}\right) r_{i0}^{b}; f' \leq 0; f\left(0\right) = r_{\max}^{b} / r_{i0}^{b}; f\left(1\right) = 1; f\left(\infty\right) = r_{\min}^{b} / r_{i0}^{b}$$

The relative top-off buffer increases with decreasing utility, but stabilizes at $r_{\max}^{b}$ for very low utility, as drivers will not want to be constrained by refilling on average too early.[2] Further, when drivers are fully confident, they will reduce their buffer to $r_{\min}^{b}$, which can be below the indicated level by the warning sign, r_{0}^{b}. To satisfy these conditions we use a simple one parameter form for f:

$$r_{iz}^{b} = \max\left[r_{\min}^{b}, r_{\max}^{b} \left(1 \big/ \left(1 + x u_{iz}^{t}\right)\right)^{\alpha^{f}} \right]; x = \left(r_{\max}^{b} / r_{i0}^{b} \right)^{1/\alpha^{f}} - 1$$

which yields, for the selected parameters:

[2] This level depends on the physical constraint of refueling elsewhere; see also equation (7) and Figure (5). From this behavioral reasonable parameters could be derived.

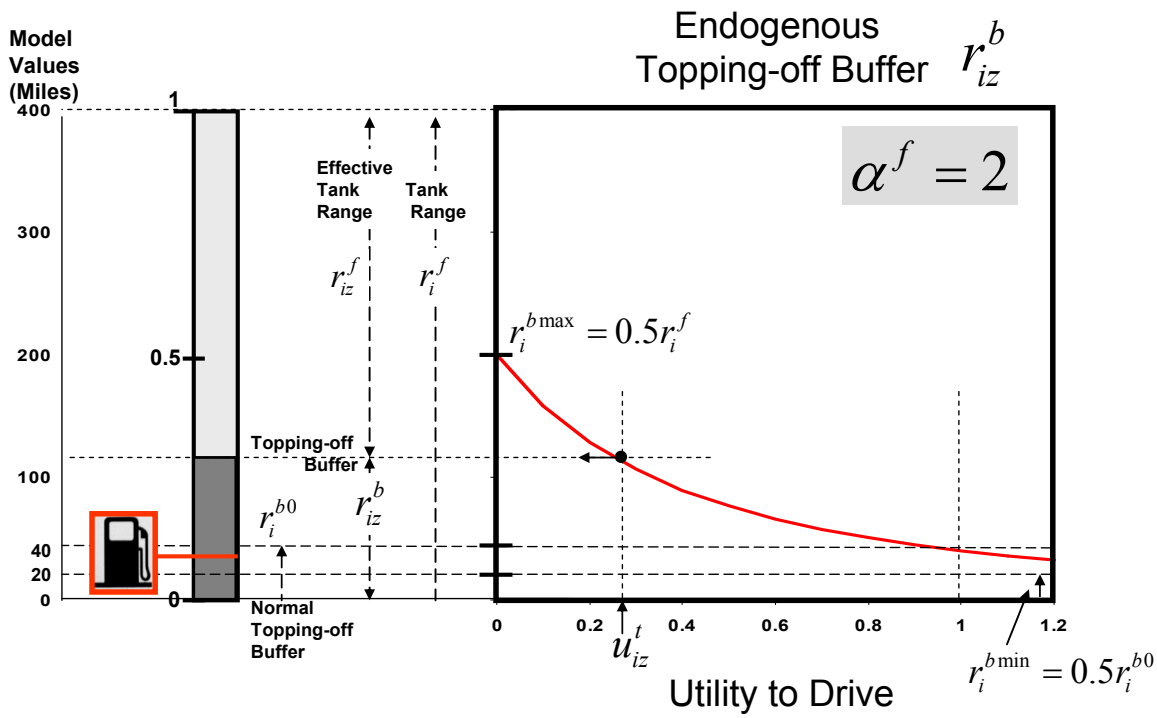

Figure A8 Endogenous topping-off buffer as a function of utility.

The reference topping-off buffer is $r_{i0}^{b} = 40$ miles (10% of the total range), $r_{\max}^{b} = 200$ miles (50% of the total tank range), and $r_{\min}^{b} = 20$ miles. We see that, for instance, the utility equals 0.27, the topping-off buffer becomes equal to 100 miles (corresponding, under current assumptions, but with uniform population and fuel station distribution, and in absence of crowing, with a station density of about 19% of the normal density).

e) Figure 14 – table for technology parameters

Table 1 Parameters for the 3 scenarios:

Parameter	Reference	Relative to reference		
		Scenario 2	Scenario 3	Scenario 4
Relative fuel efficiency	20 miles/gallon	3	3	3
Relative tank size	20 gallons/tank	0.33	0.33	0.25
Relative pump refill rate	400 gallons/hour	1	0.25	0.25
Relative fixed cost (at four pumps)	50,000 $/refill point	1	0.25	0.40
Relative wholesale fuel cost	1.65 gallons/hour	1	1	1.25

5 Stipulations

This section provides additional comments and clarifications on assumptions, or on connections.

a) Sensitivity parameters for trip efforts

The Essay describes how several attributes are brought together to detremine the trip one's utility to drive u_{iz}, (Figure 5). Below I discuss how the relative weights can be interpreted and validated.

The elasticity of one's utility to drive u_{iz}, to a change effort component c, with c={drive time,out of fuel risk, refueling service}, when all attributes $a^{f}_{zz'c}$ are at their reference level $a^{f*}_{zz'c}$ equals:

$$\varepsilon_{u-a^{fc}_{zz'}} = \frac{a^{f*}_{zz'c}}{u}\frac{du}{da^{f}_{zz'c}}\bigg|_{a_{zz'c}=a_{zz'c}} = \beta^{t}\,\frac{\phi_{izz'}w_{c}a^{f*}_{zz'c}}{a^{t*}_{zz'}} \qquad \text{(A14)}$$

Where $a_{zz'}^{t0}$ is the shortest trip effort between *z* and *z'* and $\phi_{izz'}$ is the normal refueling frequency. Further, The reference effort, $a_{zz'}^{t*}$, equals the reference trip time, plus the frequency of refueling multiplied with the reference levels for each attribute:

$$a_{zz'}^{t*} = a_{zz'}^{t0} + \phi_{izz'} \sum_c w_c a_c^{f0} \tag{A15}$$

With a_c^{f0} being the acceptable level (e.g. a_2^{f0}=0, we don't accept out of fuels). For example, ignoring the role of out of fuel and refueling time, we see that the actual elasticity of utility to drive depends on the search time for fuel, relative to the normal travel time for a trip, times the refueling frequency, times weight of finding fuel, and the elasticity to trip effort:

$$\varepsilon_{u/a_{zz'}^{fd}} = \beta^t \phi_{izz'} w^d \frac{\tau_{zz'}^{fd*}}{\tau_{zz'}^{d*}}$$

Note further that the elasticity of utility to refueling in total equals:

$$\varepsilon_{u-a_{zz'}^{f}} = \frac{a_{zz'}^{f}}{u} \frac{du}{da_{zz'}^{f}}\bigg|_{a_{zz'}^{f}=a_{zz'}^{f*}} = \beta^t \frac{\phi_{izz'} a_{zz'}^{f*}}{a_{zz'}^{t0*} + \phi_{izz'} a_{zz'}^{f*}} \left(= \beta^t \sum_c \varepsilon_{u-a_{zz'}^{fc}} \right) \tag{A16}$$

and the elasticity of utility to a change of a component, at the normal level, relative to the elasticity of utility to a change of another component is a direct measure of their relative weight:

$$\varepsilon_{u-a_{zz'}^{fx}} \Big/ \varepsilon_{u-a_{zz'}^{fx}} = w^x a_{zz'}^{f*x} \Big/ \sum_x w^x a_{zz'}^{f*x} .$$

Together this gives an interpretation of the relative importance of the attributes, with respect to each other and compared to the trip effort as a whole, determined at some useful reference point, e.g. at 30% station density of current. At that level, the out of fuel risk might be very low, say 1%, but its weight can be very large.

We now set the weight for searching for fuel equal to the value of time ν^t, divided by the a parameter that measures how time of getting fuel is weighted against spending effort /time driving towards a destination, $\gamma^f : w^d = \nu^t / \gamma^f$. Similarly, for out of fuel risk: $w^r = \nu^r / \gamma^f$, while the weight for service time is equal to that of searching fuel, corrected for a parameter that measures the weight of time waiting for fuel, with that of $w^d = \gamma^s w^d$.

6 Model and analysis documentation

The model and analyses can be replicated from the information provided in the Essay and the first two sections in the Appendix. However, analysis involved several steps and different tools. For instance, the population distribution for the proper gridsize was derived in Excel, while the static trip distributions (trip generation) were calculated in Matlab, using also the population information. Next each was uploaded in Vensim for simulation. Model source code and instruction for replication of the analysis can be downloaded from

http://web.mit.edu/jjrs/www/Thesis Documentation.htm

7 References

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