# *Alternative fuel vehicles turning the corner?:* A product lifecycle model with heterogeneous technologies


Jeroen Struben

jjrs@mit.edu

MIT Sloan School of Management, Cambridge MA 02142


August 2006

## Technical appendix

# 1 Introduction

The model described in this Essay is designed to capture the technology trajectory of and competition among multiple types of alternative vehicles, along with the evolution of the ICE fleet. For example, the model can be configured to represent ICE and alternatives such as ICE-electric hybrid, CNG, HFCV, biodiesel, E85 flexfuel, and electric vehicles. However, the Essay makes a number of simplifying assumptions that allow us to explore the global dynamics of the system. In this appendix I discuss additional components of the full model, highlighting those structures required to capture the competition among multiple alternative platforms, with their more particular context. This appendix provides also additional information to accompany the model and the analysis of Essay 3. Each subsection is pointed to from a paragraph within the Essay.

Sections group issues by:

2 **Elaborations on the model** that provide details on expressions that were not fully expanded due to space limitations (in particular we discuss functional forms).

2 **Stipulations** that provided notes on, additional motivations for, or insight into the model or analysis.

3 **Boundary constraints considered**, providing information about tests that were done by including additional behavioral and physical constraints. They partially reinforce, or otherwise dampen the dynamics, without affecting the fundamental insights of the model.

4 **Model and analysis documentation.** Essay 3, in combination with the section that elaborates on the model provides sufficient information to replicate the model. The Essay provides sufficient information to replicate the analysis.

**5 References**

## 2 Elaborations on the model

This section elaborates segments of the model that were highlighted in the paper but not fully expanded due to space limitations. These elaborations include in particular selected functional forms for functions that were provided in general form in the model, or more detailed decision structures

### a) Resource allocation

The process of resource allocation was discussed in the model (Equation 9 and 10, and discussion). Here we first provide the same process in more detail and for more. **Figure 1** illustrates this process in more detail, in particular how in the model, at different levels, competition for limited resources plays out. Constrained by allocations at more aggregate levels we derive: i) a share of total revenues going to R&D, $\sigma_j^{rd}$; ii) the share of total R&D resources of platform *j*, that managers or system integrators dedicate to the various modules $\sigma_{m,j}^{rd}, \sum_m \sigma_{m,j}^{rd} = 1$; iii) the share of total R&D resources of platform *j*, module *m* that chief engineers dedicates to activity *w* $\sigma_{jmw}^{rd}, \sum_w \sigma_{jmw}^{rd} = 1$; iv) the share of total R&D resources of platform *j*, within module *m*, process *w,* that managers dedicate to internal knowledge accumulation, $\sigma^{rd}_{j,jmw}$, as opposed to spillovers $\sigma^{rd}_{j,jmw} = 1 - \sigma^{rd}_{-j,jmw}$; and finally, v) the share of total R&D spillover resources of platform $j$, R&D process $w$, within module $m$, that engineers dedicate to extracting knowledge from platform $i \neq j$, $\sigma^{rd}_{i,jmw}$, $\sum_{i \neq j} \sigma^{rd}_{i,jmw} = 1$.

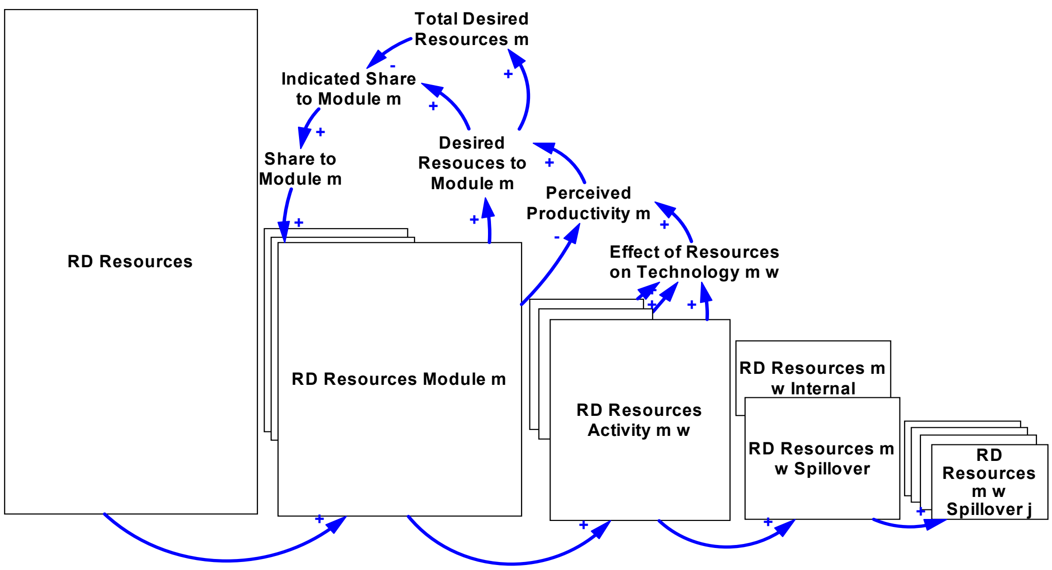


**Figure 1** R&D Resource Allocation (boxes) throughout the decision making chain, performance metric for each decision (top) and decision making process in detail for resource allocation to module *m*.

Structurally each decision process is identical. Figure 4 illustrates this resource allocation process, and in detail for one point in the hierarchy. We follow this here. We label a decision point *d*, assigning lower numbers to points up the hierarchy. With the set of potential allocations at $x_d$ being $\{x\}_d$, the share that $x_{d+1}$ receives from source $x_d$ is $\sigma^{rd}_{x_d, x_{d+1}}$. This share adjusts to its indicated share $\sigma^{rd*}_{x_d}$ over adjustment time $\tau^{rd}_d$:

$$\frac{d\sigma^{rd}_{x_d,x_{d+1}}}{dt} = \left(\sigma^{rd*}_{x_d,x_{d+1}} - \sigma^{rd}_{x_d,x_{d+1}}\right) \Big/ \tau^{rd}_d \qquad \text{(A.1)}$$

In the case of the example of **Figure 1**, this is the share of total RD resources goes to each module *m*. The adjustment time is the result of bureaucratic - and information gathering delays, depends thus on complexity, and can be different at different decision points.

The indicated share $\sigma^{rd*}_{x_d,x_{d+1}}$ is the outcome of the continuous bargaining for, given, scarce resources $R_{x_d}$ at each decision point *d*, and equals desired resources $R^{rd\circ}_{x_{d+1}}$ divided by the resources others credibly bargain for:

$$\sigma^{rd*}_{x_d,x_{d+1}} = R^{rd\circ}_{x_{d+1}} \Big/ \sum_{\{x\}_{d+1}} R^{rd\circ}_{x_{d+1}} \qquad \text{(A.2)}$$

Stakeholders at $x_d$ can credibly bargain for more resources when expected returns $\varsigma^{rd\sim}_{x_d}$ exceed the reference value at this decision point, $\varsigma^{rd\sim}_{\{x\}_d}$ :

$$R^{rd\circ}_{x_d} = f\left(\varsigma^{rd\sim}_{x_d} \Big/ \varsigma^{rd\sim}_{ref,\{x\}_d}\right) R^{rd}_{x_d};\, f' \geq 0;\, f \geq 0;\, f\left(1\right) = 1 \qquad \text{(A.3)}$$

Note that $R_{x_d} = \sigma^{rd}_{x_{d-1},x_d} R_{x_{d-1}}$ .

## b) Expected return of effort

An R&D resource allocation task involves by nature attempts to explore using some form of forward looking. I assume that for the assessment of the return on investment involves decision makers attempt to understand, at least locally, the structural factors that influence their improvement efforts. For example, one can be interested in the returns in her platform *j*'s knowledge base for activity *w* $K_{jw}$, deriving from resources dedicated to extracting knowledge from platform *i*,. She will seek information about the constituents of knowledge accumulation: i) the relevance of the source to total knowledge $\kappa^*_{ijw}$ ; ii) the perceived potential accumulating rate of spillover knowledge $\gamma^*_{ijw}$ ; and, iv) the productivity of resources $r^*_{ijw}$ . This would imply for the expected returns on effort:

$$\varsigma^*_{ijw} = \gamma^*_{ijw} k^*_{ijw} r^*_{ijw} \tag{A.4}$$

The assessment comprises activities as market research, interpretation of reports, study of patents, evaluation of historic results, study of journals, and information exchanged over coffee, in seminars, and during golf matches. Such assessments do not yield perfect information about all factors, takes time, and are subject to information processing constraints. Therefore we assume that decision makers: a) understand the correct structural factors, but simplify their world by assuming that during their assessment that the environment remains constant; b) it takes time to learn about the state of the environment and the parameters. Thus, to assess the return on effort for collecting knowledge from another platform, one has a perception of the knowledge of the other platform that one assumes to remain constant during the planning horizon. Further, one updates the perception of the knowledge base, but this takes time.

***Assumptions about decision makers' available information***

We now will illustrate what decision makers more generally need to know under to allocate their resources equal to the marginal return on effort, at least for some bounding set of assumptions. For the purpose of capturing the decisions related to resource allocation, there are two types of activities. Some resources are typically adjusted reactively, for instance, reallocation of resources for spillovers between source platforms. Others involve longer term anticipation, such as adjustment of resources across modules. In the first case improving perceived returns on effort can be captured by assuming that the rate of accumulation of performance indicator P will be adjusted. In the second case the NPV of P over a planning horizon $\tau_p$ will be improved.

We now will use specific examples to illustrate what decision makers need to know in order to have their target return on effort equal the marginal return on effort.

***Example: process improvement and product innovation***

With the change rate of a performance indicator being $\dot{P}_i = \frac{dP_i}{dX_i}\frac{dX_i}{dt}$, the direct adjustment of resources would require:

$$\frac{d\dot{P}_i}{dR_i} = \frac{dP_i}{dX_i}\frac{d\dot{X}_i}{dR_i}$$

We first determine the marginal return of knowledge accumulation, thus $P \equiv K_{jw}$, in terms of resource allocation to activity *w*, thus $R_{jw}$, which implies:

$$\frac{d\dot{K}_{jw}}{dR_{jjw}} = \frac{dK_{jw}}{dK_{jjw}}\frac{d\dot{K}_{jjw}}{dR_{jjw}}$$

And with Equation (6) and the CES relation in the Essay:

$$\dot{K}_{jjw} = \varepsilon^{i}_{jw}\left(R_{jw}/R_0\right)^{\eta^i_w}\Gamma_w$$
$$K^{e}_{jw} = \left[\kappa_{jjw}\left(K_{jjw}/K^0_w\right)^{-\rho^k_{jw}} + \left(K_{\sim jw}/K^0_w\right)^{-\rho^k_{jw}}\right]^{-1/\rho^k_{jw}}$$

where $K_{\sim jw}$ is the total spillover knowledge, $K_{\sim jw} \equiv K_w^0 \left[ \sum_{i \neq j} \kappa_{ijw} \left( K_{ijw} / K_w^0 \right)^{-\rho_{jw}^k} \right]^{-1/\rho_{jw}^k}$

We get:

$$\frac{d\overset{\bullet}{K}_{jw}}{dR_{jjw}} = \eta_w^i \kappa_{jjw} \left( \frac{K_{jw}^e}{K_{jjw}} \right)^{1+\rho} \frac{\overset{\bullet}{K}_{jjw}}{R_{jw}}$$

And attaining the optimal NPV, holding the environment constant would require:

$$\left. \frac{dP_i}{dR_i} \right|_{\tau_p} = \left( \frac{dP_i}{dK_i} \left. \frac{dK_i}{dR_i} \right|_{\tau_p} \right) \tag{A.5}$$

Maximization of Net Present Value yields, in this case the same type of relationship, because there is constant returns in accumulation of internal knowledge $K_{jjw}$.

In this case, the true values of these factors that determine the marginal return on effort are, for the productivity $\gamma_{jjw} = \varepsilon_{jw}^i \Gamma_w$, which is simply the ease at which they accumulate more knowledge, $r_{jjw} = \eta_w^i \left( R_{jw} / R_0 \right)^{\eta_w^i - 1}$, which gives the slope return on adding more resources. For instance, if the budget is a factor above than what it should be for all to be effectively spend, $r_{jjw}$ does not increase anymore. The last term, the relevance of knowledge equals $k_{jjw} = \kappa_{jjw} \left( K_{jw}^e / K_{jjw} \right)^{1+\rho}$ . A higher factor share indicates more relevance. Acknowledging this will correspond with the notion that producers of HFCVs will expect more from observing EVs, than from biodiesels. Further, if the elasticity parameter is infinite, the distribution parameter equals 0 and the whole expression is identical to its factor share. That is, when knowledge is additive, we always look towards those sources that have larger factor contributions. However, when substitutability of knowledge is less than perfect, we increasingly expect to benefit from other sources as well.

An important point of this exposition is that decision makers will use heuristics that correspond with chunks of a structure that provides a locally optimal solution, and as a whole allows them to get reasonably close to that exact contributions.

Formulating decision making chunks at each stage, whether based on the current rates or NPV, yields similar types of structures at each level. This is how the decisions have been formulated in the model. There are important conditions, one of them is that many environmental factors are held constant in the resource allocation decisions, and thus the definition of "local" is quite narrow (and defined for this purpose above). Further, there are significant time delays in learning them. These two conditions assure that the decision structure for resource allocation conforms to the perspectives of bounded rationality.

## c) Learning about productivity

Relevant knowledge, input factors, elasticity of substitution, and platform specific quality are learned over time. Biases by investors towards tested technology have important dynamics implications for the transitions. We capture this in the expanded model by allowing, for instance perceived knowledge of others, $K_{j,ijmw}^{\sim}$ adjusts to the indicated level over time $\tau^{K}$ :

$$dK_{ijmw}^{\sim} \big/ dt = \left( K_{ijmw} - K_{ijmw}^{\sim} \right) \big/ \tau^{K}$$

A more sophisticated formulation would have the learning rate depend on attention as a function of exposure and interest, but for assessing its basic impact, this will suffice.

## d) Platform- versus firm-level learning-by-doing

Equation 7 in the Essay discusses the learning-by-doing effect on process technology. Learning-by-doing can be seen as partly occurring at platform level (fast spillovers between firms for the same platform) and partly at the firm level. The number of firms changes considerably over the lifecycle of a technology, in particular, after a swift ramp-up, the number of firms tend to peak, followed by a shakeout. This would imply that the effective learning, at platform level, on is much slower early on.

It is useful to be able to capture this. I capture this by introducing the effective sales $s_j^n$ for learning in the equation:

$$\varepsilon_{j2}^i = \left(s_j^n / s_0\right)^{\eta^s}$$

Where the effective sales $s_j^n$ is a function of the total platform sales and the maturity of the platform and is represented by the share of the platform sales divided by the effective number of producers, for learning, $n_j$;

$$s_j^n = \left(s_j^n / n_j\right)$$

The effective producers assumed to decline with the maturity of the industry:

$$n_j = w_j^n n^{new} + \left(1 - w_j^n\right) n^{mat}$$

The weight of $n^{new}$ declines with total platform sales:

$$w_j^n = f^n\left(s_j/s_0\right); f\left(0\right)=0; f'\geq 0; f\left(1\right)=0.5; f\left(>>1\right)=1;$$

The weight starts at 1, and increases with total sales (ignoring the first ramp-up), saturating at 0. A sensible shape is the S curve. Here we use the standard logistic curve, with the inflection point at reference sales for learning $s_0^n$:

$$w_j^n = \exp\left(\beta^n\left[\left(s_j/s_0^n\right)-1/2\right]\right)\Big/1+\exp\left(\beta^n\left[\left(s_j/s_0^n\right)-1/2\right]\right)$$

This structure is switched off for the AFV analysis, assuming that for these dynamics very few firms will be in competition ($\beta^n = 0 \rightarrow n_j = 1$). This is valid for the basic analysis. However, in general analysis includes simulations with new industry formation. For those cases I set $n^{mat}=2$, $n^{new}=20$, $\alpha^n = -1$ and set $s_0^n$ equal $s_0$, or 20% of the total potential market. The effect of this is that learning by doing is moderately slowed down in the first decade or two in the industry.

## e) Vehicle choice and nesting

This focus of this Essay is on multi platform competition. Endogenous platform entrance is one of the dynamics that are captured. In the basic formulation of consumer choice between the available platforms (equation 13), when new vehicle types or a set of platforms are introduced, demand elasticity to the number of platforms is constant. This assumes the existence of a powerful feedback loop that is in reality much weaker: an increase in the variety and number of models, does not necessarily increase aggregate utility of "the vehicle" proportionally. That is, total increase in demand is generated depends on the correlation (or substitutability) of preference across a range of products in the choice outcome. Capturing is important to generate consistent dynamics, for example in the case of endogenous platform entrance.

Here I describe the formulation of this. I also include how to incorporate familiarity with a platform in this formulation (a consumer's familiarity with a platform, through processes of social exposure is discussed in Essay 1). The basis is a nested logit formulation. The share that drivers of platform *i* replacing their vehicle allocate to platform *j*, $\sigma_{ij}^{d}$, involves a nested decision process (Ben-Akiva 1973). A share of the discarded vehicles from platform *i* is replaced by *j*, $\sigma_{ij}^{r}$, conditional upon an earlier choice of replacing the vehicle at all $\sigma_{i}^{r}$:

$$\sigma_{ij}^{d} = \sigma_{i}^{r}\sigma_{ij}^{r} \tag{A.6}$$

For a replacement decision, all vehicle platforms form a "nest" whose utility is compared to an unspecified alternative:

$$\sigma_{i}^{r} = \frac{u_{i}^{ve}}{u_{i}^{ve} + u^{oe}} \tag{A.7}$$

An increase in the variety of models does not necessarily increase aggregate utility of "the vehicle nest" proportionally. That is, utility of the nest depends on the correlation (or substitutability) of preference across a range of products in the choice outcome (not necessarily in direct relation to the different platforms). To capture this we introduce a scaled parameter $\mu \equiv 1/(1-\chi)$ with $\chi$, $0 \le \chi \le 1$, being the correlation parameter for consumer choice with respect to the platforms within the nests (further intuition is provided following equation (A.9), the nest utility is:

$$u_i^{ve} = \left[ \sum_j u_{ij}^{ve} \right]^{1/\mu} \tag{A.8}$$

While the effective utilities for the various platforms $u_{ij}^{ve}$ are the perceived utility with each platform $u_{ij}^{v}$ adjusted for their correlation, multiplied with familiarity $F_{ij}$ of the population with the various choices:

$$u_{ij}^{ve} = F_{ij} \left( u_{ij}^{v} \right)^{\mu} \tag{A.9}$$

Utility, $u_{ij}^{v}$, depends on vehicle attributes for platform *j*, as perceived by driver *i*. For an aggregate population average familiarity $F_{ij}$ varies over the interval [0, 1].

The correlation parameter can now be interpreted as follows, with $\chi \rightarrow 0$, the case of no correlation, platforms are perceived by the consumers as fully distinct and overall "vehicle utility" rises linearly with number of platforms. For $\chi \rightarrow 1$, full correlation, vehicle platforms are perceived to be identical, and the perceived utility equals that of the most superior. For instance, in the case of n identical products, with only different prices, all demand goes to the cheapest product. Lowering price for a more expensive product, while still being above the most affordable, has no effect on market shares, nor on the overall demand. Neither extreme is behaviorally appropriate. Further, dynamically, $\chi$ controls a potentially very strong feedback, between demand and the introduction of new platforms (with maximum strength at the default, no correlation, case $\chi = 1$). In addition, $\chi$ is arguably a function of the technological heterogeneity of products on the market. That is however not the point we want to make here. In this paper we assume that the consumer only cares about performance, not so much about distinctiveness between them. Thus, in this model, $\chi$ is constant between 0 and 1.

The formulation of equation (A.6)-(A.9) is equivalent to the compact general nested formulations (Ben-Akiva and Lerman (1985), Ben-Akiva 1973), frequently used in transportation decision making models (e.g. Brownstone and Small (1989)), industrial organization literatures (e.g. Anderson and Palma (1992) regarding multi product firms, Berry et al. (1995) regarding the automobile industry). We can write $\sigma_{ij}^{d}$ as:

$$\sigma_{ij}^{d} = \sigma_{ij}^{r}\sigma_{i}^{r} = \frac{u_{i}^{ve}}{u^{oe} + u_{i}^{ve}}\frac{u_{ij}^{ve}}{u_{i}^{ve}} = \frac{F_{ij}\left(u_{ij}^{ve}\right)^{\rho}}{\left[\sum_{j} F_{ij}\left(u_{ij}^{ve}\right)^{\rho}\right]^{1/\rho} + u^{oe}}$$

In the nested logit model, $1 \le \mu \le \infty$ is the scale parameter for the MNL associated with choice between alternatives within the nest (in our case the vehicles). For $\mu \to 1$, corresponding to $\chi \to 0,$ the function converges to a standard MNL, while for $\mu \to \infty$,or $\chi \to 1$, the model is a perfect nest.

The formulation of platform utility is also consistent with general Constant Elasticity of Substitution Production Function (CES-PF) (McFadden 1963), the functional form used elsewhere in the paper:

$$K_{i} = \left[\sum_{j} \kappa_{ij}\left(K_{ij}\right)^{\rho}\right]^{1/\rho}$$

In this expression $\rho = 1 - \varsigma/\varsigma$, with $\varsigma$ the elasticity of substitution between products.

In this formulation $\mu = -\rho$. Note that the range specified for vehicle choices implies an elasticity between $-\infty < \varsigma < 0$, while elasticities on the supply side specify the positive range (complementary goods imply $\varsigma$ between 0 and 1, and substitutes $1 \le \varsigma < \infty$ ).

In the model $\chi$ is set to 0.5 throughout.

## f) External scale effects

The general formulation of the aggregate scale effect, as a function of the installed base share $\sigma_j^v = V_j \big/ V^T$ , is:

$$a_{j3} \equiv \varepsilon_j^s = f\left(\sigma_j^v\right); f' \ge 0; f\left(\infty\right) = 1; f\left(\sigma_{ref}^v\right) = \varepsilon_{ref}^s$$

We use the three parameter logistic curve to generate the patterns of Figure 7. To do so we control the value of at the inflection point, and set the fixed the rest to the selected slope at that point. This results in:

$$\varepsilon_j^s \equiv \min{}^s + \frac{\left(1 - \min{}^s\right)}{1 + \left(f_j^s - \alpha^s\right) \big/ \alpha^s \exp\left[-\beta^s \nu^s \left(\frac{\sigma_j^v - \sigma_{ref}^v}{\sigma_{ref}^v}\right)\right]} \qquad (10)$$

For this curve, which is we set the scale factor, as a measure for the scale effect, and fix the slope at the reference point. $\nu^s$ is a scaling parameter to equalize the slope at the reference point and equals $\nu^s = \left(f_j^s / \alpha^s\right) \min{}^s$, and $\min{}^s = \left(f_j^s \left(1 - \alpha^s\right) + \alpha^s\right) \Big/ f_j^{s2}$ , $\alpha^s$ is an offset parameter to determine the minimum, that is, where $\sigma_j^v$. At full penetration all scale effects work maximally to its advantage. For the default settings I use an installed base 5% of the fleet, $\sigma^{v}_{ref} = 0.05$ and sensitivity parameter $\beta^{s} = 1$, which measures the slope at the reference installed base share; $\alpha^{s} = 0.8$. See also Appendix 2f of Essay 2 for a discussion on generating logistic curves.

## 3 Stipulations

### a) Generalization to multiple attributes and modules

This section discusses the more general structure of which this model is a special case. In particular detailed vehicle attributes, . In the analysis these structures where all switched off, but for more detailed analysis and insights they, or parts of them, can be switched on.

***Overview of expanded chain***

The model as specified in the paper provides all the structure necessary to generate the key insights derived in this paper. However, in order to consistently simulate a wide range of behaviors, and more intuitive patterns we include additional structure. Figure A1 shows the chain of decisions and technological chain, between resource allocation for R&D and consumer choice regarding the technologies.

| segment | variable | indices | operation |
|---|---|---|---|
| **Consumer choice** | $\sigma_j$ Market share | Platform j<br>Attribute l | Nested Logit Model |
| | $u_j$ Utility | Platform j | |
| | $a_{jl}$ Attribute | Platform j<br>Attribute l | Mapping of module technology on attributes: {m,x} → {l} |
| **Technological Performance** | $\theta_{jmx}$ Relative technology | Platform j<br>Function x | Mapping of activity type on Performance/cost: {w} → {x}<br>Normalization of technology |
| | $T^{e}_{jmw}$ Effective technology | Platform j<br>Module m<br>Activity type w | Complementarity across Activities {w'} → {w} |
| | $T_{jmw}$ Technology | Platform j<br>Module m<br>Activity type w | Diminishing returns |
| **Knowledge Accumulation** | $K^{e}_{jmw}$ Effective knowledge | Platform j<br>Module m<br>Activity type w | CES function |
| | $K_{ijmw}$ Knowledge Input | Source platform i<br>Target platform j<br>Module m<br>Activity type w | Learning-by-doing, R&D, and spillovers; |
| **Resource Allocation** | $R_{ijmw}$ Resources | Source platform i<br>Target platform j<br>Module m<br>Activity type w | Improve marginal return on effort |

**Figure A1** Diagrammed representation of chain of decisions and technological change for expanded model.

Consumer choice contains a nested logit model capturing the notion of substitutability of choice across platforms. Figure A2 shows the indices used in the expanded model. They include, from bottom to top: platforms $j$; modules $m$, the most important level at which technological change and spillovers occur; activity type $w$, that specifies whether technology advances derive from product- or process improvements; function $x$ that allows to differentiate relevance of product and process improvements to either cost or performance; and attribute $l$ that captures dimensions of merit from the perspective of a consumer.

| | | |
|---|---|---|
| attribute | $l \in \{1,...,\mathrm{L}\}$ | e.g. Price, Power, Operating Cost, Reliability. |
| function | $x \in \{\text{cost,performance}\}$ | |
| activity type | $w \in \{\text{process,product}\}$ | |
| module | $m \in \{1,...,\mathrm{M}\}$ | e.g. Body, Brakesystem, Powertrain, Wiring. |
| platform | $i, j \in \{1,..., N\}$ | e.g. ICE, HFCV, HEV, EV, CNG. |

**Figure A2** Indices used in the expanded model.

Attributes derive their state from the technology performance at the module level (Figure A1), while the current unit costs at each module determine the price attribute of the vehicle. Further down the chain, another distinction is that the effective technology captures notion of complementarity between activities: advances at the product level will make process advances obsolete. We will now describe these adjustments.

**Multiple dimensionality of choice attributes**

The perceived utility of a platform captures the aggregate of perceived attractiveness of a platform across various dimensions of merit, for which we define the attribute set that includes price, vehicle range, power etc... With $a_{ijl}$ being the state of the $l^{th}$ attribute of platform *j* as perceived by drivers of platform *i*, its perceived utility from that platform equals:

$$u_{ij} = u^* \exp\left[\sum_l \beta_l \left(a_{ijl} / a_l^* - 1\right)\right] \qquad \text{(A.11)}$$

where $\beta_l$ is the sensitivity of utility to a change in the attribute. Struben (2006a) discusses the various channels through which consumers learn about and experience performance.

The performance of a user-attribute is established through technological advances with producers of platform $j$. Further, a technology is multidimensional. Vehicles comprise of modules that include for instance the powertrain, suspension, controls and the body. This means for different attributes, different modules $m$ are determinants of the performance. We follow a two stage production function (McFadden 1963) to specify the attribute performance in relation to the knowledge produced at module level. We first describe in general formulations how we capture the dependence of attribute performance on knowledge, and follow that up with an example.

An attribute's performance comprises a fixed component, and one that depends on the current state of the technology.

$$a_{jl} = a_{jl}^{0} + a_{jl}^{v} \qquad \text{(A.12)}$$

$a_{jl}^{0}$ is the initial attribute independent of module level improvements through R&D, and other endogenous processes. This is the performance level that is attained at start-up depends for instance on the state of the complementary technologies, and can therefore differ per platform.

We assume that substitutability between modules' technology is maintained, independent of the rate of progress. We can thus use the standard constant elasticity of substitution (CES) function (Arrow et al. 1961), with multi inputs (McFadden 1963).[1] The CES approach to the multi input substitution problem is convenient, leads to simple estimation

[1] For multi inputs, the exact elasticity of substitution between two inputs is not easily to categorize, and several definitions exist. This is not a problem for our purpose.

methods and is widely used (see also Solow 1967). The behavioral characteristics are discussed further in the analysis section.

Then, performance of attribute *l* is a function of the relative technology of module *m*, and function $x, \theta_{jmx}$. The index *x* represents either performance, or cost. How a technology $\theta_{jmx}$ impacts an attribute, depends on its factor contributions to, or relevance for, attribute *l*, $\kappa_{jmlx}$. Then:

$$a_{jl}^{v} = a_{jl}^{1}\left(\sum_{m,x}\kappa_{jmxl}\left(\theta_{jmx}\right)^{-\rho_j^a}\right)^{-1/\rho_j^a} \tag{A.13}$$

where $\rho_j^a = (1 - \zeta_j^a)/\zeta_j^a$ is the substitution parameter and $\zeta_j^a$, the elasticity of substitution between the effectiveness of the technology between different modules for attribute $j$. The attribute associated with vehicle price map strictly on the cost index of x, while all others strictly map on performance. As technologies are substitutes, the range of the elasticity is confined to $1 < \varsigma_j^a < \infty$. Further, $a_{jl}^1$ is the scale, or efficiency parameter for the attribute and is scaled such that $\sum_{m,x}\kappa_{jmlx} = 1$. The distribution parameters $\kappa_{jmlx}$ define the relative importance of each module *m* to attribute $j$. Then, by construction, when all module technologies' effectiveness equal unity, the fixed share in the total attribute state equals $a_{jl}^0 \big/ \left(a_{jl}^0 + a_{jl}^1\right)$, which provides an interpretation for $a_{jl}^0$. Finally, we have constrained the aggregate performance to constant returns to scale with respect to the total effective technology. That is, the function has an implicit degree of homogeneity parameter that is set to 1.

Finally, the performance and cost are found from the technology as produced through process and product improvement. Index *w* represents different activities that allow improving a technology, such as product innovation, process improvement. Here we define strictly *w={*product innovation, process innovation}:

$$\theta_{jmx} = \sum_{w} \alpha_{jmwx} \theta_{jmw}; \sum_{x} \alpha_{jmwx} = 1 \qquad \text{(A.14)}$$

Where $\alpha_{jmwx}$ represents the share of the improvements in the technology of module m, through activity w (product or process), contributing to function x (price or performance).

***Illustration: from knowledge, to cost, to vehicle price***

Figure A1 illustrated the how chain of decisions and technological relations connect the state of an attribute to knowledge at the module/activity type level $K_{jmw}$. In the exposition we just went through, we saw that this chain can be compactly captured formally, through a two-stage CES PF. The following example serves to illustrate this more clearly. The chain of connections comprises, first, the effect of the various modules on the attribute state and, second, the effect of the various sources of knowledge to improving the technology at the modular level. I use the vehicle price attribute as an example, with $a_{j1} \equiv p_j$. I select vehicle price deliberately. We have an intuition how price is connected to cost that improves especially through learning (and scale economies). By showing that also this set of relations fits in this structure, I hope to improve our intuition of it.

Following the CES expression, using index 1 of *x* for cost, we must get to:

$$p_j = p_j^1 \left( \sum\nolimits_{m1} \kappa_{jm1} \theta_{jm1}^{e \ -\rho_{jm}} \right)^{-1/\rho_{jm}} + p_j^0; \qquad \text{(A.15)}$$

The interpretation of the variables must become clear in the end. Further, from the bottom-up, unit costs are the sum of the system level unit cost $c_j^{u0}$ and the cost incurred for producing modules, $c_{jm}^u$:

$$c_j^u = \left( \sum\nolimits_m c_{jm}^u \right) + c_j^{u0} \qquad \text{(A.16)}$$

Unit costs are prone to learning by doing and/or scale economy effects, $\varepsilon_{jm}^c$, thus: [2]

$$c_{jm}^u = c_{jm}^{u0} \varepsilon_{jm}^c \qquad \text{(A.17)}$$

But only a fraction $f_{jm}^s$ is variable with respect to scale, $\varepsilon_{jm}^s$, and $f_{jm}^s$ is subject to experience through learning-by-doing $\varepsilon_{jm}^e$:

$$\varepsilon_{jlm}^c = \left(1 - f_{jm}^s\right)\left(1 - f_{jm}^e\right) + f_{jm}^s \varepsilon_{jm}^s + f_{jm}^e \varepsilon_{jm}^e \qquad \text{(A.18)}$$

For simplification we ignore from here any internal scale economies. With $p_j = \left(1 + m_j\right) c_j$ and the derivation of the unit cost above, we now rewrite the price and derive, and interpret, the two components in (A.15):

$$p_j^1 = \left(1 + m_j\right) c_{0j}^{uve} = c_{0j}^{uve} \sum_m f_{jm}^e c_{0jm}; \sigma_{jm} = \frac{f_{jm}^e c_{0jm}}{c_{0j}^{uve}};$$

$$p_j^0 = \left(1 + m_j\right) c_{0j}^e; c_{0j}^e = \sum_m \left(1 - f_{jm}^e\right) c_{0jm} + c_{0j}$$

With $c_{0j}^{uve}$ being the part of the total unit cost subject to learning, when inputs for learning are equal to their normal levels. We see that the interpretation of the fixed component corresponds with the one provided in equation (A.12) and (A.13). $p_j^1$, the efficiency or

---

[2] Implicitly, for purpose of analytical clarity, we assume here that system level costs are not subject to learning/innovation improvement. This can easily be relaxed.

scale parameter, is the variable price, when all are equal to their normal values, and $p_j^0 / \left(p_j^0 + p_j^1\right)$ is the fixed share when all are equal to their normal values.

Further, $\theta_{jmx} = \sum_w \alpha_{jmwx} f\left(\varepsilon_{jmw}^e\right)$; with typically $\alpha_{jm21}$ large (most improvements from process improvement lead to cost improvements) and $\alpha_{jm21} > \alpha_{jm11}$ (most, but certainly not all, cost improvements come from learning by doing).

Also from bottom-up, the learning-by-doing relation also gives us:

$\varepsilon_{jmw}^e = \sigma_{jmw} \left(\frac{K_{jmw}}{K_0}\right)^{\lambda_{jmw}^e}$ and as cost decrease at diminishing rate with embedded knowledge, thus, $0 < \lambda_{jmw}^e < 1$. Thus in order for the attribute vehicle price expression to hold, $\theta_{jmw} = {\varepsilon_{jmw}^e}^{-1}$, or $-1 < \lambda_{jmw} < 0$ in equation (13) and substitution parameter $\rho$=-1. This corresponds with the elasticity of substitution being infinite. This is intuitive: we are indifferent to the sources of cost reductions. Further, in the case of vehicle price, an input factor share must be interpreted as the relative contribution of each module in terms of variable unit cost when technology is equal to normal values.

We end this exposition with the following question (and examination of it): Are technology level returns to scale are independent of the number of modules? That is, how can we avoid that dynamics are affected when we aggregate or disaggregate?

The dynamics are not affected. This follows directly from equation (A.13). First, a hypothetical case: splitting the drive train into two modules into n parts that are in fact independent, implies that the distribution parameter for each sub-module is smaller. In the case of two equal sub modules the distribution parameters are 50% of the distribution parameter of the whole module $\kappa_{jm'wl} = 0.5\kappa_{jmwl}$. The CES function is indifferent to this reconfiguration. However, in this case, the state of the technology for each must now increase at the same rate as the whole, with half the resources required. This implies that the reference resources for the sub modules are equal to half of that of the full module: $R^0_{m'} = 0.5R^0_m$. The same explanation holds when we generalize to a larger number of sub modules that have varying contribution. This also implies that, if we are interested in more basic dynamics, we can aggregate multiple modules into one, following the same procedure, without impacting the fundamental dynamics.

***Effective technology***

The complementarity between activities is captured in the net progress rate of effective technology $T^e_{jw}$ that depends on the progress rate of the total technology of all activities $w'$. For instance, complementing a radically new body will make previous process technology obsolete. Capturing this is important when we examine the interaction between novel and mature technologies. For instance, mature platforms can be expected to be conservative with innovating.

Growth of the effective technology follows that of the cumulative technology, but is adjusted for the obsolescence rate that results from other activities. With $\Gamma^e_{jW}$ being the vector of the growth rate of the effective technology, with its $w^{th}$ element defined as, and $\Gamma_{jW}$ being a similar growth rate vector for the technology $T_{jw}$ :[3]

$$\frac{dT^e_{jw}}{dt} = \sum_{w'} \varepsilon^t_{jww'} g_{jw'} T_{jw'}$$

The growth rate $g_{jw'} \equiv \left( dT_{jw'} / dt \right) / T_{jw'}$. By definition, the diagonal terms are unitary. Representing the usually negative effect of improvements in *w'* on activity *w,* the lower triangular terms are bounded by $-1 \le \varepsilon^t_{jww'} \le 0$, while the upper triangular terms are zero. Thus, with product and process innovation:

$$\mathrm{E}^t_j = \begin{bmatrix} 1 & 0 \\ \varepsilon^t_{j21} & 1 \end{bmatrix} \qquad \text{(A.19)}$$

In the analysis of the Essay, I ignore any overlap and thus, $\varepsilon^t_{j21} = 0$.

***Spillover potential***

The process knowledge related factor shares are by construction equal to unity for internal knowledge accumulation. However, for spillovers the factor contribution depends on the amount of technology of *j* that is currently embodied in the technology *i*, thus, for $i \neq j$:

$$\begin{aligned} &\frac{d\theta^\kappa_{ijw}}{dt} = \sum \varepsilon^t_{jww'} f\left( K_{ijw'} / K^0_{w'} \right) g_{iw} \theta_{jw} \\ &f(0) = 0, f(1) = \kappa^0_{w'}; f \le 1; f' > 0 \\ &\kappa_{ijw} = \sum_{w' \neq w} \theta^\kappa_{ijw'} / \theta^e_{jw} + \left( 1 - \sum_{w' \neq w} \varepsilon^t_{jww'} \right) \kappa^0_{ijw} \end{aligned} \qquad \text{(A.20)}$$

[3] This could also be represented by a co-flow structure, but in this case this construction seems more intuitive.

In the analysis, the factor overlap is equal to zero, therefore the spillover potential for process improvement is independent of the product technology, and $\kappa_{ijw}^{0} = \kappa_{ij}^{0}\ \forall w$.

In the analysis of the Essay, I ignore any overlap and thus, $\varepsilon_{j21}^{t} = 0$.

## b) The locus of diminishing returns

The process of accumulation of knowledge, improving technology, performance and increasing attractiveness is subject to increasing returns. In the model we limited diminishing returns to technology in an increase of total knowledge, while the attribute state has constant returns to technological change, and total knowledge has constant returns to knowledge accumulation. However, in real life it is hard to distinguish between them. Here we show that the return to scale parameter can be transferred among these three, without affecting the main dynamics. First, note that returns to scale is maintained across a constant returns function. For instance between attribute and technology, ignoring the function and activity indices, we have:

$$a_l = a^0 \left( \sum_{m=1}^{M} \kappa_m \left( \left[ \frac{T_m}{T^0} \right]^{\eta} \right)^{-\rho} \right)^{-1/\rho} \approx a^0 \kappa_m^{-(1-\eta)/\rho} \left( \sum_{m=1}^{M} \kappa_m \left( \frac{T_m}{T^0} \right)^{-\rho} \right)^{-\eta/\rho} \equiv a_l^{\ e}.$$

Subsequently, we can say:

$$a_0^{\ e} = \left( a_0\ \kappa_m^{-1/\rho} \right)^{1-\eta} \left( \frac{a_l^{*}}{a_0} \right)^{\eta},$$

where $a_l^* = a^0 \left( \sum_{m=1}^{M} \kappa_m \left( \frac{T_m}{T^0} \right)^{-\rho} \right)^{-1/\rho}$ is the constant returns equivalent of technology. Thus, by approximation we can shift the constant returns parameter, only having to correct by a constant. The approximation is exact, when $\kappa$ is identical for all m.

More important, diminishing returns to knowledge accumulation for source *i* implies: $dK_i / dt = \varepsilon_i^k \gamma_i ; \varepsilon_i^k = \left( K_i / K^0 \right)^{\eta^k}$ , with $\eta^k$ . We can rewrite this, through an intermediate variable $K_i^{'} \equiv \left( K_i / K^0 \right)^{1/\nu}$, such that $K_i = K^0 \left( \frac{K_i^{'}}{K^0} \right)^{\nu}$ and $\frac{dK_i^{'}}{dt} = \gamma_i \left( \frac{R_i}{R^0} \right)^{\eta^R} K^0 ; \nu = 1/1 - \eta^K$ . Further, total knowledge accumulation is also a CES function of all the various sources. Thus, we can convert diminishing returns to knowledge accumulation for each individual to diminishing returns at the level of the technology, by letting $\eta^K = \left( \eta^T - 1 \right) / \eta^T$ (thus, for diminishing returns, $\eta^K$ is negative, which makes sense: the accumulation of knowledge decreases with an increase of knowledge).

This exposition can further be expanded to include consumer choice sensitivity to a change in the attribute, and the overall elasticity of demand (including market saturation effects). Doing this will give insights under what condition the technological progress as a whole (temporarily) exhibits increasing, constant, or diminishing returns to scale.

## c) Optimal Resource Allocation

***Proposition 1*** Resource allocation decisions are asymptotically optimal within the planning horizon, holding the environment constant, including the technology of other platforms.

***Proof:*** Appendix 2A showed the decision structure and also that the perceived return on effort, $\varsigma_{x_d}^{rd\sim}$, was plausibly set equal to the marginal return on effort. Further, in equilibrium, indicated shares are equal to actual. Then (Appendix 2a), $R_{x_d} = \sigma_{x_{d-1},x_d}^{rd} R_{x_{d-1}}$, and $\varsigma_{x_d}^{rd\sim} = f\left( dP_{x_d}^{\tau_d^p} \big/ dR_{x_d} \right)$ yield:

$$\sigma_{x_d,x_{d+1}}^{rd} = f\left( \varsigma_{x_d}^{rd\sim} \big/ \varsigma_{ref,\{x\}_d}^{rd\sim} \right) R_{x_d}^{rd} \bigg/ \sum_{\{x\}_{d+1}} R_{x_{d+1}}^{rd\circ} = f\left( dP_{x_{d+1}}^{\tau_d^p} \big/ dR_{x_d} \big/ \varsigma_{ref,x_d}^{rd\sim} \right) \sigma_{x_d,x_{d+1}}^{rd} R_{\{x\}_{d+1}}^{rd} \bigg/ \sum_{\{x\}_{d+1}} R_{x_{d+1}}^{rd\circ}$$

And with *f* smooth and non-decreasing, we get:

$$\frac{\sigma_{x_d,x_{d+1}}^{rd}}{\sigma_{x_d',x_{d+1}}^{rd}} = \frac{f\left( dP_{x_{d+1}}^{\tau_d^p} \big/ dR_{x_d} \big/ \varsigma_{\{x\}_d}^{rd\sim} \right) \sigma_{x_d,x_{d+1}}^{rd} R_{\{x\}_{d+1}}^{rd}}{f\left( dP_{x_{d+1}}^{\tau_d^p} \big/ dR_{x_d} \big/ \varsigma_{\{x\}_d}^{rd\sim} \right) \sigma_{x'_d,x_{d+1}}^{rd} R_{\{x\}_{d+1}}^{rd}} \Rightarrow \left( dP_{x_d}^{\tau_d^p} \big/ dR_{x_d} \right) = \left( dP_{x'_d}^{\tau_d^p} \big/ dR_{x'_d} \right)$$

Thus, in equilibrium, the marginal returns on effort of all allocations are identical. Since the costs of resources are identical across resources, this implies optimal allocation of resources.

### *A more formal derivation*

Here we derive more formally that the preceding statement is valid. Assume a production function that improves performance indicator P, with various forms of Inputs $K_i$, with cost $C_i=c_iR_i$. Then, maximizing returns yields:

$$P\left(\overrightarrow{K(R)}\right)-\vec{C} \tag{A.21}$$

This implies that allocation of resources is optimal if:[4]

$$\frac{dP}{dR_i}=\frac{c_i}{c_j}\frac{dP}{dR_j}\forall i,j \tag{A.22}$$

Where $dP/dR_i=\left(dP/dK_i\right)\left(dK_i/dR_i\right)$, if the marginal productivity in resources is multiplicatively separable in those resources, $dP/dR_i=p_i f_i\left(R_i\right)$, then the optimal resource allocation equals:

$$\frac{dP/dR_i}{dP/dR_j}=\frac{p_i f_i\left(R_i\right)}{p_j f_j\left(R_j\right)}=\frac{c_i}{c_j}\Rightarrow R_j^*=f_j^{-1}\left(\frac{p_i c_j}{p_j c_i}f_i\left(R_i^*\right)\right) \tag{A.23}$$

And

$$\sigma_i=\frac{R_i}{\sum_j R_j}\Rightarrow\sigma_i^*=\frac{1}{1+\frac{1}{R_i^*}\sum_{j\neq i}f_j^{-1}\left(\frac{p_i c_j}{p_j c_i}f_i\left(R_i^*\right)\right)} \tag{A.24}$$

When the functional forms are identical this simplifies to:

$$\sigma_i^*=\frac{1}{\sum_j f^{-1}\left(\frac{p_j}{c_j}\frac{c_i}{p_i}\right)}=\frac{f^{-1}\left(c_i/p_i\right)}{\sum_j f^{-1}\left(c_j/p_j\right)} \tag{A.25}$$

Note that this implies, as expected, when the production function is linear in R, it is optimal to allocate all resources to the one with the highest marginal productivity. Further, in equilibrium, the desired share equals the desired resources, $\sigma_i^d=\sigma_i^*$, with

[4] We take the Paretian profit-maximization hypothesis in which only prices are fixed and conditional on diminishing marginal productivities. This is a not unlimitedly strong but general assumption.

$$\sigma_i^d = \frac{R_i^d}{\sum_j R_j^d} \tag{A.26}$$

and $R_i^d = R_i\, f\left(x_i\right)$.

In equilibrium, $\sum_j R_j^d = f\left(x_i\right)\sum_j R_j \Rightarrow f\left(x_i\right) = f\left(x_j\right);\forall i, j$

Thus, when $x_i \propto$ the marginal return on effort, $dP/dR_i$, we reach the equilibrium where shares are optimal.

As an example, assume the following multi input CES production function, as is specified for knowledge accumulation:

$$P = P^0\left(\sum_{i=1}^{I}\kappa_i\left(\frac{K_i}{K^0}\right)^{-\rho}\right)^{-\eta^S/\rho} \tag{A.27}$$

In this expression $\rho = 1-\varsigma/\varsigma$, with $\varsigma$ the elasticity of substitution between products, and $p^0 \equiv P^0/K^0$ is the price of P. Then,

$$\frac{dP}{dK_i} = \eta^S \frac{\kappa_i\left(\frac{K_i}{K^0}\right)^{-\rho}}{\sum_{i=1}^{I}\kappa_i\left(\frac{K_i}{K^0}\right)^{-\rho}}\frac{P}{K_i};\varepsilon_{P/K_i} = \eta^S \frac{\kappa_i\left(\frac{K_i}{K^0}\right)^{-\rho}}{\sum_{i=1}^{I}\kappa_i\left(\frac{K_i}{K^0}\right)^{-\rho}} \tag{A.28}$$

Assume now the following relationship $R_i = K_i$, (e.g. forms of labor productive capital vs output). The optimal share equals:

$$\frac{dP}{dK_i}/\frac{dP}{dK_j} = \frac{\kappa_i}{\kappa_j}\left(\frac{K_i}{K_j}\right)^{-(\rho+1)} = \frac{c_i}{c_j} \Rightarrow \frac{K_i}{K_j} = \left(\frac{c_j\kappa_i}{c_i\kappa_j}\right)^{1/(\rho+1)} \Rightarrow \sigma_i^* = \frac{\left(\kappa_i/c_i\right)^{1/(\rho+1)}}{\sum_j\left(\kappa_j/c_j\right)^{1/(\rho+1)}} \tag{A.29}$$

Which is identical to equation (A.25).

Again, when the PF grows linearly with resources $\left(\rho=-1\right)$, the marginal productivity between allocation to i and the others is a fixed ratio, say $\left(1+\alpha\right)$ and in equilibrium, all shares will go to the one with largest marginal productivity:

$$\sigma_i^* = \frac{\left(1+\alpha\right)\sigma_i}{\left(1+\alpha\right)\sigma_i + \sigma_{-i}} \Rightarrow \sigma_i^{eq} = 1$$

## 4 Boundary constraints considered

These involved boundary constraints to which the model is tested against. I will discuss briefly what role they play in the analysis and where they influenced dynamics, sometimes in a significant way.

### a) Capacity adjustment, backlogs and churn

Capacity adjustment assures robust dynamics during strong demand growth. Further, capacity adjustment is another balancing constraint on growth, relevant to many technologies. Japanese automakers face significant delays in meeting demands for their hybrids. Further, significant backlogs can have more side-effects involves churn and suppression of potential demand, as those who consider such a platform, will now abstain form selecting it. As the social behavior regarding this is hard to assess, a mismatch of supply and demand can severely hurt transition dynamics.

To simplify analysis, there are no adjustment costs, but it does take adjustment time $\tau^c$ to reach desired capacity $C_j^*$. Desired capacity equals current, adjusted for signals from demand:

$$C_j^* = \varepsilon_j^c C_j \tag{A.30}$$

Where and $\varepsilon_j^c$ is the effect of utilization on capacity adjustments:

$$\varepsilon_{js}^c = f^c\left(\left[\tau_j^d - \tau^{d*}\right]/\tau^{d*}\right); f^c\,' > 0; f^c\left(0\right) = 1; \tag{A.31}$$

$\tau^{d*}$ and $\tau_j^d$ are the desired and current delivery time for platform $j$. The current delivery time is given by Little's law by backlog and capacity:

$$\tau_j^d = B_j/C_j \tag{A.32}$$

The backlog structure is modeled explicitly to retain dynamic consistency when demand and supply are in significant imbalance, but also to allow for churning dynamics. Backlogs grow with initial purchase decisions $s_j^{k*}$ and churn from others $b_j^{ci}$, and decline with actual sales, at delivery, $s_j^k$, and total churn to other platforms $b_j^{co}$:

$$\frac{dB_j}{dt} = s_j^{k*} - s_j^k + b_j^{ci} - b_j^{co} \tag{A.33}$$

The indicated sales, under capacity constraint $s_j^{k*}$ is equal to the sales rate discussed in the paper. However, in perceived utility for each platform is adjusted, to include an extra attribute that captures the effect of perceived wait time on attractiveness to buy, $a_j^b = \tau_j^w$ and $a^{b*} = \tau_{ref}^w$. Actual sales under capacity constraints result from deliveries to those in the backlogs at delivery rate $\tau_j^d$, $s_j^k = B/\tau_j^d$. Further, those who are in the backlog churn when their experienced wait time $\tau_j^w$ is much larger than their expected wait time, when they decided to purchase $\tau_j^{w*}$:

$$b_j^{co} = \lambda_{ref}^c f\left(\tau_j^w / \tau_j^{w*}\right); f' \geq 0; f\left(0\right) = 0; f\left(1\right) = 1 \tag{A.34}$$

Expected weight time of those who are in the backlog, $\tau_j^{w*}$ and their experienced wait time, $\tau_j^w$, are traced through a co-flow structure (Sterman 2000; see also Appendix ** of Essay 1). Finally, the perceived backlog, also feeds into the initial purchase decision, thus backlog is another attribute at purchase, with a negative elasticity.

## b) Endogenous elasticity of substitution

It depends on the state of the technology how newly acquired knowledge contributes to the total technology. In the early stages a technology trajectory is malleable. Alternative solutions can easily be incorporated, while substitutability is low. As technology accumulates, standards emerge, flexibility decreases, which means that substitutability increases. Including this formulations allows exploring the fundamental dynamics consistently over a rich set of relevant environments. For instance, the competition between that include. For instance, incorporating this effect amplifies the fundamental dynamics.

The functional form of Equation (5) in the Essay connects to this through the substitution parameter: a low substitution parameter, say -1, implies that it is optimal to allocate all resources to those knowledge sources with the highest factor shares. A substitution parameter of 0, implies that it is optimal to build up knowledge proportional to the factor shares, and the effective knowledge is very sensitive to an increase in knowledge sources. While the substitution parameter is intimately linked with the elasticity parameter, via $\rho_{jw}^{k} = \left(1-\varsigma_{jw}\right)/\varsigma_{jw}$ , the elasticity parameter does not necessarily yield the elasticity of substitution between the knowledge between platform, when there are more than two platforms. However, following the reasoning above, the decrease of the substitution parameter is a good representation of platform maturity.

Capturing this formally through the elasticity parameter of knowledge exchange between that of platform *j* and that *i*, $\varsigma_{jw} = \varepsilon_{jw}^{\varsigma}\varsigma_{w}^{\min}$ , we get.

$$\varepsilon_{jw}^{\varsigma} = f\left(\theta_{jw}\right); f\left(0\right) > 0; f\left(1\right) = 1; f' \geq 0 \qquad \text{(A.35)}$$

We imposed these conditions for the logical arguments of maturation of the technology. However, under these conditions of a non-decreasing elasticity parameter $\varsigma_{jw}$, we can arrive at the same intuition more formally:

***Proposition 2:*** For novel technologies, the effective technology exhibits increasing returns in the number of platforms. However in the long-run equilibrium, technologies exhibit neutral returns to the number platforms, even under infinite market and constant entrance probabilities.

Intuition: spillovers are a central mechanism for growth of knowledge, especially in the early stages of a product lifecycle. For a novel technology, knowledge is incomplete and thus has complementarities with that within other technologies. As a technology matures, knowledge is increasingly complex, providing more incentive to exploit the most productive aspects.

***Proof:*** Assume the a fortiori case in which spillover rate among platforms is infinite and free, so all resources are allocated to internal knowledge development, and we don't have to impose optimal allocation of resources.

Starting with $K_{jm}^{n} = K_{0jm}\left(\sum_{i=1}^{M}\kappa_{ijm}\left(k_{ijm}\right)^{-\rho_{ijm}^{M}}\right)^{-1/\rho_{jm}^{M}}$. Then, introducing a new platform *n+1*, with instantaneous spillover to platform *j* implies:

$$K_{jm}^{n+1} = K_{0jm}^{\{n\}}\left(\sum_{i=1}^{n}\kappa_{ijm}\left(k_{ijm}\right)^{-\rho_{ijm}^{M}} + \kappa_{n+1,jm}\left(k_{n+1,jm}\right)^{-\rho_{ijm}^{M}}\right)^{-1/\rho_{jm}^{M}} \tag{A.36}$$

The main insights are derived when we rewrite this, so that the first order effect gets captured in the scale parameter $K_{0jm}$. Hereto we defining a set of distribution parameters $\kappa'_{i,jm}$, such that $\sum_{i=1}^{n+1}\kappa'_{ijm} = \sum_{i=1}^{n}\kappa_{ijm}$. Then we can rewrite (A.36) to

$K_{jm} = K_{0jm}^{\{n+1\}}\left(\sum_{i=1}^{n+1}\kappa_{i,jm}'\left(k_{ijm}\right)^{-\rho_{jm}^{M}}\right)^{-1/\rho_{jm}^{M}}$. With:

$$\frac{K_{0jm}^{\{n+1\}}}{K_{0jm}^{\{n\}}} = \left(\sum_{i=1}^{n+1}\kappa'_{i,jm} \Big/ \sum_{i=1}^{n}\kappa_{ijm}\right)^{-1/\rho_{jm}^{M}} \tag{A.37}$$

For immature technologies and very small number of technologies, or, elasticity of substation close to 1, $\rho_{jm}^{M} \to 0$, and *n* small, the increasing returns are very large. However, once the entrants increase, *n* large, and the technology matures elasticity $\rho_{jm}^{M} \to -1$, the effect diminishes fully. Also, effect of entrance on knowledge growth raises with the distribution parameters. New entrants have less knowledge, so (A.37) bounds the direct knowledge. Other second order effects are also balancing, for instance, the market share goes down, reducing revenues and profits, and resource allocation for all the incumbent platforms. Generally the overlap (or quality) becomes cannot be maintained, corresponding with on average smaller spillover factors $\kappa$, further reducing the returns to the number of entrants, for incumbents. This is in particular the case for more mature technologies, for which marginal benefit of an increase in knowledge increases linearly with the input factor.

Thus, spillover is a central mechanism for growth of knowledge, especially in the early stages of a product lifecycle. For novel technologies, knowledge is incomplete and thus has complementarities with others. As technology matures, knowledge is increasingly substitutable, providing more incentive to exploit the most productive aspects. As a corollary to this, platforms with more mature knowledge fixate on fewer candidates for sources of spillover. Another interpretation of this is that with reduction of uncertainty the knowledge allocation is more accurate – closer to the concept of Jovanovic (1982) that companies only borrow from the leader.

### c) Product experience

Product improvement productivity increases with effective experience in R&D. This captures an additional feedback loop that will extend the time for new technologies to catch up. This is included by making the productivity of product innovations endogenous, tracing experience $E_{j1}$ that accumulates historic resources allocation:

$$\varepsilon_{j1}^{r} = \left(E_{j1} / E_{0}\right)^{\eta^{e}} \qquad (A.38)$$
$$\frac{dE_{j1}}{dt} = R_{j1}$$

where $E_0$ is the reference experience at which relative productivity is equal to 1.

## d) Markups

Markups adjust to desired levels $m_j^*$ over adjustment time $\tau^m$. Desired markups are equal to a reference markup, adjusted through pressure from market level prices $\varepsilon_j^m$

$$m_j^* = \varepsilon_j^m m_{ref} \qquad (A.39)$$

Pressure to decrease (increase) markups result from a discrepancy between price $p_j = \left(1 + m_j\right) c_j$ and the market level, relevant for platform *j*, $p_j^m$:

$$\varepsilon_j^m = f\left(p_j / p_j^m\right); f' < 0; f(0) = 0; f(1) = 1; f(\infty) = \varepsilon_{\max}^m \qquad (A.40)$$

The perceived relevant market price adjusts to the actual relevant market price $p_j^{*m}$ with adjustment time $\tau^p$. This model ignores potential product differentiation with respect to consumer choice; therefore the indicated relevant market price is the price of all platforms weighted by their market shares:

$$p_j^{*m} = \sum_{j'} \sigma_{j'} p_{j'} \qquad (A.41)$$

Thus, in the long run the market tends to produce at unit costs of the cheapest producing platform.

Throughout the analysis I hold markups fixed at 0.2.

## e) Scale economies within a platform

The role of scale economies are important to consider. First they

We include two forms of scale economies. First, important economies of scale, internal to the production, and act at the modular level, $\varepsilon_{jm}^{s}$. Other scale economies are aggregated and modeled as a function of the existing installed base and introduced in the analysis section. Here we specify the internal scale economies:

$$\varepsilon_{jm}^{s} = f\left(s_j / s^0\right); f' \leq 0; f(\infty) = 1; f(1) = 1 \qquad \text{(A.42)}$$

The selected function is a standard power law, where cost improves as $f(x) = x^{\gamma_m^s}$. The scale exponent $\gamma_m^s$ is calculated from the assumed fractional cost improvement per doubling of sales, $(1 + \Delta) = (2s_0/s_0)^{\gamma}$, or $\gamma = \ln(1+ \Delta)/\ln(2)$. For analysis a 30% scale curve, $\Delta = 0.3$, is the default, corresponding with the scale effect parameter $\gamma^s = 0.379$. The section that discusses c) Optimal Resource Allocation shows how scale economies feed into the cost equation.

In the analyses of the Essay the scale effect parameter is set to 0.

## 5 Model and analysis documentation

The model and analyses can be replicated from the information provided in the Essay and the first section in the Appendix. In addition model source code and analysis documentation can be downloaded from

http://web.mit.edu/jjrs/www/Thesis Documentation.htm

## 6 References


Ben-Akiva, M. E. and S. R. Lerman (1985). Discrete choice analysis : theory and application to travel demand. Cambridge, Mass., MIT Press.

Berry, S., J. Levinsohn, et al. (1995). "Automobile Prices in Market Equilibrium." Econometrica **63**(4): 841-890.

Brownstone, D. and K. A. Small (1989). "Efficient Estimation of Nested Logit-Models." Journal of Business & Economic Statistics **7**(1): 67-74.