Minimal scenarios for leptogenesis and CP violation

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The relation between leptogenesis and CP violation at low energies is analyzed in detail in the framework of the minimal seesaw mechanism. Working, without loss of generality, in a weak basis where both the charged lepton and the right-handed Majorana mass matrices are diagonal and real, we consider a convenient generic parametrization of the Dirac neutrino Yukawa coupling matrix and identify the necessary condition which has to be satisfied in order to establish a direct link between leptogenesis and CP violation at low energies. In the context of the LMA solution of the solar neutrino problem, we present minimal scenarios which allow for the full determination of the cosmological baryon asymmetry and the strength of CP violation in neutrino oscillations. Some specific realizations of these minimal scenarios are considered. The question of the relative sign between the baryon asymmetry and CP violation at low energies is also discussed.

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I. INTRODUCTION

One of the most exciting recent developments in particle physics is the discovery of neutrino oscillations pointed out by the Sudbury Neutrino Observatory [1] and confirmed by the Super-Kamiokande experiment [2]. Neutrino oscillations provide evidence for non-vanishing neutrino masses and mixings, with the novel feature that large leptonic mixing angles are required, in contrast with what happens in the quark sector. Indeed, the combined results from these experiments suggest that, in addition to the large mixing angle required by the atmospheric neutrino data, another large angle should be present in the leptonic sector. This leads to the so-called large mixing angle (LMA) solution of the solar neutrino problem which turns out to be presently the most favored scenario for the explanation of the solar neutrino deficit. From a theoretical point of view, understanding the large leptonic mixing is still an unresolved mystery for which a considerable number of solutions have been proposed [3]. On the other hand, the appearance of neutrino masses much smaller than those of charged leptons is elegantly explained through the seesaw mechanism [4] which can be implemented by extending the standard model (SM) particle content with right-handed neutrinos. These can be easily accommodated in grand unified theories (GUT) where they appear on equal footing with the other SM particles.

The heavy singlet neutrino states can also play an important role in cosmology, namely, in the explanation of the observed cosmological baryon asymmetry. During the last few years, the data collected from the acoustic peaks in the cosmic microwave background radiation [5] has allowed us to obtain a precise measurement of the baryon asymmetry of the universe (BAU). The Microwave Anisotropy Probe (MAP) experiment [6] and the Planck satellite [7] planned for the near future should further improve this result. At the present time, the measurement of the baryon-to-entropy ratio $Y_B/n_B$ is

$$0.7 \times 10^{-10} \leq Y_B / n_B \leq 1.0 \times 10^{-10}.$$  \hfill (1)

Leptogenesis is one of the most attractive mechanisms to generate the BAU. As first suggested by Fukugita and Yanagida [8], the key ingredient in leptogenesis are the heavy Majorana neutrinos which, once included in the SM, can give rise to a primordial lepton asymmetry through their out-of-equilibrium decays. This lepton asymmetry is subsequently reprocessed into a net baryon asymmetry by the anomalous sphaleron processes.

In spite of being attractive and successful, leptogenesis turns out to be extremely difficult or even impossible to test experimentally in a direct way. This difficulty is obviously related to the large masses of the heavy Majorana neutrino singlets. Nevertheless, the joint analysis of leptogenesis and low-energy neutrino phenomenology can be viewed as an indirect way of testing it and here the experimental results from neutrino oscillation experiments such as those related to the search of leptonic CP violation in the future long-baseline neutrino experiments are extremely valuable [9].

In this paper, we will address the question of linking the amount and sign of the BAU to low-energy neutrino experiments, namely to the sign and strength of the CP asymmetries measured through neutrino oscillations. Our analysis is performed in the weak basis (WB) where the charged lepton mass matrix $m_l$ and the right-handed Majorana matrix $M_R$ are both real and diagonal. In this WB, all CP-violating phases are contained in the Dirac neutrino mass matrix $m_D$. The matrix $m_D$ is arbitrary and complex, but since three of its nine phases can be eliminated through rephasing, one is...
left with six independent physical CP-violating phases. In order to study the link between the BAU generated through leptogenesis and CP violation at low energies, it is crucial to use a convenient parametrization of \( m_D \). We shall make use of the fact that any arbitrary complex matrix can, without loss of generality, be written as the product of a unitary matrix \( U \) and a lower triangular matrix \( Y_\Delta \). We show that \( U \) contains three phases which do not contribute to leptogenesis, while the other three phases contained in \( Y_\Delta \) contribute to both leptogenesis and low-energy CP violation. As a result, a necessary condition for having a link between leptogenesis and low-energy CP breaking is that the matrix \( U \) contains no phases, the simplest choice being obviously \( U = 1 \). Within this class of Dirac neutrino mass matrices, we perform a search of the minimal scenarios where not only a good fit of low-energy neutrino data is obtained but also a link between the observed size and sign of the BAU and the strength of CP violation observable at low energies through neutrino oscillations can be established.

II. GENERAL FRAMEWORK

We work in the framework of a minimal extension of the SM which consists of adding to the standard spectrum one right-handed neutrino per generation. Before gauge symmetry breaking, the leptonic couplings to the SM Higgs doublet \( \phi \) can be written as

\[
L_Y = -Y_{\nu \phi}^T \bar{v}_L^0 \phi_R^0 - Y_{\nu \phi}^T \phi_R^0 \bar{v}_L^0 + \text{H.c.},
\]

where \( \phi = i \tau_2 \phi^* \). After spontaneous gauge symmetry breaking the leptonic mass terms are given by

\[
L_m = - \left[ \frac{1}{2} \bar{v}_R^0 M_D v_R^0 + \frac{1}{2} \bar{v}_R^0 M_C M_R v_R^0 + \bar{v}_R^0 m_i^0 \right] + \text{H.c.},
\]

\[
= - \left[ \frac{1}{2} \bar{v}_R^0 M_D M^* n_L + \bar{v}_R^0 m_i^\ell_R \right] + \text{H.c.},
\]

where \( m_D = v Y_{\nu} \) is the Dirac neutrino mass matrix with \( v = \langle \phi^0 \rangle / \sqrt{2} \approx 174 \text{ GeV} \), \( M_D \) and \( m_i^0 = v Y_{\nu} \) denote the right-handed Majorana neutrino and charged lepton mass matrices, respectively, and \( n_L = (\nu_L^0, (\nu_R^0)^c) \). Among all the terms, only the right-handed neutrino Majorana mass term is SU(2) \( \times U(1) \) invariant and, as a result, the typical scale of \( M_R \) can be much above the electroweak symmetry breaking scale \( v \), thus leading to naturally small left-handed Majorana neutrino masses of the order \( m_i^0 / M_R \) through the seesaw mechanism. In terms of weak-basis eigenstates the leptonic charged current interactions are given by

\[
L_W = - \frac{g}{\sqrt{2}} W_{\mu}^+ v_L^0 \gamma^\rho \nu^\rho_L + \text{H.c.}.
\]

It is clear from Eqs. (3) and (4) that it is possible to choose, without loss of generality, a weak basis where both \( m_i \) and \( M_R \) are diagonal, real and positive. Note that in this WB, \( m_D \) is a general complex matrix which contains all the information on CP-violating phases. Since in the present framework there is no \( \Delta L = 2 \) mass term of the form \( \frac{1}{2} \bar{v}_L^0 C M_D v_R^0 \), the total number of CP-violating phases for \( n \) generations is given by \( n(n-1) \) \cite{10} which are all contained in \( m_D \) in this special weak basis.\(^1\)

We recall that the full \( 6 \times 6 \) neutrino mass matrix \( M \) is diagonalized via the transformation:

\[
V^T M^* V = D,
\]

where \( D = \text{diag}(m_1, m_2, m_3, M_1, M_2, M_3) \), with \( m_i \) and \( M_i \) denoting the physical masses of the light and heavy Majorana neutrinos, respectively. It is convenient to write \( V \) and \( D \) in the following form, together with the definition of \( \mathcal{M} \):

\[
V = \begin{pmatrix} K & Q \\ S & T \end{pmatrix}, \quad D = \begin{pmatrix} d_{\nu} & 0 \\ 0 & D_R \end{pmatrix}, \quad \mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_T^D & M_R \end{pmatrix}.
\]

From Eq. (5) one obtains, to an excellent approximation, the seesaw formula:

\[
d_{\nu} = -K^\dagger m_D m_R^{-1} m_T^D K = K^\dagger \mathcal{M}_{\nu} K^*,
\]

where \( \mathcal{M}_{\nu} \) is the usual light neutrino effective mass matrix. The leptonic charged-current interactions are given by

\[
- \frac{g}{\sqrt{2}} (\bar{\nu}_L \gamma^\mu K \nu_L + \bar{\nu}_L \gamma^\mu Q N_L) W^\mu + \text{H.c.},
\]

CP violation in neutrino oscillations

It has been shown \cite{12} that the strength of CP violation at low energies, observable for example through neutrino oscillations, can be obtained from the following low-energy WB invariant:

\[
T_{\text{CP}} = \text{Tr}[H_{\nu} H_{\ell}^T]^3 = 6i \Delta_{21} \Delta_{31} \Delta_{32} \text{Im}[\langle H_{\nu}, H_{\ell} \rangle_{23} (\langle H_{\nu}, H_{\ell} \rangle_{13})],
\]

where \( H_{\nu} = M^\dagger_{\nu} \mathcal{M}_{\nu}^\dagger \), \( H_{\ell} = m_{\ell} m_D^\dagger \) and \( \Delta_{il} = (m_i^2 - m_j^2) \) with analogous expressions for \( \Delta_{21} \), \( \Delta_{32} \). This relation can be computed in any weak basis. The low-energy invariant (9) is sensitive to the Dirac-type phase \( \delta \) and vanishes for \( \delta = 0 \). On the other hand, it does not depend on the Majorana

\(^1\)The counting of independent CP-violating phases for the general case, where besides \( m_D \) and \( M_R \) there is also a left-handed Majorana mass term at tree level, has been discussed in Ref. \cite{11}.
where the $\Delta m_{ij}$’s are the usual light neutrino mass squared differences and $J_{CP}$ is the imaginary part of an invariant quartet appearing in the difference of the CP-conjugated neutrino oscillation probabilities $P(\nu_e \rightarrow \nu_\mu) - \bar{P}(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$. One can easily get

$$J_{CP} = \text{Im}[(U_\nu)^*_i(U_\nu)^*_j(U_\nu)^*_k(U_\nu)^*_l] = \frac{1}{8} \sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23})\cos(\theta_{13})\sin \delta,$$

(11)

where the $\theta_{ij}$ are the mixing angles appearing in the standard parametrization adopted in [13]. Alternatively, one can use Eq. (10) and write

$$J_{CP} = -\frac{\text{Im}[(U_\nu)^*_i(U_\nu)^*_j(U_\nu)^*_k(U_\nu)^*_l]}{\Delta m^2_{12}\Delta m^2_{23}\Delta m^2_{31}}.$$

(12)

This expression has the advantage of allowing the computation of the low-energy CP invariant without resorting to the mixing matrix $U_\nu$.

It is also possible to write WB invariants useful to leptogenesis [12] as well as WB invariant conditions for CP conservation in the leptonic sector relevant in specific frameworks [11,14].

### III. CP ASYMMETRIES IN HEAVY MAJORANA NEUTRINO DECAYS

The starting point in leptogenesis scenarios is the CP asymmetry generated through the interference between tree-level and one-loop heavy Majorana neutrino decay diagrams. In the simplest extension of the SM, such diagrams correspond to the decay of the Majorana neutrino into a lepton and a Higgs boson. Considering the decay of one heavy Majorana neutrino $N_j$, this asymmetry is given by

$$\varepsilon_j = \frac{\Gamma(N_j \rightarrow \ell \phi) - \Gamma(N_j \rightarrow \ell \phi^\dagger)}{\Gamma(N_j \rightarrow \ell \phi) + \Gamma(N_j \rightarrow \ell \phi^\dagger)}.$$  

(13)

In terms of the Dirac neutrino Yukawas the CP asymmetry (13) is [15]

$$\varepsilon_j = \frac{1}{8\pi(Y^\nu_{\ell}Y^\nu)^*_j} \sum_{i+j} \text{Im}[(Y^\nu_{\ell}Y^\nu)^*_j]\frac{M^2_j}{M^2_j},$$  

(14)

where the index $j$ is not summed over in $(Y^\nu_{\ell}Y^\nu)^*_j$. The loop function $f(x)$ includes the one-loop vertex and self-energy corrections to the heavy neutrino decay amplitudes,

$$f(x) = \sqrt{x} \left[ (1 + x) \ln \left( \frac{x}{1 + x} \right) + \frac{2 - x}{1 - x} \right].$$  

(15)

From Eq. (14) it can be readily seen that the CP asymmetries are only sensitive to the CP-violating phases appearing in $Y^\nu_{\ell}Y^\nu$ (or equivalently in $m_\beta m_\gamma$) in the WB where $M_R$ and $m_\ell$ are diagonal.

#### A. Hierarchical case: $M_1 < M_2 < M_3$

In the hierarchical case $M_1 < M_2 < M_3$, only the decay of the lightest heavy neutrino $N_1$ is relevant for leptogenesis, provided the interactions of $N_1$ are in thermal equilibrium at the time $N_{2,3}$ decay, so that the asymmetries produced by the latter are erased before $N_1$ decays. In this situation, it is sufficient to take into account the CP asymmetry $\varepsilon_1$. Since in the limit $x \gg 1$ the function $f(x)$ can be approximated by

$$f(x) = -3/(2\sqrt{x}),$$

we have from Eq. (14)

$$\varepsilon_1 = -\frac{3}{16\pi(Y^\nu_{\ell}Y^\nu)^*_1} \sum_{k=2,3} \text{Im}[(Y^\nu_{\ell}Y^\nu)^*_k] \frac{M^2_k}{M^2_k}.$$  

(16)

which can be recast in the form [16]

$$\varepsilon_1 = -\frac{3M_1}{16\pi} \text{Im}[(Y^\nu_{\ell}Y^\nu_D R^{-1} Y Y^\nu)^*_1] \frac{M^2_1}{(Y^\nu_{\ell}Y^\nu)^*_1},$$  

(17)

using the seesaw relation given in Eq. (7).

#### B. Two-fold quasi-degeneracy: $M_1 = M_2 \ll M_3$

In the context of thermal leptogenesis, when the observed baryon asymmetry is generated through the decays of the lightest heavy Majorana neutrino $N_1$, there exists an upper bound on the CP asymmetry $\varepsilon_1$ which directly depends on the mass of the lightest neutrino $M_1$ [17]. In turn, such a bound implies a lower bound on the lightest mass $M_1$, typically $M_1 \approx 10^8$ GeV. The latter bound is however barely compatible with the reheating temperature bound $T_R \approx 10^6 \sim 10^9$ GeV required in several supergravity models in order to avoid a gravitino overproduction [19]. To overcome this problem one can consider, for instance, the decays of two heavy neutrinos which are quasi-degenerate in mass, $M_1 \approx M_2$. In this case, the CP asymmetries $\varepsilon_1$ are enhanced due to self-energy contributions [21] and the required baryon asymmetry can be produced by right-handed heavy neutrinos.

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2 This approximation can be reasonably used for $x \approx 15$.

3 A more stringent constraint, $M_1 \approx 10^{10}$ GeV, is obtained in [18].

4 Another possible solution to the gravitino problem is to consider non-thermal production mechanisms [20]. Since in these cases the condition $M_1 < T_R$ is not required, the gravitino problem is easily avoided once heavy particles can be created with a relatively low reheating temperature without threatening big bang nucleosynthesis.
with moderate masses $M_1 = M_2 \approx 10^8$ GeV. Moreover, it has been shown that in the presence of small Dirac-type leptonic mixing at high energies and GUT-inspired Dirac neutrino Yukawa couplings, the heavy Majorana neutrino degeneracy is compatible with the LMA solar solution [22].

Let us assume that the heavy Majorana neutrinos $N_1$ and $N_2$ are quasi-degenerate. It is useful to define the parameter $\delta_N$ which represents the degree of degeneracy between the masses $M_1$ and $M_2$ as

$$\delta_N = \frac{M_2}{M_1} - 1. \quad (18)$$

Since $M_1 = M_2$, we expect $\delta_N \ll 1$. For the perturbative approach to remain valid, the tree-level decay width $\Gamma_i$ for each of the heavy Majorana neutrinos must be much smaller than the mass difference between them. This is translated into the relations

$$\Gamma_i = \frac{(H_{\nu})_{ii} M_i}{8 \pi} \ll M_2 - M_1 = \delta_N M_1, \quad i = 1, 2, \quad (19)$$

where $H_{\nu} = Y_{\nu}^T Y_{\nu}$. From this equation we can find the following lower bound for $\delta_N$:

$$\delta_N \approx \max \left\{ \frac{(H_{\nu})_{ii} M_i}{8 \pi M_1} \right\}_{i=1,2}. \quad (20)$$

Assuming that this criterion is verified, the $CP$ asymmetries $\varepsilon_i$ can be obtained combining Eqs. (14) and (18). We find

$$\varepsilon_1 = - \frac{1}{8 \pi (H_{\nu})_{11}} \left[ \text{Im} (H_{\nu})_{21}^2 / (1 + \delta_N)^2 \right]$$

$$- \frac{3}{2} \text{Im} (H_{\nu})_{31} M_1 / M_3,$$}

$$\varepsilon_2 = - \frac{1}{8 \pi (H_{\nu})_{22}} \left[ \text{Im} (H_{\nu})_{12}^2 / (1 + \delta_N)^2 \right]$$

$$- \frac{3}{2} \text{Im} (H_{\nu})_{32} M_2 / M_3. \quad (21)$$

Taking into account that for $\delta_N \ll 1$ the function $f[(1 + \delta_N)^\pm 2]$ can be approximated by $\frac{1}{2} \delta_N$, we obtain

$$\varepsilon_j = \frac{1}{16 \pi (H_{\nu})_{jj}} \left[ \text{Im} (H_{\nu})_{21}^2 / \delta_N + 3 \text{Im} (H_{\nu})_{31} M_1 / M_3 \right],$$

$$j = 1, 2. \quad (23)$$

Typically, the term proportional to $M_1 / M_3$ can be neglected and in this case $\varepsilon_1$ and $\varepsilon_2$ have the same sign. This aspect turns out to be relevant for the discussion on the relative sign between the BAU and low-energy leptonic $CP$ violation.

**IV. ON THE CONNECTION BETWEEN LEPTOGENESIS AND LOW-ENERGY CP VIOLATION**

In this section we analyze the possible link between $CP$ violation at low energies, measurable for example through neutrino oscillations, and leptogenesis. The possibility of such a connection has been previously analyzed in the literature [12,23,24]. Nevertheless, we find it worthwhile presenting here a thorough discussion on the subject. In particular, we will address the following questions:

If the strength of $CP$ violation at low energies in neutrino oscillations is measured, what can one infer about the viability or non-viability of leptogenesis?

From the sign of the BAU, can one predict the sign of the $CP$ asymmetries at low energies, namely the sign of $J_{CP}$?

We will show that having an explicit parametrization of $m_D$ (or equivalently of $Y_{\nu} = m_D / v$) is crucial not only to determine which phases are responsible for leptogenesis and which ones are relevant for leptonic $CP$ violation at low energies, but also to analyze the relationship between these two phenomena.

From the available neutrino oscillation data, one obtains some information on the effective neutrino mass matrix $M_{\nu}$ which can be decomposed in the following way:

$$U_{\nu} d_{\nu} U_{\nu}^T = M_{\nu} = L L^T, \quad L = \text{im} D_{R}^{-1/2}. \quad (24)$$

The extraction of $L$ from $M_{\nu}$ suffers from an intrinsic ambiguity [25] in the sense that, given a particular solution $L_0$ of Eq. (24), the matrix $L = L_0 R$ will also satisfy this equation, provided that $R$ is an arbitrary orthogonal complex matrix, $R \in O(3, C)$, i.e. $R R^T = 1$. It is useful to take as a reference solution $L_0 = U_{\nu} d_{\nu}^{1/2}$, so that

$$L = U_{\nu} d_{\nu}^{1/2} R. \quad (25)$$

Since three of the phases of $m_D$ can be eliminated, the matrix $L$ has 15 independent parameters. The parametrization of $L$ given in Eq. (25) has the interesting feature that all its parameters are conveniently distributed among $U_{\nu}$, $d_{\nu}$, and $R$, which contain 6 (3 angles + 3 phases), 3 and 6 (3 angles + 3 phases) independent parameters, respectively. Of the 18 parameters present in the Lagrangian of the fundamental theory described by $m_D$ and $D_{R}$, only 9 appear at low energy in $M_{\nu}$ through the seesaw mechanism. To further disentangle $m_D$ from $D_{R}$ in $L$, one needs the 3 remaining inputs, namely the three heavy Majorana masses of $D_{R}$. As for the meaning of the information encoded in $R$, it turns out that the pattern of this matrix has a suggestive interpretation in terms of the different rôles played by the heavy neutrinos in the seesaw mechanism. In fact, $R$ can be viewed as a dominance matrix [26] since it gives the weights of each heavy Majorana neutrino in the determination of the different light neutrino masses $m_1$ [27]. The fact that $R_{ij}^2$ are weights for $m_i$ is quite obvious due to the orthogonality of $R$.
\[ m_i = \sum_j m_j R^2_{ij}, \]  
\( (26) \)

On the other hand, since \( U^\dagger m_R = -i d_{\nu}^{1/2} R D^1_R \), the single contribution \( m_j R^2_{ij} \) is also given by

\[ m_j R^2_{ij} = - \frac{(U^\dagger m_R)^2_{ij}}{M_j} = \frac{X_{ij}}{M_j}. \]  
\( (27) \)

Therefore, once \( U_{\nu} \) is fixed, each weight \( R^2_{ij} \) just depends on the mass \( M_j \) of the \( j \)-th heavy Majorana neutrino and on its couplings with the left-handed neutrinos \( (m_D)_{ij} \). Thus, the contribution of each heavy neutrino to \( m_i \) is well defined and expressed by the weight \( \text{Re}(R^2_{ij}) \). One may roughly say that the heavy Majorana neutrino with mass \( M_j \) dominates\(^6\) in \( m_i \) if

\[ \frac{|\text{Re}(X_{ij})|}{M_j} > \frac{|\text{Re}(X_{ik})|}{M_k}, \quad k \neq j \]  
\( (28) \)

which implies \( |\text{Re}(R^2_{ij})| > |\text{Re}(R^2_{ik})| \). So that, if one of the heavy Majorana neutrino gives the dominant contribution to one of the masses \( m_i \), this information is encoded in the structure of \( R \). The interpretation in terms of weights is straightforward for the rotational part of \( R \). However, one has to be careful because in the presence of the three boosts (controlled by the three phases) the weights \( R^2_{ij} \) are not necessarily real and positive. Although this situation is more subtle, the above dominance arguments still hold.

Coming back to the connection between leptogenesis and low-energy data, it is important to note that \( U_{\nu} \) does not appear in the relevant combination for leptogenesis \( Y_P U_{\nu} \), in the same way as \( R \) does not appear in \( \mathcal{M}_L \). Indeed, one has

\[ m_P m_D = D^1_R R^1_{\nu} R D^1_R. \]  
\( (29) \)

From the above discussion, it follows that it is possible to write \( m_P \) in the form \( m_D = -i U_{\nu} d_{\nu}^{1/2} R D^1_R \) in such a way that leptogenesis and the low-energy neutrino data (contained in \( \mathcal{M}_L \)) depend on two independent sets of \( CP \)-violating phases, respectively those in \( R \) and those in \( U_{\nu} \). In particular, one may have viable leptogenesis even in the limit where there are no \( CP \)-violating phases (neither Dirac nor Majorana) in \( U_{\nu} \) and hence, no \( CP \) violation at low energies [28]. Therefore, in general it is not possible to establish a link between low-energy \( CP \) violation and leptogenesis. This connection is model dependent: it can be drawn only by specifying a particular \( ansatz \) for the fundamental parameters of the seesaw, \( m_D \) and \( D_R \), as will be done in the following sections.

The relevance of the matrix \( R \) for leptogenesis can be rendered even more explicit [27] by rewriting the \( \varepsilon_1 \) asymmetry by means of Eq. (29) and defining \( R_{ij} = |R_{ij}| e^{i \phi_{ij}/2} \), \( \Delta m^2_\odot = \Delta m^2_{21} \) and \( \Delta m^2_{\odot} = \Delta m^2_{32} \). In the case of hierarchical heavy Majorana neutrinos, say \( M_1 \ll M_2 \ll M_3 \) one obtains

\[ \varepsilon_1 \approx \frac{3}{16 \pi \nu^2} \frac{M_1 \Delta m^2_{31} |R_{31}|^2 \sin \varphi_{31} - \Delta m^2_{21} |R_{21}|^2 \sin \varphi_{21}}{m_1 |R_{11}|^2 + m_2 |R_{21}|^2 + m_3 |R_{31}|^2}, \]  
\( (30) \)

and we recover what one would have expected by intuition, namely that the physical quantities involved in determining \( \varepsilon_1 \) are just \( M_1 \), the spectrum of the light neutrinos, \( m_i \), and the first column of \( R \), which expresses the composition of the lightest heavy Majorana neutrino in terms of the light neutrino masses \( m_i \). In the case \( M_1 = M_2 = M_3 \), similar expressions hold,

\[ \varepsilon_j \approx \frac{1}{16 \pi \nu^2} \frac{M_1 M_2 \text{Im}(R_{11}^* R_{21})}{M_2 - M_1} \frac{R_{21}}{(R^1_d, R^1_l)}, \quad j = 1, 2, \]  
\( (31) \)

where now also the mass \( M_2 \) and the second column of \( R \) are involved. A detailed study of the relevance of the matrix \( R \) for leptogenesis is under way [31].

As stressed before, different \( ansätze \) for \( R \) have no direct impact on \( CP \) violation at low energy; the impact is in a sense indirect because \( R \) specifies if dominance of some heavy Majorana neutrino is at work in the seesaw mechanism [26].

In conclusion, the link between leptogenesis and low-energy \( CP \) violation can only be established in the framework of specific \( ansätze \) for the leptonic mass terms of the Lagrangian. We shall derive a necessary condition for such a link to exist. In order to obtain this connection, it is convenient to use a triangular parametrization for \( m_D \), which we describe next.

### Triangular parametrization

It can be easily shown that any arbitrary complex matrix can be written as the product of a unitary matrix \( U \) with a lower triangular matrix \( Y_{\Delta} \). In particular, the Dirac neutrino mass matrix can be written as

\[ m_D = U Y_{\Delta}, \]  
\( (32) \)

with \( Y_{\Delta} \) of the form

\[ Y_{\Delta} = \begin{pmatrix} y_{11} & 0 & 0 \\ y_{21} e^{i \phi_{21}} & y_{22} & 0 \\ y_{31} e^{i \phi_{31}} & y_{32} e^{i \phi_{32}} & y_{33} \end{pmatrix}, \]  
\( (33) \)

where \( y_{ij} \) are real positive numbers. Since \( U \) is unitary, in general it contains six phases. However, three of these phases can be rephased away by a simultaneous phase transformation on \( \nu^0_L, \tilde{\nu}^0_L \), which leaves the leptonic charged current invariant. Under this transformation, \( m_P \rightarrow P \gamma m_D \), with \( P \equiv \text{diag}(e^{i \epsilon_1}, e^{i \epsilon_2}, e^{i \epsilon_3}) \). Furthermore, \( Y_{\Delta} \) defined in Eq. (33) can be written as

\[ Y_{\Delta} = P \tilde{Y}_{\Delta} P^\dagger, \]  
\( (34) \)

\(^6\)See for instance Refs. [29,30] for other approaches to the dominance mechanism.
where \( P_\beta = \text{diag}(1,e^{i\beta_1},e^{i\beta_2}) \) with \( \beta_1 = -\phi_{21}, \beta_2 = -\phi_{31} \) and
\[
\hat{Y}_\Delta = \begin{pmatrix}
y_{11} & 0 & 0 \\
y_{21} & y_{22} & 0 \\
y_{31} & y_{32}e^{i\sigma} & y_{33}
\end{pmatrix},
\]
with \( \sigma = \phi_{32} - \phi_{31} + \phi_{21} \). It follows then from Eqs. (32) and (34) that the matrix \( m_D \) can be decomposed in the form
\[
m_D = v U_\rho P_\alpha \hat{Y}_\Delta P_\beta,
\]
where \( P_\alpha = \text{diag}(1,e^{i\alpha_1},e^{i\alpha_2}) \) and \( U_\rho \) is a unitary matrix containing only one phase \( \rho \). Therefore, in the WB where \( M_\nu \) and \( M_R \) are diagonal and real, the phases \( \rho, \alpha_1, \alpha_2, \sigma, \beta_1 \) and \( \beta_2 \) are the only physical phases characterizing \( CP \) violation in the leptonic sector. The phases relevant for leptogenesis are those contained in \( m_D \). From Eqs. (34)–(36) we conclude that these phases are \( \sigma, \beta_1 \) and \( \beta_2 \), which are linear combinations of the phases \( \phi_{ij} \). On the other hand, all the six phases of \( m_D \) contribute to the three phases of the effective neutrino mass matrix at low energies [12] which in turn controls \( CP \) violation in neutrino oscillations. Since the phases \( \alpha_1, \alpha_2 \) and \( \rho \) do not contribute to leptogenesis, it is clear that a necessary condition for a direct link between leptogenesis and low-energy \( CP \) violation to exist is the requirement that the matrix \( U \) in Eq. (32) contains no \( CP \)-violating phases. Note that, although the above condition was derived in a specific WB and using the parametrization of Eq. (32), it can be applied to any model. This is due to the fact that starting from arbitrary leptonic mass matrices, one can always make WB transformations to render \( M_\ell \) and \( M_R \) diagonal, while \( m_D \) has the form of Eq. (32). A specific class of models which satisfy the above necessary condition in a trivial way are those for which \( U = 1 \), leading to \( m_\rho = v Y_\Delta \). This condition is necessary but not sufficient to allow for a prediction of the sign of the \( CP \) asymmetry in neutrino oscillations, given the observed sign of the BAU together with the low-energy data. Therefore, we will consider next a more restrictive class of matrices \( m_D \) of this form and we will show that, in an appropriate limiting case, our structures for \( m_D \) lead to the ones assumed by Frampton, Glashow and Yanagida in [24].

V. MINIMAL SCENARIOS

From the analysis carried out in the previous section, it becomes clear that the computation of the cosmological baryon asymmetry \( Y_B \) in leptogenesis scenarios strongly depends on the Yukawa structure of the Dirac neutrino mass term and on the heavy Majorana neutrino mass spectrum. Moreover, if one assumes that the seesaw mechanism is responsible for the smallness of the neutrino masses, then the connection between the baryon asymmetry and low-energy neutrino physics is unavoidable. In fact, this constitutes an advantage for the leptogenesis mechanism when compared to other baryogenesis scenarios. In the context of supersymmetric extensions of the SM it is possible (although not always simple) to combine the study of leptogenesis and neutrino physics with other physical phenomena like flavor-violating decays [18,32]. In general, this analysis does not give us definite answers, yet it may help to discriminate among certain neutrino mass models.

Recently, a considerable amount of work has been done aiming at relating viable leptogenesis to all the available low-energy neutrino data coming from solar, atmospheric and reactor experiments [33]. Roughly speaking, two different approaches to the problem are to be found. The first one is based on the computation of the baryon asymmetry as a function of the lightest heavy Majorana neutrino mass \( M_1 \), the \( CP \) asymmetry \( e_1 \) and the so-called effective neutrino mass \( \bar{m}_1 = (m_\rho m_D)_{11}/M_1 \) [34]. By solving the Boltzmann equations, this kind of analysis provides valuable information on the ranges of these parameters that lead to an acceptable value of \( Y_B \). The weak point of this procedure lies on the fact that the input information depends on quantities which are not sensitive to the full structure of \( Y_\nu \) and \( M_R \) and, therefore, no further conclusions can be drawn about the class of models which can lead to acceptable values of the input parameters referred above. In fact, the values of \( M_1 \), \( \bar{m}_1 \) and \( e_1 \) should not be taken as independent parameters. The second approach is based upon initial assumptions on the structure of \( Y_\nu \) and \( M_R \) at high energies which are fixed by recurring to theoretical arguments like for example grand unified theories or flavor symmetries. Although in this framework some generality is lost, it has the advantage that one can compute the generated baryon asymmetry and, simultaneously, perform a low-energy neutrino data analysis. It is precisely the latter approach that we shall follow in the present work.

In this section we present a class of minimal scenarios for leptogenesis based on the triangular decomposition of \( Y_\nu \) given in Eqs. (32) and (33). Namely, we would like to answer the question of how simple can the structure of the Dirac neutrino Yukawa coupling matrix be in order not only to get an acceptable value of the baryon asymmetry but also to accommodate the neutrino data provided by the atmospheric, solar and reactor neutrino experiments. In particular, we require the non-vanishing of the \( CP \) asymmetry generated in the decays of the lightest heavy Majorana neutrino, since the final value of the baryon asymmetry crucially depends on this quantity. Throughout our analysis we shall also consider the predictions on the \( CP \)-violating effects at low energies.

In the previous section we have seen that the Dirac neutrino Yukawa coupling matrix \( Y_\nu \) can be decomposed into the product of a unitary matrix \( U \) and a lower-triangular matrix \( Y_\Delta \) [cf. Eq. (32)]. It was also shown that the \( CP \) asymmetries \( e_1 \) generated in the heavy Majorana neutrino decays do not depend on the matrix \( U \). In the special case \( U = 1 \) it is possible to establish the connection between leptogenesis, low-energy \( CP \) violation and neutrino mixing, since the same phases affect these phenomena. We classify this scenario as a minimal scenario for leptogenesis and \( CP \) violation in the sense that the \( CP \)-violating sources that do not contribute to leptogenesis are neglected. On the other hand, if \( U \neq 1 \) this connection is not trivial. Therefore, from now on we will
consider the case \(U=1\) which implies the following simple structure for the Dirac neutrino mass matrix:

\[
m_D = v Y_\Delta = v \begin{pmatrix}
  y_{11} & 0 & 0 \\
  y_{21} e^{i \phi_{21}} & y_{22} & 0 \\
  y_{31} e^{i \phi_{31}} & y_{32} e^{i \phi_{32}} & y_{33}
\end{pmatrix}
\]  

(37)

Then, from Eq. (14) the \(CP\) asymmetry generated in the decay of the heavy Majorana neutrino \(N_j\) is

\[
e_j = -\frac{1}{8\pi(H_\Delta)^i_j} \sum_{i\neq j} \text{Im}[H_\Delta^i_j] \gamma_{ij},
\]

(38)

where

\[
H_\Delta = Y_\Delta^\dagger Y_\Delta, \quad f_{ij} = f \left( \frac{M_i}{M_j^2} \right).
\]

(39)

It follows from Eqs. (38)–(41) that, in principle, one can obtain simultaneously viable values for the \(CP\) asymmetries \(e_j\) and a phenomenologically acceptable effective neutrino mass matrix in order to reproduce the solar, atmospheric and reactor neutrino data. This can be achieved by consistently choosing the values of the free parameters \(y_{ij}\), \(M_i\) and \(\phi_{ij}\). Yet, a closer look at Eqs. (38)–(40) shows that there are terms contributing to \(e_j\) which vanish independently from the others. This means that a non-vanishing value of \(e_j\) can be guaranteed even for simpler structures for \(Y_\nu\), which can be obtained from \(Y_\Delta\) assuming additional zero entries in the lower triangle.\(^7\) The results are given in Table I where we present the textures constructed from \(Y_\Delta\) by neglecting one (textures I–III) and two (textures IV–VI) off-diagonal entries. The form of the effective neutrino mass matrix \(M_\nu\) and the expressions for the \(CP\) asymmetries \(e_{1,2}\) for each case are also given.\(^8\)

Let us first discuss textures IV–VI. For these three textures the effective neutrino mass matrix \(M_\nu\) predicts a complete decoupling of one light neutrino from the other two.

\(^7\)Notice however that the vanishing of diagonal elements in \(Y_\Delta\) would imply \(\det(m_D) = 0\) and consequently \(\det(M_\nu) = 0\), leading to the existence of massless light neutrinos.

\(^8\)In commonly used language, textures I–III and IV–VI belong to the classes of four and five texture zero matrices, respectively. For a complete discussion on seesaw realization of texture-zero mass matrices see e.g. [35] and for its implications at low energies see e.g. [36].

with \(f(x)\) defined in Eq. (15).

From Eqs. (37) and (39) we readily obtain

\[
\text{Im}(H_\Delta^2) = y_{21}^2 y_{22}^2 \sin(2 \phi_{21}) + 2y_{21} y_{22} y_{31} y_{32} \sin \theta_1 + y_{31}^2 y_{32}^2 \sin \theta_2.
\]

\[
\text{Im}(H_\Delta^3) = y_{31}^2 y_{33}^2 \sin(2 \phi_{31}),
\]

\[\text{Im}(H_\Delta^4) = y_{32}^2 y_{33}^2 \sin(2 \phi_{32}),
\]

(40)

with \(\theta_1 = \phi_{21} + \phi_{31} - \phi_{32}\) and \(\theta_2 = 2(\phi_{31} - \phi_{32})\).

All the information about neutrino masses and mixing is fully contained in the effective neutrino mass matrix \(M_\nu\) which is determined through the seesaw formula given by Eq. (7). In this case

\[
M_\nu = \frac{v^2}{M_1} \begin{pmatrix}
  y_{11}^2 & y_{11} y_{21} e^{i \phi_{21}} & y_{11} y_{31} e^{i \phi_{31}} \\
  y_{11} y_{21} e^{i \phi_{21}} & y_{21}^2 e^{2i \phi_{21}} + y_{22}^2 & y_{21} y_{31} e^{i(\phi_{31} + \phi_{21})} + y_{22} y_{32} M_1 e^{i \phi_{32}} \\
  y_{11} y_{31} e^{i \phi_{31}} & y_{21} y_{31} e^{i(\phi_{31} + \phi_{21})} + y_{22} y_{32} M_1 e^{i \phi_{32}} & y_{31}^2 e^{2i \phi_{31}} + y_{32}^2 + y_{33}^2 M_1 + y_{32} M_1 e^{2i \phi_{32}}
\end{pmatrix}
\]

(41)

This is in disagreement with the available neutrino data which indicates that, in the framework of the LMA solution, only one neutrino mixing angle should be small, namely \(\theta_{13}\), instead of two as predicted by textures IV–VI. Furthermore, texture VI predicts a vanishing value for the \(CP\) asymmetry in the decay of the lightest heavy Majorana neutrino, implying \(Y_\nu = 0\) in the case of hierarchical heavy Majorana neutrinos. Texture III can also be excluded on the grounds that it cannot account for the large solar angle. To illustrate this, let us write for this texture the matrix \(L\) given in Eq. (24) as

\[
L = im_D D_R^{1/2} = \begin{pmatrix}
  z_1 & 0 & 0 \\
  z_2 & y_2 & 0 \\
  z_3 & 0 & x_3
\end{pmatrix}
\]

(42)

where \(z_1\), \(y_2\), and \(x_3\) are real and positive [26]. We also take \(z_2\) and \(x_3\) real to simplify the discussion. Considering first the hierarchical case \(\Delta m_{32}^2 = m_3\), a large atmospheric angle and \(m_2 < m_3\) naturally arise if \(z_2 \sim z_3 > y_2, x_3\). More precisely, \(\tan \theta_{23} = z_2 / z_3\). In addition, this implies

\[
\tan \theta_{13} = \frac{z_1}{\sqrt{z_2^2 + z_3^2}} = t_{13},
\]

(43)

which has to satisfy the CHOOZ bound, \(t_{13} \leq 0.2\) [37]. The solar angle is approximately given by
TABLE I. Minimal textures based on the triangular decomposition of $m_D$ and their respective light neutrino mass matrix $M_\nu$ and CP-asymmetries $\epsilon_1$ and $\epsilon_2$.

<table>
<thead>
<tr>
<th>Texture</th>
<th>$Y_{\Delta}$</th>
<th>$M_{\nu}/v^3 M_\nu$</th>
<th>$\epsilon_{1,2}$</th>
</tr>
</thead>
</table>
| I       | $\left( \begin{array}{ccc}
    y_{11} & 0 & 0 \\
    y_{21} e^{i \phi_{21}} & y_{22} & 0 \\
    0 & y_{32} e^{i \phi_{32}} & y_{33}
\end{array} \right)$ | $\begin{array}{ccc}
    y_1^2 & y_{11} y_{21} e^{i \phi_{21}} & 0 \\
    0 & y_{11} y_{21} e^{i \phi_{21}} + y_{22} y_{32} e^{i \phi_{32}} & y_{22} y_{32} e^{i \phi_{32}} \\
    0 & y_{22} y_{32} e^{i \phi_{32}} & y_{33}^2 + y_{33} y_{32} e^{i \phi_{32}}
\end{array}$ | $\begin{array}{c}
    \epsilon_1 = -\frac{y_{33}^2 y_{32}^2 \sin(2 \phi_{32})}{8 \pi (y_{32}^2 + y_{33}^2)} \\
    \epsilon_2 = -\frac{y_{33}^2 y_{32}^2 \sin(2 \phi_{32})}{8 \pi (y_{32}^2 + y_{33}^2)}
\end{array}$ |
| II      | $\left( \begin{array}{ccc}
    y_{11} & 0 & 0 \\
    0 & y_{22} & 0 \\
    y_{31} e^{i \phi_{31}} & y_{32} e^{i \phi_{32}} & y_{33}
\end{array} \right)$ | $\begin{array}{ccc}
    y_1^2 & 0 & y_{11} y_{31} e^{i \phi_{31}} \\
    0 & y_{22}^2 M_2 M_3 & y_{22} y_{32} M_2 e^{i \phi_{32}} \\
    y_{31} y_{32} e^{i \phi_{31}} + y_{32} y_{33} M_2 + y_{33} y_{32} M_2 e^{i \phi_{32}}
\end{array}$ | $\begin{array}{c}
    \epsilon_1 = -\frac{y_{33}^2 y_{32}^2 \sin(2 \phi_{32})}{8 \pi (y_{32}^2 + y_{33}^2)} \\
    \epsilon_2 = -\frac{y_{33}^2 y_{32}^2 \sin(2 \phi_{32})}{8 \pi (y_{32}^2 + y_{33}^2)}
\end{array}$ |
| III     | $\left( \begin{array}{ccc}
    y_{11} & 0 & 0 \\
    y_{21} e^{i \phi_{21}} & y_{22} & 0 \\
    y_{31} e^{i \phi_{31}} & 0 & y_{33}
\end{array} \right)$ | $\begin{array}{ccc}
    y_1^2 & y_{11} y_{21} e^{i \phi_{21}} & y_{11} y_{31} e^{i \phi_{31}} \\
    y_{21} y_{21} e^{i \phi_{21}} + y_{22} y_{32} e^{i \phi_{32}} & y_{21} y_{31} e^{i \phi_{31}} + y_{22} y_{32} e^{i \phi_{32}} \\
    y_{31} y_{32} e^{i \phi_{31}} + y_{32} y_{33} M_2 + y_{33} y_{32} M_2 e^{i \phi_{32}}
\end{array}$ | $\begin{array}{c}
    \epsilon_1 = -\frac{y_{33}^2 y_{32}^2 \sin(2 \phi_{32})}{8 \pi (y_{32}^2 + y_{33}^2)} \\
    \epsilon_2 = -\frac{y_{33}^2 y_{32}^2 \sin(2 \phi_{32})}{8 \pi (y_{32}^2 + y_{33}^2)}
\end{array}$ |
| IV      | $\left( \begin{array}{ccc}
    y_{11} & 0 & 0 \\
    y_{21} e^{i \phi_{21}} & y_{22} & 0 \\
    0 & 0 & y_{33}
\end{array} \right)$ | $\begin{array}{ccc}
    y_1^2 & y_{11} y_{21} e^{i \phi_{21}} & 0 \\
    y_{11} y_{21} e^{i \phi_{21}} + y_{22} y_{32} e^{i \phi_{32}} & y_{11} y_{31} e^{i \phi_{31}} + y_{22} y_{32} e^{i \phi_{32}} \\
    0 & y_{33}^2 M_2 M_3 + y_{33} y_{32} M_2 e^{i \phi_{32}}
\end{array}$ | $\begin{array}{c}
    \epsilon_1 = -\frac{y_{33}^2 y_{32}^2 \sin(2 \phi_{32})}{8 \pi (y_{32}^2 + y_{33}^2)} \\
    \epsilon_2 = -\frac{y_{33}^2 y_{32}^2 \sin(2 \phi_{32})}{8 \pi (y_{32}^2 + y_{33}^2)}
\end{array}$ |
| V       | $\left( \begin{array}{ccc}
    y_{11} & 0 & 0 \\
    0 & y_{22} & 0 \\
    y_{31} e^{i \phi_{31}} & 0 & y_{33}
\end{array} \right)$ | $\begin{array}{ccc}
    y_1^2 & 0 & y_{11} y_{31} e^{i \phi_{31}} \\
    0 & y_{22}^2 M_3 & y_{22} y_{32} M_2 e^{i \phi_{32}} \\
    y_{31} y_{32} e^{i \phi_{31}} + y_{32} y_{33} M_2 + y_{33} y_{32} M_2 e^{i \phi_{32}}
\end{array}$ | $\begin{array}{c}
    \epsilon_1 = -\frac{y_{33}^2 y_{32}^2 \sin(2 \phi_{32})}{8 \pi (y_{32}^2 + y_{33}^2)} \\
    \epsilon_2 = -\frac{y_{33}^2 y_{32}^2 \sin(2 \phi_{32})}{8 \pi (y_{32}^2 + y_{33}^2)}
\end{array}$ |
| VI      | $\left( \begin{array}{ccc}
    y_{11} & 0 & 0 \\
    0 & y_{22} & 0 \\
    0 & y_{32} e^{i \phi_{32}} & y_{33}
\end{array} \right)$ | $\begin{array}{ccc}
    y_1^2 & 0 & 0 \\
    0 & y_{22}^2 M_2 & y_{22} y_{32} M_2 e^{i \phi_{32}} \\
    0 & y_{22} y_{32} M_2 e^{i \phi_{32}} & y_{33}^2 M_2 + y_{33} y_{32} M_2 e^{i \phi_{32}}
\end{array}$ | $\begin{array}{c}
    \epsilon_1 = 0 \\
    \epsilon_2 = 0
\end{array}$ |
MINIMAL SCENARIOS FOR LEPTOGENESIS AND CP...

\[
\tan(2\theta_{12}) \approx 2 \frac{t_{13} \beta \cos \theta_{23} + \sin \theta_{23}}{1 - t_{13} \beta} = B_{12}(\beta, t_{13}),
\]

where \( \beta = \max(\tan(\theta_{12}), \cot \theta_{23}) \). The upper bound \( B_{12}(\beta, t_{13} = 0.2) \) is an increasing function of \( \beta \). Experimentally 0.7 \( \leq \beta \leq 1.7 \), so that \( B_{12}(\beta) \approx B_{12}(1.7, 0.2) = 4 \). This corresponds to \( \tan^2 \theta_{12} \approx 0.3 \), while the fitted LMA solution requires \( \tan^2 \theta_{12} \approx 0.7 \). We thus conclude that type III cannot account for the observed large solar angle in the hierarchical case. Moreover, it turns out that it cannot also account for the pattern of \( M_\nu \) required by inverse hierarchy since, in this case, the atmospheric oscillation fit would require \( \varphi_1 \varphi_2 = \varphi_1 \varphi_3 = \frac{\pi}{2} \) and \( \varphi_2, \varphi_3, \varphi_4 \ll 1 \). Finally, the degenerate case can be accommodated only by tuning the elements of \( L \). In view of this, from now on we will focus our analysis only on textures I and II.

Another interesting fact which comes out of the observation of Table I is that the phase content for textures I and II can be further reduced. Indeed, it is straightforward to show that with only one phase in \( \varphi_\Delta \) it is possible to obtain a non-vanishing \( CP \) asymmetry \( \varepsilon_1 \).

As it has been stated in Sec. II, the strength of \( CP \) violation at low energies is sensitive to the value of the \( CP \) invariant \( J_{CP} \) given by Eq. (12). For the textures I and II we get

\[
J_{CP}^I = \frac{y_{12}^2 y_{33}^2 + y_{12}^2 y_{33}^2 + y_{12}^2 y_{33}^2 + y_{12}^2 y_{33}^2}{M_1^2 M_2^2 \Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2} \left[ (y_{12}^2 y_{33}^2 + y_{12}^2 y_{33}^2 + y_{12}^2 y_{33}^2) \times \sin(2 \phi_{21}) - y_{22} y_{33}^2 M_3 \sin(2 \phi_{32}) \right. \\
+ \left. y_{33}^2 (y_{12}^2 + y_{33}^2) M_2 M_3 \sin^2(2 \phi_{21} - \phi_{32}) \right],
\]

\[
J_{CP}^II = \frac{y_{11}^2 y_{22}^2 y_{33}^2 + y_{11}^2 y_{22}^2 y_{33}^2 + y_{11}^2 y_{22}^2 y_{33}^2}{M_1^2 M_2^2 \Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2} \left[ (y_{22}^2 y_{33}^2 + y_{11}^2 y_{22}^2 + y_{11}^2 y_{22}^2) \times \sin[2(\phi_{32} - \phi_{31})] + y_{22} y_{33}^2 M_1 M_3 \sin(2 \phi_{32}) \right. \\
- \left. y_{11}^2 M_3 \sin(2 \phi_{31}) \right].
\]

From Table I and Eqs. (45) it is clear that we can have both \( \varepsilon_1 \neq 0 \) and \( J_{CP} \neq 0 \) with a single non-vanishing phase.

**On the relative sign between \( Y_B \) and \( J_{CP} \)**

It has recently been pointed out in [24] that the relative sign between \( CP \) violation and the baryon asymmetry can be predicted in a specific class of models. In our framework, it is clear from Table I and Eqs. (45) that the relative sign between the low-energy invariant \( J_{CP} \) and the \( CP \) asymmetries \( \varepsilon_{1,2} \) cannot be predicted without specifying the values of the heavy neutrino masses \( M_i \) and the Dirac Yukawa couplings. This is mainly due to the fact that at least one of these quantities depends on both phases appearing in the Dirac neutrino Yukawa coupling matrix for textures I and II. Therefore, in order to establish a direct connection between the sign of the \( CP \) asymmetries \( \varepsilon_i \) and the low-energy \( CP \) invariant \( J_{CP} \) further assumptions are required. For instance, considering the case \( \phi_{31} = 0 \) and \( M_1 / M_2 \ll M_3 / M_1 \) so that the terms proportional to \( f_{31} \) can be safely neglected in \( \varepsilon_i \) (see Table I), we obtain from Eqs. (45) and Table I the relative signs given in Table II.

It is well known that in the case of hierarchical heavy neutrinos, the sign of \( Y_B \) is fixed by the sign of the \( CP \) asymmetry generated in the decay of the lightest heavy neutrino, let us say \( \varepsilon_L \). For \( Y_B \) to be positive, as required by the observations, one must have \( \varepsilon_L < 0 \). Thus, the measurement of the sign of \( CP \) violation at low-energy could in principle exclude some of the heavy Majorana neutrino mass regimes presented in Table II. As an example, let us assume that \( J_{CP} \neq 0 \). In this case, the requirement \( \varepsilon_L < 0 \) would exclude the mass regime \( M_2 / M_1 < x \) for texture I and \( M_1 / M_2 < x \) for texture II.

Finally, it is interesting to note that the Dirac neutrino Yukawa matrices considered in [24] correspond to our textures I and II with \( y_{33} = 0 \) and have the remarkable feature that the number of real parameters equals the number of masses and mixing angles. In this limit the heavy Majorana neutrino \( N_3 \) completely decouples, rendering this situation phenomenologically equivalent to the two heavy neutrino case considered in [24]. Namely, it can be easily seen that for \( y_{33} = 0 \) the phase \( \varphi_{32} \) can be rephased away and consequently the \( CP \) asymmetries \( \varepsilon_{1,2} \) and the low-energy \( CP \) invariant \( J_{CP} \) will only depend on the phases \( \varphi_{21} \) and \( \varphi_{31} \) for textures I and II, respectively. This means that the examples considered in [24] are special cases of our framework which is motivated by the condition that \( m_D \) does not contain any phase that would only contribute to low-energy \( CP \) violation. It is clear from Table II that, in general, even by keeping only

<table>
<thead>
<tr>
<th>( M_1 / M_2 )</th>
<th>( x = M_1 / M_2 &lt; 1 )</th>
<th>( x = M_2 / M_1 &lt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_{1,2} )</td>
<td>( J_{CP}^I )</td>
<td>( J_{CP}^II )</td>
</tr>
<tr>
<td>( \varepsilon_1 )</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( \varepsilon_2 )</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( \varepsilon_1 )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \varepsilon_2 )</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

*For the sake of more generality, we do not assume the mass-ordering \( M_1 < M_2 \) in this discussion.*
one phase the sign of $e_j \cdot J_{CP}$ depends on the particular Yukawa texture and hierarchy between the heavy Majorana neutrinos.

VI. EXAMPLES

In this section we present some examples of the minimal textures discussed in Sec. V and proceed to the study of their implications to low-energy physics as well as to the computation of the baryon asymmetry $Y_B$ through the numerical solution of the Boltzmann equations as described in the Appendix. We will only consider cases that lead to the LMA solution of the solar neutrino problem, which means that the neutrino mixing angles and the squared mass differences lie in the typical ranges:

\[
\begin{align*}
2.5 \times 10^{-5} \text{ eV}^2 &\leq \Delta m_{12}^2 \leq 3.4 \times 10^{-4} \text{ eV}^2, \\
0.24 &\leq \tan^2 \theta_{12} \leq 0.89, \\
1.4 \times 10^{-3} \text{ eV}^2 &\leq \Delta m_{13}^2 \leq 6.0 \times 10^{-3} \text{ eV}^2, \\
0.40 &\leq \tan^2 \theta_{13} \leq 3.0,
\end{align*}
\]

(46)

with the solar and atmospheric mixing angles being identified as $\theta_{12} = \theta_{21}$, $\theta_{13} = \theta_{32}$, respectively, in the standard parametrization [13]. The $\theta_{13}$ mixing angle is at present constrained by reactor neutrino experiments to satisfy $|\sin \theta_{13}| = |U_{e3}| \leq 0.2$ [37].

A. Hierarchical heavy Majorana neutrinos ($M_1 < M_2 \ll M_3$)

The first example is a realization of the texture I given in Table I with $\phi_{32} = 0$. The entries of the Dirac neutrino Yukawa coupling and the right-handed neutrino mass matrices are chosen to be of order

\[
Y_\nu = \frac{\Lambda_D}{v} \begin{pmatrix}
e^6 & 0 & 0 \\
e^{e^{i \phi_{21}}} & e^4 & 0 \\
0 & e^4 & 1
\end{pmatrix},
\]

\[
D_R = \Lambda_R \text{diag}(e^{12}, e^{10}, 1),
\]

(47)

where $e < 1$ is a small parameter. For our numerical estimates we consider the typical Dirac neutrino mass scale to be $\Lambda_D \approx 100$ GeV, which corresponds approximately to the top quark mass at the GUT scale [38]. The neutrino mass matrix $M_\nu$ is then given by

\[
M_\nu = \frac{\Lambda^2_D}{\epsilon^2 \Lambda_R} \begin{pmatrix}
e^2 & e^{e^{i \phi_{21}}} & 0 \\
e^{e^{i \phi_{21}}} & 1 + e^{2 \epsilon \phi_{21}} & 1 \\
0 & 1 & 1 + \epsilon^2
\end{pmatrix}.
\]

(48)

In this particular case, considering for the moment $\phi_{21} = 0$, one gets\footnote{We have checked that the light neutrino masses $m_i$ and mixing angles $\theta_{ij}$ are not sensitive to the phase $\phi_{21}$.}

\[
m_1 = \frac{\Lambda^2_D}{2 \Lambda_R} (2 - \sqrt{2}), \quad m_2 = \frac{\Lambda^2_D}{2 \Lambda_R} (2 + \sqrt{2}),
\]

\[
m_3 = \frac{\Lambda^2_D}{e^4 \Lambda_R} (2 + \epsilon^2),
\]

(49)

leading to

\[
\Delta m_{21}^2 = \frac{2 \sqrt{2} \Lambda^4_D}{\Lambda^2_R}, \quad \Delta m_{32}^2 = \frac{4 \Lambda^4_D}{e^4 \Lambda^2_R}.
\]

(50)

The requirement

\[
\left| \frac{\Delta m_{21}^2}{\Delta m_{32}^2} \right| = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} \approx \frac{e^4}{\sqrt{2}},
\]

(51)

forces $\epsilon$ to be in the range $0.3 \leq \epsilon \leq 0.7$. For the neutrino mixing angles we have

\[
\tan^2 \theta_{12} \approx 1 - \frac{e^2}{2 \sqrt{2}} \quad \text{and} \quad \tan^2 \theta_{23} \approx 1, \quad |U_{e3}| \approx \frac{e^2}{2 \sqrt{2}}.
\]

(52)

which are in good agreement with the data taking into account the range of $\epsilon$. The matrix $V_L$ which corresponds to the left-handed rotation involved in the diagonalization of $m_D$ can be viewed as the equivalent in the leptonic sector of the quark mixing matrix $V_{CKM}$. This matrix is obtained by diagonalizing the Hermitian matrix $Y_\nu Y_\nu^\dagger$,

\[
V_L = \begin{pmatrix}
1 - e^4/2 & e^4 - e^{12}/2 & 0 \\
-e^{12} + e^{12}/2 & 1 - e^8/2 & e^8 \\
e^{12} & -e^8 & 1
\end{pmatrix}.
\]

(53)

This is in fact an interesting result in the sense that we are in the presence of a typical large mixing out of small mixing situation, where the large neutrino mixing is generated through the seesaw mechanism [29,39]. The scale $\Lambda_R$, or equivalently the mass $M_3$ of the heaviest Majorana neutrino, is determined by requiring $\Delta m_{32}^2$ to be in the experimental range given in Eq. (46). We find

\[
M_3 = \Lambda_R \approx \frac{2 \Lambda^2_D}{e^2 \sqrt{\Delta m_{12}}} \approx \frac{2 \Lambda^2_D}{\sqrt{\Delta m_{12}}} = 8 \times 10^{15} \text{ GeV},
\]

(54)

and Eq. (47) implies $M_1 = 1.3 \times 10^{11}$ GeV and $M_2 \approx 8.4 \times 10^{15}$ GeV, for $\epsilon = 0.4$.

From the expression of the $CP$ asymmetry $e_1^\dagger$ given in Table I, we obtain
randomly included $\mathcal{O}(1)$ coefficients\footnote{For illustration, we have taken the $\mathcal{O}(1)$ coefficients in the range $[0.9,1.3]$. We notice however that the results are not too sensitive to slight variations of this range.} for the non-vanishing entries of $Y_\nu$ and taken $\phi_{21}=\pi/4$. The spreading of the points in the figure, due to the random variation of the coefficients, shows that the textures are stable under these perturbations.

In order to compute the value of the baryon asymmetry we proceed to the numerical solution of the Boltzmann equations as described in the Appendix, taking the initial conditions: $Y_{N_1}^0=0$, $Y_{N_2}^0=0$. The results are presented in Fig. 2 where we have plotted $Y_{N_1}$, $Y_{N_2}$ and $Y_B$ as functions of the parameter $\epsilon=M_1/T$ for given values of the CP asymmetries, heavy Majorana neutrino masses and the lightest neutrino mass. The predicted value for the final baryon asymmetry is $Y_B=9\times10^{-11}$, which is inside the observational range (1).

Our next example is a particular case of the texture II presented in Table I with $\phi_{32}=0$. The Dirac neutrino Yukawa coupling and heavy neutrino mass matrices are chosen in this case to be of order

\begin{equation}
Y_\nu=\frac{\Lambda_D}{v}\begin{pmatrix}
\epsilon^5 & 0 & 0 \\
0 & \epsilon^3 & 0 \\
\epsilon^5\epsilon^{i\phi_{31}} & \epsilon^3 & 1
\end{pmatrix},\quad D_R=\Lambda_D\text{diag}(\epsilon^{10},\epsilon^8,1),
\end{equation}

leading to the following light neutrino mass matrix

\begin{equation}
\begin{pmatrix}
M_1^2 \\
M_2^2 \\
M_3^2
\end{pmatrix} = \text{ diag}(2.5,11.0,13.0) \, \text{GeV}^2,
\end{equation}
The predictions for the $M_{ij}$'s and neutrino mixing angles are similar to the ones given in Eqs. (50) and (52). Moreover, from Eqs. (54) and (58) and for $\epsilon = 0.4$, we get the following heavy Majorana neutrino masses: $M_1 = 9 \times 10^{11}$ GeV, $M_2 = 6 \times 10^{12}$ GeV, $M_3 = 8 \times 10^{15}$ GeV. The left-handed matrix $V_L$ is given in this case by

$$V_L = \begin{pmatrix} -1 & -e^{10} & e^{10} \\ -e^{10} & 1 & e^6 \\ e^{10} & -e^6 & 1 \end{pmatrix}. \quad (60)$$

Finally, the CP asymmetry can be obtained from the expression of $\epsilon_1^R$ presented in Table I with $\phi_{32} = 0$. Taking into account the form of $Y_{\nu}$ and $D_R$ as in Eqs. (58), we obtain

$$\epsilon_1 = \frac{3y_{31}^2\sin(2\phi_{31})}{16\pi(y_{11}^2 + y_{31}^2)} \left( \frac{y_{32}^2 M_1}{y_{33}^2 M_2} + \frac{y_{33}^2 M_1}{y_{31}^2 M_3} \right) \approx \frac{3\Lambda_2^2 e^8}{32\pi v^2} \sin(2\phi_{31}),$$

where the last estimate has been obtained assuming $\epsilon = 0.4$. It can also be shown that the CP invariant $J_{CP}$ reads [see Eq. (45)],

$$J_{CP} = -\frac{y_{11}^2 y_{22}^2 y_{31}^2 y_{32}^2}{M_1^2 M_3^2} \frac{v^12}{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2} \left[ y_{22}^2 y_{31}^2 + y_{11}^2 (y_{22}^2 + y_{32}^2) \right] + \frac{y_{11}^3 y_{32}^3}{M_2 M_3} \sin(2\phi_{31}),$$

which leads to the approximate result

$$J_{CP} = -\frac{3\epsilon^2 \sin(2\phi_{31})}{32 \sqrt{2}} \approx -1.1 \times 10^{-2} \sin(2\phi_{31}). \quad (63)$$
Notice that $e_1$ and $\mathcal{J}_{CP}$ have opposite signs in this case, once again in agreement with Table II.

In Fig. 3 we present the same numerical analysis as in Fig. 1, but for the case where $Y_\nu$ and $D_R$ are defined through Eqs. (58). We find good agreement between our approximate analytical results and the numerical ones. The integration of the Boltzmann equations is plotted in Fig. 4. The value for the final baryon asymmetry is $Y_B \approx 8 \times 10^{-11}$.

### B. Two-fold quasi-degeneracy ($M_1=M_2\neq M_3$)

As an example, let us consider the texture of type I given in Table I and assume that $M_1=M_2\neq M_3$. The Hermitian matrix $H_\Delta = Y_\Delta Y_\Delta^\dagger$ is given by

$$H_\Delta = 
\begin{pmatrix}
\begin{array}{ccc}
 y_{11}^2 + y_{21}^2 & y_{21} y_{22} e^{-i\phi_{21}} & 0 \\
y_{21} y_{22}^* e^{i\phi_{21}} & y_{22}^2 + y_{32}^2 & y_{32} y_{33} e^{-i\phi_{32}} \\
0 & y_{32} y_{33}^* e^{i\phi_{32}} & y_{33}^2
\end{array}
\end{pmatrix}.
$$

Taking into account the requirement of Eq. (20) we can get the range of validity of the parameter $\delta_N$:

$$\delta_N \gg \frac{1}{8\pi} \max\{y_{11}^2 + y_{21}^2, y_{22}^2 + y_{32}^2\}.$$  

In the limit $\delta_N \to 0$, Eq. (48) is recovered. Since $\delta_N \ll 1$ the results for the neutrino masses and mixing parameters are practically the same as the ones given in Eqs. (49)–(52). The same is expected for the estimate of the $CP$ invariant $\mathcal{J}_{CP}$ defined in Eq. (66). The main differences reside obviously in the heavy Majorana neutrino mass spectrum and on the values of the $CP$ asymmetries, since now the relation $M_1 \neq M_2$ is no longer valid. From Eq. (23) we obtain

$$
e_1 = \frac{\epsilon_1^2 (1+\delta_N)}{\epsilon_1^2 (1+\delta_N) e^{i\phi_{21}}} = \frac{1.3 \times 10^{-12}}{\delta_N} \sin(2\phi_{21}),$$

$$
\epsilon_2 = \frac{\epsilon_2^2 (1+\delta_N)}{\epsilon_2^2 (1+\delta_N) e^{i\phi_{21}}} = \frac{1.1 \times 10^{-13}}{\delta_N} \sin(2\phi_{21}).
$$

The results of the numerical integration of the Boltzmann equations for this case are presented in Fig. 5. A realistic value for $Y_B$ is also generated in this case.

Before we end this section, it is worthwhile to comment on the possible effects of quantum corrections to the effective neutrino mass matrix $\mathcal{M}_\nu$. This discussion turns out to be relevant since in the examples considered above we have taken the effective neutrino mass matrix at the heavy neutrino decoupling scale. Although a detailed treatment would require a renormalization group analysis for the effective neutrino mass operator, one can employ the simple analytical treatment considered by many authors in the literature (see for example Refs. [22,40]). Following this, we recall that the effective neutrino mass matrix at the electroweak scale $m_Z$ can be related to the one at the heavy neutrino decoupling scale $\Lambda_R$ as

$$\mathcal{M}_\nu(m_Z) \sim \text{diag}(1,1,1 + \epsilon_\nu) \mathcal{M}_\nu(\Lambda_R) \text{diag}(1,1,1 + \epsilon_\nu),$$

where, neglecting the running of the charged lepton Yukawa couplings,

$$\epsilon_\nu \approx \frac{3y_\tau^2 \ln \left(\frac{\Lambda_R}{m_Z}\right)}{32\pi^2 \ln \left(\frac{\Lambda_R}{m_Z}\right)}.$$

Taking $y_\tau = m_\tau / v$ and $\Lambda_R \approx 10^{16}$ GeV, we obtain $\epsilon_\nu \approx 10^{-5}$. Considering $\mathcal{M}_\nu(\Lambda_R)$ as given in Eq. (48) we would get, from Eq. (71),

$$\mathcal{M}_\nu(m_Z) \approx \text{diag}(1,1,1 + \epsilon_\nu) \mathcal{M}_\nu(\Lambda_R) \text{diag}(1,1,1 + \epsilon_\nu).$$

Using Eqs. (23), the $CP$ asymmetries are given in this case by

$$\epsilon_1 = \frac{y_{21}^2 y_{22}^* \sin(2\phi_{21})}{16\pi \delta_N (y_{11}^2 + y_{21}^2)},$$

$$\epsilon_2 = \frac{1}{16\pi} \left[ \frac{y_{21}^2 y_{22}^* \sin(2\phi_{21}) + 3y_{32}^2 y_{33}^* \sin(2\phi_{32})}{\delta_N (y_{22}^2 + y_{32}^2)} \right].$$

Let us now consider the following simple realization of $Y_\nu$ and $D_R$:

$$Y_\nu = \Lambda_D \begin{pmatrix}
\epsilon^{10} & 0 & 0 \\
0 & \epsilon^{i\phi_{21}} & \epsilon^{9} \\
0 & \epsilon^{9} & 1
\end{pmatrix},$$

$$D_R = \Lambda_R \text{diag}(\epsilon^{10}, 1 + \delta_N, \epsilon^{20}),$$

with $\delta_N \ll 1$. From Eq. (65) we get in this case for $\epsilon = 0.3$:

$$\delta_N \approx \frac{\Lambda_R^2}{4\pi v^2} \approx 10^{-11}.$$
Since example considered in Eq. (58) with \( \phi \eta_3 = \pi/4 \) and \( \epsilon = 0.4 \). The values of the heavy Majorana neutrino masses \( M \), the CP asymmetries \( \epsilon_{1,2} \) and the lightest neutrino mass \( m_1 \) are consistent with the ones presented in Fig. 3 for this value of \( \epsilon \). The predicted value for the final baryon asymmetry is \( Y_B = 8 \times 10^{-11} \).

\[
\frac{\Delta m^2_{21}}{\Delta m^2_{32}} = \frac{(1 - \epsilon_x) \epsilon_x^4}{\sqrt{2}}, \quad \tan^2 \theta_{12} = 1 - \frac{\epsilon_x^2}{2\sqrt{2}} - \sqrt{2} \epsilon_x,
\]

\[
\tan^2 \theta_{23} = 1 - 2 \epsilon_x, \quad |U_{e3}| = \frac{\epsilon_x^2}{2\sqrt{2}} \left( 1 - \frac{3}{2} \epsilon_x \right). \tag{73}
\]

Since \( \epsilon_x \ll 1 \), the results given by Eqs. (51) and (52) are not significantly altered. The same conclusions are drawn for the example considered in Eq. (59). Thus, we conclude that the effects of quantum corrections due to the renormalization of the neutrino mass operator can be safely neglected in our case.

VII. CONCLUDING REMARKS

We have analyzed, in the context of the minimal seesaw mechanism, the link between leptogenesis and CP violation at low energies. In particular, it was shown that, in order to present a thorough discussion on this question, it is convenient to work in the WB where both the charged lepton and right-handed Majorana neutrino mass matrices are diagonal and real, and to write, without loss of generality, the Dirac neutrino mass matrix as the product of a unitary matrix and a lower triangular matrix. From the analysis of the phases that contribute to leptogenesis and low-energy CP violation, we have identified a necessary condition which is required in order to establish a link between these two phenomena. We have studied a class of models which satisfy the above necessary condition in the simplest way, namely those where the Dirac neutrino mass matrix is of the triangular form. By choosing this structure the number of physical parameters in the theory is reduced then enhancing its predictability. In this case there are only three CP-violating phases which contribute both to leptogenesis and CP violation at low energies. We have then studied the minimal scenarios where a correct value of the baryon-to-entropy ratio can be generated, while accounting for all the low-energy neutrino data in the context of the LMA solution. Moreover, the examples considered in Sec. VI predict the existence of low-energy CP-violating effects within the range of sensitivity of the future long-baseline neutrino oscillation experiments. In fact, it is a remarkable feature of these scenarios that the solutions viable for leptogenesis are precisely those which maximize \( J_{CP} \). The question of relating the observed sign of the baryon asymmetry to the sign of the leptonic CP violation, measurable at low energies through neutrino oscillations, was also considered. Namely, we have concluded that, within the minimal scenarios presented, this relation crucially depends on the heavy Majorana neutrino mass spectrum. We remark that a full discussion of this aspect requires the computation of the BAU since, besides the prediction of the relative sign between the BAU and \( J_{CP} \), the determination of \( Y_B \) is of extreme importance to infer about the viability of a given model.

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APPENDIX: BOLTZMANN EQUATIONS

The computation of the cosmological baryon asymmetry \( Y_B \) involves the solution of the full set of Boltzmann equa-
tions which governs the time evolution of the right-handed neutrino number densities \( Y_{N_j} \) and the generated lepton asymmetry \( Y_L \). These quantities depend not only on the physics occurring in the thermal bath but also on the universe expansion. In the SM framework extended with heavy right-handed neutrinos the physical processes relevant to the generation of the baryon asymmetry are typically the \( N_i \) decays and inverse decays into Higgs bosons and leptons, the \( \Delta L=1 \) scatterings involving the top quark and \( \Delta L=2 \) scatterings with virtual \( N_i \) and \( N_i-N_j \) scatterings [41,42]. The initially produced lepton asymmetry \( Y_L \) is converted into a net baryon asymmetry \( Y_B \) through the \((B+L)\)-violating sphaleron processes. One finds the relation

\[
Y_B = \xi Y_{B-L} = \frac{\xi}{\xi - 1} Y_L. \tag{A1}
\]

For the SM with three heavy Majorana neutrinos \( \xi = 1/3 \) and therefore \( Y_B = Y_{B-L}/3 = -Y_L/2 \).

The Boltzmann equations for the \( N_i \) number densities and the \( Y_{B-L} \) asymmetry in terms of the dimensionless parameter \( z = M_1/T \), where \( T \) is the temperature, can be written in the form

\[
\begin{align*}
\frac{dY_{N_j}}{dz} &= -\frac{z}{Hs(z)} \left( \gamma_D^j + \gamma^0_j \right) \left( \frac{Y_{N_j}}{Y_{N_j}^{eq}} - 1 \right) + 3 \sum_{i=1}^3 \gamma_{NN}^{ij} \left( \frac{Y_{N_i}^{eq} Y_{N_j}}{Y_{N_j}^{eq} Y_{N_i}} - 1 \right), \\
\frac{dY_{B-L}}{dz} &= -\frac{z}{Hs(z)} \left( \sum_{j=1}^3 c_j \gamma_D^j \left( \frac{Y_{N_j}^{eq}}{Y_{N_j}^{eq}} - 1 \right) + \gamma^W \frac{Y_{B-L}}{Y_{B-L}^{eq}} \right), \tag{A2}
\end{align*}
\]

where \( H \) is the Hubble parameter evaluated at \( z = 1 \) and \( s(z) \) is the entropy density given by

\[
H = \sqrt{\frac{4\pi g_*}{45} \frac{M_p^2}{M_1^2}} \quad s(z) = \frac{2\pi^2 g_*}{45} \frac{M_1^3}{z^3}, \tag{A3}
\]

respectively. Here, \( g_* = 106.75 \) is the effective number of relativistic degrees of freedom and \( M_p = 1.2 \times 10^{19} \) GeV is the Planck mass. The equilibrium number density of a particle \( i \) with mass \( m_i \) is given by

\[
Y_i^{eq}(z) = \frac{45}{4\pi^4 g_*} \frac{m_i}{M_1^3} \left( \frac{m_i}{M_1} \right)^2 K_2 \left( \frac{m_i}{M_1} z \right), \tag{A4}
\]

where \( g_* \) denote the internal degrees of freedom of the corresponding particle (\( g_N = 2 \), \( g_t = 4 \)) and \( K_n(x) \) are the modified Bessel functions. The \( \gamma \)'s are the reaction densities for the different processes. For the decays one has

\[
\gamma_j^D = \frac{M_1 M_j^3}{8 \pi^2 z^2} (Y^D \nu)_{jj} K_1 \left( \frac{z m_j}{M_1} \right). \tag{A5}
\]

The reaction densities for the \( \Delta L = 1 \) processes with the top quark and for the \( \Delta L = 2 \) scatterings \( N_i - N_j \) can be written in the following way:

\[
\gamma_j^D = \gamma_j^{(1)} + 4 \gamma_j^{(2)}, \quad \gamma_j^{NN} = \gamma_{NN}^{(1)} + \gamma_{NN}^{(2)}, \tag{A6}
\]

respectively. Finally, \( \gamma^W \) accounts for the washout processes:

\[
\gamma^W = \sum_{j=1}^3 \left( \frac{1}{2} \gamma_j^D + \frac{Y_{N_i}}{Y_{N_j}^{eq}} \gamma_j^{(1)} + 2 \gamma_j^{(2)} \right) \left( \gamma_j^{(1)} + 2 \gamma_j^{(2)} \right). \tag{A7}
\]

Each of the above reaction densities can be computed through the corresponding reduced cross section \( \hat{\sigma}^{(i)}(x) \):

\[
\gamma^{(i)}(z) = \frac{M_1^4}{64 \pi^4 z^2 (m_{n}^2 + m_{\bar{n}}^2)/M_1^2} \frac{\hat{\sigma}^{(i)}(x) \sqrt{x}}{x} K_1(z \sqrt{x}) dx, \tag{A8}
\]

where \( x = s/M_1^2 \), with \( s \) the center-of-mass energy squared and \( m_{a,b} \) the masses of the initial particle states. All the relevant reduced cross sections are summarized below. For a more detailed presentation the reader is addressed to Ref. [42].

We write the reduced cross sections as functions of the parameter \( x \), the Hermitian matrix \( H_{ji} = Y_{ji}^\dagger Y_{ji} \) and the quantities \( a_j \) and \( c_j \) defined as

\[
a_j = \left( \frac{M_1}{M_1} \right)^2, \quad c_j = \left( \frac{\Gamma_j}{M_1} \right)^2 = a_j (H_{ji})_{ji}^2 \frac{M_1^2}{64 \pi^2}, \tag{A9}
\]

where \( \Gamma_j \) is the decay rate defined in Eq. (19).

For the \( \Delta L = 1 \) processes involving interactions with quarks we have

\[
\hat{\sigma}^{(1)}(x) = \hat{\sigma}(N_j + \ell \leftrightarrow q + \bar{u}) = 3 \alpha_u (H_{ij})_{ji} \left( \frac{x - a_j}{x} \right)^2, \tag{A10}
\]

\[
\hat{\sigma}^{(2)}(x) = \hat{\sigma}(N_j + u \leftrightarrow \ell + q) = \hat{\sigma}(N_j + q \leftrightarrow \bar{u} + \ell) = 3 \alpha_u (H_{ij})_{ji} x - a_j \left[ 1 + \frac{a_h - a_j}{x - a_j + a_h} \right] x - a_j \ln \left( \frac{x - a_j + a_h}{a_h} \right), \tag{A11}
\]

where \( \alpha_u \) (introduced to regularize the infrared divergencies) and \( \alpha_u \) are given by

\[
a_h = \left( \frac{\mu}{M_1} \right)^2, \quad \alpha_u = \frac{\text{Tr}(Y_u^\dagger Y_u)}{4 \pi} = \frac{m_t^2}{4 \pi v^2}. \tag{A12}
\]
respectively. The mass parameter $\mu$ is chosen to be $\mu = 800$ GeV as in Refs. [41,42]. The reduced cross sections relative to the $N_i - N_j$ scatterings are

$$\tilde{\sigma}^{(1)}_{N_iN_j} = \tilde{\sigma}(N_iN_j \rightarrow \ell + \bar{\ell})$$

$$= \frac{1}{2\pi}\left[ (H_{\nu})_{i} (H_{\nu})_{j} \left[ \frac{\sqrt{\lambda_{ij}}}{x} + \frac{a_j + a_i}{2x} \right] L_{ij} \right] - \text{Re}(H_{\nu}^{2})_{ij} \frac{\sqrt{a_i a_j}}{x-a_i-a_j} L_{ij}, \quad (A13)$$

$$\tilde{\sigma}^{(2)}_{N_iN_j} = \tilde{\sigma}(N_iN_j \rightarrow \phi + \phi)$$

$$= \frac{1}{2\pi}\left[ (H_{\nu})_{i} (H_{\nu})_{j} \left[ \frac{L_{ij}}{2} - \frac{\sqrt{\lambda_{ij}}}{x} \right] - \text{Re}(H_{\nu}^{2})_{ij} \frac{\sqrt{a_i a_j} (a_i + a_j)}{x(x-a_i-a_j)^{2}} L_{ij} \right], \quad (A14)$$

where the functions $L_{ij}$ and $\lambda_{ij}$ read as

$$L_{ij} = 2 \ln \left( \frac{x + \sqrt{x^2 - 2a_i a_j}}{2 \sqrt{a_i a_j}} \right),$$

$$\lambda_{ij} = x^2 - 2(xa_i + a_j) + (a_i - a_j)^2. \quad (A15)$$

Since the heavy Majorana neutrinos are unstable particles, one has to replace the usual fermion propagators by the off-shell propagators $D_i(x)$ in the computation of the amplitudes for the right-handed neutrino mediated processes:

$$\frac{1}{D_j} = \frac{1}{D_j(x)} = \frac{x - a_j}{(x - a_j)^2 + a_j c_j} \quad (A16).$$

For the $\Delta L = 2$ processes the reduced cross sections read then

$$\tilde{\sigma}^{(1)}_{N_iN_j} = \tilde{\sigma}(\ell + \phi \rightarrow \ell + \phi)$$

$$= \sum_{j=1}^{3} (H_{\nu})_{ij}^2 A_{ij} \left[ 1 + \sum_{n=1}^{3} \text{Re}(H_{\nu}^{2})_{nj} B_{nj} \right] \quad (A17)$$

where

$$A_{ij} = \frac{1}{2\pi}\left[ \frac{x}{x + a_j} + \frac{a_j}{2D_j} \left[ a_j + a_j \left( x + a_j \right) \right] \right],$$

$$B_{nj} = \frac{\sqrt{a_i a_j}}{x-a_i-a_j} \left[ \frac{1}{D_j} + \frac{x-a_j}{D_j} \right] \left[ \frac{1}{D_n} + \frac{x-a_n}{D_n} \right]. \quad (A18)$$

In our analysis we have computed numerically the reaction densities through Eq. (A8) and the above definitions of the reduced cross sections. Nevertheless, useful analytical approximations can be obtained for specific temperature regimes [42].


MINIMAL SCENARIOS FOR LEPTOGENESIS AND $CP$ . . .