Comparing two formulations for the ARM problem

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Outline

1. Introduction
   - Description of Problem
   - The Area Restriction Model (ARM)

2. Two Integer Programming Approaches for ARM
   - Cell Approach
   - Cluster Approach

3. Comparing the two Approaches
   - Modeling Advantages of the Cluster Approach
   - Computational Advantages of Each Approach
Obtain Harvest Schedule that Maximizes Profit Subject to Clear Cut Limitations and Side Constraints

- Environmental regulations set Maximum Area Constraints:
  - Reasons include wildlife habitat, scenic beauty, etc.
  - Maximum Clear Cut Area: 40+ to 120+ acres.

- Side constraints include:
  - Timber Volume Flow Constraints.
  - Average Ending Age.
ARM Includes Aggregation of Cells in the Problem

- Forest composed of small management units (Cells).
- Cluster = Groups of adjacent cells.
- Feasible Cluster = Area-complying clusters.
- Solution is group of non-adjacent feasible clusters.
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Cell Approach Forbids Infeasible Clusters

- One variable per cell.
- Cover/Path Constraints forbid harvesting (Minimal) Infeasible Clusters. (McDill et al. 2002)
- Strengthening:
  - Crowe et al. 2003 *Clique* Constraints.
  - Gunn and Richards 2005 *Stand Centered* Const.
  - Tóth et al. 2005 *Lifted Cover* Const.
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Strengthening:
Introduction

Two Integer Programming Approaches for ARM

Comparing the two Approaches

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Cluster Approach Easily Allows for Extra Modeling Requirements

- **Fixed Harvesting Costs:**
  - Modify objective coefficients in cluster approach.
  - Not clear how to do in cell approach.

- **Average area clear-cut constraints:**
  - Implemented as linear constraints in cluster approach.
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Control Over Clusters Creation Allows to Restrict Clear Cut Shapes

- Easy to forbid inconvenient cluster shapes:
  - U shaped clusters.
  - Long and thin clusters.
  - etc.

- Minimum Cluster Size.
  - Often fixed costs hard to quantify.
  - Imposed for economic reasons.
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Description of Forest Instances

- **Buttercreek**
  - 351 nodes and 662 arcs. Max area 120.
  - Feasible clusters $\leq$ 8 nodes, cliques $\leq$ 4 nodes.

- **El Dorado**
  - 1,363 nodes and 3,609 arcs. Max area 120.
  - Feasible clusters $\leq$ 7 nodes, cliques $\leq$ 4 nodes.

- **Shulkell**
  - 1,039 nodes and 2,065 arcs. Max area 40.
  - Feasible clusters $\leq$ 13 nodes, cliques $\leq$ 4 nodes.

- **Lemon Creek (Partial URM)**
  - 6,624 nodes and 18,048 arcs. Max area 40.
  - Feasible clusters $\leq$ 5 nodes, cliques $\leq$ 4 nodes.

- 3, 5 and 12 period instances with volume and ending age constraints. Solved with CPLEX 9 for 10,000 seconds. 0.01\% GAP considered Optimal.
Sizes of Formulations

- Maximum # of cells in a feasible cluster is the key:
  - Can grow if cells become smaller.
  - Can grow if Maximum Area grows.
- For fixed maximum # of cells in a feasible cluster both formulations grow polynomially.
- If maximum # of cells in a feasible cluster is not fixed both formulations can grow exponentially.
- Cell Approach: Size driven by Constraints = Path/Cover.
- Cluster Approach: Size driven by Variables = Feasible Clusters.
- Experiment: Plot Path/Cover and Feasible Clusters v/s Maximum Area.
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Sizes of Formulations are Comparable

El Dorado

Maximal Cliques
Feasible Clusters
Cover Constraints (Path)

# of elements in family vs. Maximum clear-cut area [acres]
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Shulkell

Maximal Cliques
Feasible Clusters
Cover Constraints (Path)
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![Graph showing the comparison of maximal cliques, feasible clusters, and cover constraints (Path) for Lemon Creek.](image)

- **Maximal Cliques** (red line)
- **Feasible Clusters** (green dotted line)
- **Cover Constraints (Path)** (blue dotted line)

The graph illustrates the relationship between the maximum clear-cut area in acres and the number of elements in the family for different formulations. The sizes of the formulations are comparable across the different approaches.
Solving the ARM Model

- Feasible solutions are easy to find:
  - CPLEX heuristic usually finds optimum (Some problems with Cluster and Vol. Constraints).
  - Many custom heuristics are available.

- Problem is proving optimality:
  - Tight LP relaxation is very important.
Problem is pure combinatorial.

Cluster formulation is far superior:
- LP relaxation is very tight.
- Solve times much better that Cell approach.
**Theorem**: LP of Cluster Formulation is Stronger than LP of Cell Formulation with Cover Constraints
Tight LP relaxation for Cluster Formulation Translates Into Fast Solve Times

![Bar chart showing solve times for different clusters](image-url)
Multi-Period with Side Constraints

- Side Constraints can be more important than area constraints.

- Both formulations perform similarly:
  - LP relaxations are similar.
  - Solve times are similar.
Multi-Period w. Side Constraints: Cluster LP Relaxation Still Tighter, but Difference is Smaller

LP relaxation theorem still holds.
Multi-Period w. Side Constraints: Similar LP Gaps Translates into Similar Solve Times.
Improvement in Objective When Removing Area Constraints (El Dorado)

![Bar chart showing improvement in objective when removing area constraints.](chart.png)
Improvement in Objective When Removing Area Constraints (El Dorado)
Side constraints can be more important than area constraints:
- Effect usually stronger for many periods.
- Area constraints are still needed.
- Cluster approach particularly sensitive to hard side constraints.

Green-up > 1 can make Area Constraints crucial again.
- Particularly important for many periods.
- INFORMS 2006.
Conclusions

- Advantages of the Cluster Approach:
  - Models problems which cell approach can not.
  - Better at area constraints aspect of the problem.

- Advantages of the Cell Approach.
  - Much less sensitive to hard side constraints.

- Other aspects of Cell Approach:
  - Strengthening can help.
  - Branch-and-cut implementation (Tóth et al. 2005).

- New Formulation: Constantino, Borges and Martins.

- More real forest instances needed. (FMOS)
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