Imposing Old-growth Patch Constraints in Forest Harvest Scheduling Models

Juan Pablo Vielma
University of Pittsburgh and IBM Watson Research Center
http://www.pitt.edu/~jvielma

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Forest Harvest Scheduling

1. Maximize profits
2. Protect Environment
Forest Harvest Scheduling

1. Maximize profits
2. Protect Environment

Schedule “stands” for harvest
Protect Environment: Connectivity

- Forbid large clear-cut areas:
  - Area Restriction Model (ARM)
  - Good IP models (6000 stands)

- Protect some contiguous areas:
  - Harder problems (400 stands)
  - Old-growth, reserve selection, wildlife corridors
IP Models for Harvest Scheduling

- Linear Constraints/Objective:
  - Profits, timber flow, ending age of forest, etc.

- Combinatorial Constraints:
  - Protect Environment

\[ y_{v,t} = \begin{cases} 
1 & \text{if stand } v \text{ is harvested in period } t. \\
0 & \text{otherwise} 
\end{cases} \]
Connectivity: Single Patch

- Rooted
- Unrooted
Connectivity: Multiple Patches

- Rooted
- Unrooted
Graph Representation of Forest
Area Restriction Model (ARM)

- Limit area of contiguous clear-cut region
- Unrooted multi-patch model:
  - Limit *maximum area* of patches
Harvested stands are clear-cut and replanted

- Stand harvested in $t$ is clear-cut only in $t$
- ARM constraints span only one period
- Stands can only be harvested once

\[ y_{v,t} = \begin{cases} 
1 & \text{if stand } v \text{ is harvested in period } t. \\
0 & \text{otherwise} 
\end{cases} \]
ARM Constraints: Forbid Sets

- Connected set of stands $C'$:
  - Area is strictly greater than maximum area
  - Minimal with respect to inclusion
ARM Constraints: Forbid Sets

- Connected set of stands \( C \):
  - Area is strictly greater than \textit{maximum area}
  - Minimal with respect to inclusion

\[
\sum_{v \in C} y_{v,t} \leq |C| - 1
\]
ARM Constraints: Forbid Sets

- Connected set of stands $C$:
  - Area is strictly greater than maximum area
  - Minimal with respect to inclusion

$$\sum_{v \in C} y_{v,t} \leq |C| - 1$$

Usually few of these sets exist
Connectivity Constraints

ARM solution = Fragmentation
Connectivity Constraints

Need Min Area Connected Patch

- 1+ connected regions:
  - minimum (average) area
  - is old-growth, contains animal population, contain water source, etc.
  - Other: shape, edge, etc.
- Force connectivity and add other constraints
Select red nodes for old-growth/reserve

Red nodes are disconnected because:

- Selected Nodes
- Disconnected Nodes

Selected Nodes: 1, 6, 10
Disconnected Nodes: 2, 3, 4, 5, 7, 8, 9, 11
Unrooted (Lack of) Connectivity

- Select red nodes for old-growth/reserve
- Red nodes are disconnected because:
  - There is a node-cut separating 1 and 11 with no selected nodes
Selected Set Is Connected if ...

- Set is connected $\iff$ pairs of nodes are connected
- Pairs are connected $\iff$ every cut separating them intersects selected nodes
Connectivity Constraints

**Selected Set Is Connected if ...**

- Set is connected $\iff$ pairs of nodes are connected
- Pairs are connected $\iff$ every cut separating them intersects selected nodes
Force Connectivity Constraints

\[ z_v = \begin{cases} 
1 & \text{if stand } v \text{ is selected to be old-growth/reserve} \\
0 & \text{otherwise} 
\end{cases} \]
Connectivity Constraints

**Force Connectivity Constraints**

\[
    z_v = \begin{cases} 
    1 & \text{if stand } v \text{ is selected to be old-growth/reserve} \\
    0 & \text{otherwise}
    \end{cases}
\]

\[
    \sum_{w \in S} z_w \geq z_u + z_v - 1 \quad \forall u, v
\]

For every cut \( S \) separating \( u \) and \( v \)
Connectivity Constraints

**Force Connectivity Constraints**

\[ z_v = \begin{cases} 
1 & \text{if stand } v \text{ is selected to be old-growth/reserve} \\
0 & \text{otherwise} 
\end{cases} \]

\[ \sum_{w \in S} z_w \geq z_u + z_v - 1 \quad \forall u, v \]

For every cut \( S \) separating \( u \) and \( v \)

\[ \sum_{w \in S} z_w \geq z_v \quad \forall v \]

For every cut \( S \) separating \( r \) and \( v \)

● Rooted: All selected stands connected to root \( r \)
Advantages and Disadvantages

- Can easily add extra requirements
  - e.g. minimum area

\[
\sum_v a_v z_v \geq A_{\text{Min}} \\
\text{where } a_v = \text{area of stand } v
\]
Advantages and Disadvantages

- Can easily add extra requirements
  - e.g. minimum area

\[
\sum_v a_v z_v \geq A_{\text{Min}}
\]

\[a_v = \text{area of stand } v\]

- Too many separating-cut constraints
- Separating the constraints is easy
Connectivity Constraints

Cutting Plane Procedure

\[ \sum_{w \in S} z_w \geq z_u + z_v - 1 \]

\( z^* = \text{current solution} \)

Find cut \( S^* \) that separates \( u, v \) and minimizes \( \sum_{w \in S^*} z_w^* \)

Use max-flow solver

For every pair \( u, v \)

check if \( z_u^* + z_v^* - 1 > 0 \)

If \( \sum_{w \in S^*} z_w^* < z_u^* + z_v^* - 1 \)

add cut
Problem Specification

1. Maximize NPV of harvest schedule s.t.:
   - ARM Constraints: maximum clear-cut
   - Volume flow constraints
   - Bound on average ending age of forest

2. Additionally:
   - Reserve 10% of forest area as a contiguous old-growth path (unrooted model)
Instances and Solvers

- 5-period instances from FMOS repository:

<table>
<thead>
<tr>
<th>Instance</th>
<th>Stands</th>
<th>Total area</th>
<th>Max CC Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Dorado</td>
<td>1363</td>
<td>52,255.5</td>
<td>120</td>
</tr>
<tr>
<td>Shulkell</td>
<td>1039</td>
<td>11,116.65</td>
<td>40</td>
</tr>
<tr>
<td>NBCL5A</td>
<td>5581</td>
<td>149,235</td>
<td>80</td>
</tr>
<tr>
<td>FLG9A</td>
<td>850</td>
<td>24,708.1</td>
<td>80</td>
</tr>
</tbody>
</table>

- CPLEX 11 on a Quad-core Xeon with 32Gb RAM
Computational Example

Results: Time limit of 4 hours

1. ARM:
   - Directly solved by CPLEX
   - 3 optimal in <400s, 1 reaches 0.03% GAP

2. ARM+old-growth
   - CPLEX based branch-and-cut: need heuristics, use of rooted formulations, “ring” cuts, etc.
   - 3 with <1% GAP, 1 with 2.2% GAP
**Economic Effect**

- **ARM**: 2-5% loss in NPV
- **ARM+old-growth**: additional loss of:
Solutions Sometime Look Good
Computational Example

Solutions Sometimes Don’t Look Good

FLG9A
Conclusions and Future Work

Done:
- Connectivity for environmental protection
- Can obtain good solutions for old-growth
- Optimization: Cost is moderate

To do:
- Optimization too "clever": snake like patches
- Some challenges in branch-and-cut