Integer Programming Approaches for Imposing Connectivity in Forest Management

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University of Chile
Introduction

Forest Management
Schedule “stands” for different uses
Connectivity in Forestry

- Clearcut size constraints
- Old growth patches
- Reserve selection
- Wildlife corridors
Where are we?

Clearcut constraint:

Peder Wikström
Introduction

Where are we?

☑ Clearcut constraint:

☐ Other connectivity?

Peder Wikström
Introduction

Types of Connectivity 1

- Rooted Multi-Patch
- Rooted Single Patch
- Unrooted Multi-Patch
- Unrooted Single Patch
Introduction

Types of Connectivity 2

- **t=1**
- **t=2**
- **t=3**

- Static Patch
- Dynamic Patch
Today: Rooted Single Patch
\[ z_{v,t} + y_{v,t} \leq 1 \quad \forall t, v \]

\{v : z_{v,t} = 1\} is connected \( \forall t \)

\[ y_{v,t} = \begin{cases} 
1 & \text{if stand } v \text{ is harvested in period } t. \\
0 & \text{otherwise} 
\end{cases} \]

\[ z_{v,t} = \begin{cases} 
1 & \text{if stand } v \text{ is old-growth or reserve in period } t \\
0 & \text{otherwise} 
\end{cases} \]

- Linear Constraints/Objective:
- Profits, timber flow, ending age of forest, etc.
IP Models?
Two Types of (Strongest) IP Models

\[ S := \left\{ x \in \mathbb{Z}^n : \sum_{i=1}^{n} |x_i| \leq 1 \right\} \]
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\end{align*} \]
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\[ S := \left\{ x \in \mathbb{Z}^n : \sum_{i=1}^{n} |x_i| \leq 1 \right\} \]

\[ \sum_{i=1}^{n} s_i x_i \leq 1 \quad \forall s \in \{-1, 1\}^n \]

\[ x_i \in \mathbb{Z} \quad \forall i \in \{1, \ldots, n\} \]

Original Space: Size = \(O(2^n)\)

\[ x_1, x_2 \in \mathbb{Z}^2 \]

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\[ x_i \in \mathbb{Z} \quad \forall i \in \{1, \ldots, n\} \]

Extended Formulation: Size = \( O(n) \)

\[ \sum_{i=1}^{n} y_i \leq 1 \]
\[ -y_i \leq x_i \leq y_i \quad \forall i \in \{1, \ldots, n\} \]
\[ x_i \in \mathbb{Z} \quad \forall i \in \{1, \ldots, n\} \]
Two Types of (Strongest) IP Models

Original Space: Size $= O(2^n)$

Extended Formulation: Size $= O(n)$
Two Types of (Strongest) IP Models

Original Space: Size = $O(2^n)$

Extended Formulation: Size = $O(n)$
Two Types of (Strongest) IP Models

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Extended Formulation: Size= \( O(n) \)

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\[ x_i \in \mathbb{Z} \quad \forall i \in \{1, \ldots, n\} \]
Using Large Formulations: Separate

\[ \max_{x \in \mathbb{Z}^n} \sum_{i=1}^{n} x_i \]

1. Initialize small rMIP
2. Solve rMIP
3. Separate Optimal \( x^* \)
4. Separated?
   - Yes: Add Cut
   - No: Done
Using Large Formulations: Separate

\[ \max_{x \in \mathbb{Z}^n} \sum_{i=1}^{n} x_i \]

\[-1 \leq x_i \leq 1 \quad \forall i \in \{1, \ldots, n\} \]

**Flowchart:**

1. Initialize small rMIP
2. Solve rMIP
3. Separate Optimal \( x^* \)
4. Separated? (Yes/No)
   - Yes: Add Cut
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  - Yes
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                                - Done
**Using Large Formulations: Separate**

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\begin{align*}
\max_{x \in \mathbb{Z}^n} & \sum_{i=1}^{n} x_i \\
-1 \leq x_i & \leq 1 \quad \forall i \in \{1, \ldots, n\} \\
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\end{align*}
\]

1. Initialize small rMIP
2. Solve rMIP
3. Add Cut
4. Separate Optimal \( x^* \)
5. Separated?
   - Yes
   - No
6. Done
Using Large Formulations: Separate

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Initialize small rMIP

Solve rMIP

Add Cut

Separate Optimal \(x^*\)

Separated?

Yes

Done

No
Using Large Formulations: Separate

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\begin{align*}
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Initialize small rMIP

Solve rMIP

Separate Optimal \(x^*\)

Separated? Yes

Add Cut

No

Done
IP Models

Using Large Formulations: Separate

\[
\max_{x \in \mathbb{Z}^n} \sum_{i=1}^{n} x_i
\]

\[-1 \leq x_i \leq 1 \quad \forall i \in \{1, \ldots, n\}\]

\[\sum_{i=1}^{n} x_i \leq 1\]

Initialize small rMIP

Solve rMIP

Separate Optimal \(x^*\)

Separated?

Add Cut

Yes

No

Done
Key is Fast Separation

\[
\max_{x \in \mathbb{Z}^n} \sum_{i=1}^{n} x_i \\
-1 \leq x_i \leq 1 \quad \forall i \in \{1, \ldots, n\} \\
\sum_{i=1}^{n} \text{sign}(x_i^*) x_i \leq 1
\]
In Practice ...

- Branch and Cut
  - Linear Programming = Solution Quality Bounds
  - Heuristics = Actual Solutions
- Modern IP for many applications:
  - Traveling Salesman, Vehicle Routing, etc.
In Practice ...

- Branch and Cut
  - Linear Programming = Solution Quality Bounds
  - Heuristics = Actual Solutions
- Modern IP for many applications:
  - Traveling Salesman, Vehicle Routing, etc.
- Still (Cool) compact extended formulations have applications
Back to Connected Forests, i.e. Trees
Connectivity Formulations in Forestry

- Compact Extended and Large Formulations:
  - Önal and Briers (2006), Önal and Wang (2008), Rebain and McDill (2003), Martins et al. (2005), Carvajal et al. (2010), etc.
Connectivity Formulations in Forestry

- Compact Extended and Large Formulations:
  - Önal and Briers (2006), Önal and Wang (2008), Rebain and McDill (2003), Martins et al. (2005), Carvajal et al. (2010), etc

- Today = Carvajal et al. Large Formulation
Graph Representation of Forest
Graph Representation of Forest
Connectivity in Forestry: Models

Graph Representation of Forest

\[ z_{v,t} = \begin{cases} 
1 & \text{if stand } v \text{ is old-growth or reserve in period } t \\
0 & \text{otherwise} 
\end{cases} \]
Rooted (Lack of) Connectivity

- **Root**: r
- **Selected Nodes**: 
- **Cut Nodes**: 

Solution:
Rooted (Lack of) Connectivity

Solution:

- Root
- Selected Nodes
- Cut Nodes
**Rooted (Lack of) Connectivity**

- **Root**: $r$
- **Selected Nodes**: Nodes 6, 9, 11
- **Cut Nodes**: Nodes 2, 4, 5

**Solution:**

$$z_6 + z_9 + z_{11} \geq z_{10}$$
**Solution:**

\[ z_6 + z_9 + z_{11} \geq z_{10} \]
Connectivity in Forestry: Models

### Rooted (Lack of) Connectivity

- **Root**
- **Selected Nodes**
- **Cut Nodes**

**Solution:**

\[ z_6 + z_9 + z_{11} \geq z_{10} \]
Rooted (Lack of) Connectivity

Solution:

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Rooted (Lack of) Connectivity

Solution:

\[ z_6 + z_9 + z_{11} \geq z_{10} \]
\[ z_2 + z_5 + z_6 \geq z_{10} \]
Connectivity in Forestry: Models

Rooted (Lack of) Connectivity

- Root
- Selected Nodes
- Cut Nodes

Solution:

\[ z_6 + z_9 + z_{11} \geq z_{10} \]
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Connectivity in Forestry: Models

Rooted (Lack of) Connectivity

Solution:

\[ z_6 + z_9 + z_{11} \geq z_{10} \]

\[ z_2 + z_5 + z_6 \geq z_{10} \]
Connectivity in Forestry: Computation

Maximum Clearcut Area (ARM)
Maximum Clearcut Area (ARM)

Partial Solution: Connected Old Growth Patch
Maximum Clearcut Area (ARM)

Partial Solution: Connected Old Growth Patch
Maximum Clearcut Area (ARM)

Partial Solution: Connected Old Growth Patch
Test Problem: Harvest Scheduling

- 3 Periods
- Maximize NPV of schedule
- Maximum clear-cut
- Volume flow
- Average ending age of forest
- **Connected** old-growth patch (rooted model)
## Connectivity in Forestry: Computation

### Instances = FMOS, Solver=CPLEX 11

<table>
<thead>
<tr>
<th>Instance</th>
<th>Stands</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Dorado</td>
<td>1363</td>
</tr>
<tr>
<td>Shulkell</td>
<td>1039</td>
</tr>
<tr>
<td>NBCL5A</td>
<td>5581</td>
</tr>
</tbody>
</table>

### Static Set

<table>
<thead>
<tr>
<th>Instance</th>
<th>Stands</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gavin</td>
<td>352</td>
</tr>
<tr>
<td>Hardwicke</td>
<td>423</td>
</tr>
</tbody>
</table>

### Dynamic Set

- **Time Limit = 4 hours**
Modern IP = Quality Bounds = GAP

- For Max, GAP = $100 \times \frac{(\text{Bound} - \text{BestSol})}{\text{BestSol}}$
- Bound =
  - Root = LP relaxation + cuts
  - Final = Best B&B node left
- BestSol = best known solution
Results for Static Patch
Results for Static Patch

- **El Dorado**: Root GAP 0%, Final GAP 0%
- **Shulkell**: Root GAP 0.75%, Final GAP 1.5%
- **NBCL5A**: Root GAP 2.25%, Final GAP 3%

- *Root Solve Time* < 3 min for all locations.
Results for Dynamic Patch
Results for Dynamic Patch

- Root GAP
- Final GAP

- Gavin
- Hardwicke

- Root Solve Time < 4 min
Connectivity in Forestry: Computation

Solutions Sometime Look Good

El Dorado
Solutions Sometime Don’t Look Good

Connectivity in Forestry: Computation

Gavin 3 Periods
Final Thoughts...

- Don’t blindly use IP:
  - Optimization can be “too” clever
  - Don’t fear “Large” formulations
  - Do use special purpose heuristics
- BIG open problem in IP: Compact extended formulation (Strongest) for general matching
- Known large formulation is similar to Önal and Wang (2008)