**Review: Complexity Theory**

\[ \text{TIME}(t(n)) = \{ A \mid A \text{ is a language decided by a deterministic 1-tape TM in } O(t(n)) \text{ time} \} \]

\[ \text{NTIME}(t(n)) = \{ A \mid A \text{ is a language decided by a non-deterministic 1-tape TM in } O(t(n)) \text{ time} \} \]

\[ P = \bigcup_{k=1}^{\infty} \text{TIME}(n^k) \text{ where the union is over all natural numbers } k \]

\[ NP = \bigcup_{k=1}^{\infty} \text{NTIME}(n^k) \text{ where the union is over all natural numbers } k \]

“quickly” = “in polynomial time”

**Polynomial Time**

\[ P = \{ L \mid L \text{ is a language that can be decided quickly} \} \]

**Non-deterministic Polynomial Time** (NOT “Not Polynomial”!)

\[ NP = \{ L \mid L \text{ is a language that, given a certificate, can be verified quickly} \} = \{ L \mid L \text{ is a language that can be decided quickly by a non-deterministic TM} \} \]

All languages in \( P \) are also in \( NP \).

The $1 million question: Are all languages in \( NP \) also in \( P \)?

A language \( L \) is **NP-complete** if:

a) \( L \) is in \( NP \)

b) Every language \( A \) in \( NP \) is polynomial-time reducible to \( L \)

**If \( B \) is NP-complete and \( B \) is in \( P \), then \( P=NP \).**

(In other words, if you can show any NP-complete problem is actually solvable in polynomial time, then all NP problems are also solvable in polynomial time.)