**Review: Miscellaneous Math Topics**

**Special Number Sets**

- **Natural numbers**: \( \mathbb{N} = \{1, 2, 3, 4, \ldots \} \)
- **Integers**: \( \mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \} \)
- **Rational numbers**: \( \mathbb{Q} = \{ \frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0 \} \)
- **Real numbers**: \( \mathbb{R} \) Includes all rational numbers and irrational numbers (which can’t be rewritten as fractions but have decimal representations: e.g. \( \pi, e, \) and \( \sqrt{2} \)).

**Proof Techniques**

- **Proof by Construction**
  To show something can be done, make something (e.g. a TM) that can do it.

- **Proof by Contradiction**
  Assume, for contradiction, the opposite of the statement you’re trying to prove. Then do stuff to reach a contradiction. Conclude that your assumption must be false after all.

- **Proof by Induction**
  Base case: Prove the statement is true for \( n=1 \)
  Inductive hypothesis: Assume that the statement is true for \( n=m \)
  Inductive case: Prove the statement is true for \( n=m+1 \), using as a given that it’s true for \( n=m \).

- **Pigeonhole Principle**
  If you have \( n \) pigeons and \( m \) holes, and \( m \leq n \), then at least one hole has two or more pigeons.

**Combinatorics**

- **Permutations**: The number of ways to choose \( r \) from \( n \) elements, where order matters.
  \[ _nP_r = \frac{n!}{(n-r)!} \]

- **Combinations**: Number of ways to choose \( r \) from \( n \) elements, where order does not matter.
  \[ _nC_r = \frac{n!}{r!(n-r)!} \]