DECIDABILITY PROBLEMS

* (1) Oh, Look, DFAs Should Be Easy…
(From Michael Sipser, Introduction to the Theory of Computation, 2nd ed., Exercise 4.1.)

True (T) or false (F)?

a)  <M, 0100> ∈ A_{DFA}

b)  <M, 011> ∈ A_{DFA}

c)  <M> ∈ A_{DFA}

d)  <M, 0100> ∈ A_{REX}

e)  <M> ∈ E_{DFA} [IMAGE HERE]

f)  <M, M> ∈ E_{Q_{DFA}}

* (2) I Feel So Empty Inside! (Or at Least, My CFG Does.)

Define E_{CFG} = \{<G> | G is a CFG and L(G) = {} \}

a)  Describe in English what the language E_{CFG} consists of.

b)  Give an example CFG G_{happy} where <G_{happy}> is in E_{CFG}.

c)  Prove that E_{CFG} is a decidable language. (Hint: Look at the proof for E_{DFA} to get an idea.)
* (3) All the King’s Horses, and All the King’s Men…

Let \( \text{ALL}_{\text{DFA}} = \{<A> | A \text{ is a DFA and } L(A) = \Sigma^*}\)

a) Describe in English what the language \( \text{ALL}_{\text{DFA}} \) consists of.

b) Give an example of a DFA \( A_{\text{stallion}} \) that is in \( \text{ALL}_{\text{DFA}} \).

c) Prove that \( \text{ALL}_{\text{DFA}} \) is decidable.
** (4) Oh Yeah, DFAs Are Related to Regular Expressions!
(From Michael Sipser, *Introduction to the Theory of Computation*, 2nd ed., Exercise 4.2.)

Consider the problem of determining whether a given DFA and a given regular expression are equivalent (i.e. recognize the same language).

a) Write this problem as a language.

b) Prove that this problem is decidable.

*/** (5) Everyone Wants to Feel Needed, Sometimes

A *useless state* in a PDA is a state that is never entered on any input string.

* a) Draw or describe a PDA containing a useless state.

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* b) Formulate as a language the problem of determining whether a PDA has any useless states.

*** c) Prove that the language in part (b) is decidable.

** (∞) To Infinity, and Beyond!

Prove that INFINITE$_{DFA} = \{ <A> \mid A$ is a DFA and $L(A)$ is an infinite language$\}$ is decidable.
**** (6) The Odd One Out

Let \( A_{\text{odd1}} = \{<M> | M \text{ is a DFA that doesn’t accept any string containing an odd number of 1s}\} \)
Prove that \( A_{\text{odd1}} \) is decidable.
(Hint: Remember that regular languages are closed under intersection.)