LOGARITHMS PROBLEMS

* (1) Convert (Logs, Not Religions)

Recall that \[ \log_b(x) = n \iff b^n = x \]

Convert between the two forms:

a) \( \log_2 8 = 3 \) \iff 

b) \( \quad \iff 2^4 = 16 \)

c) \( \log_{10} 1000 = 3 \) \iff 

d) \( \quad \iff 10^2 = 100 \)

e) \( \log_3 27 = 3 \) \iff 

** (2) Abraham Lincoln Was Born in a Log Cabin

Compute the values of the following logarithms:

a) \( \log_2 8 = \)

b) \( \log_7 1 = \)

c) \( \log_{132} 1 = \)

d) \( \log_7 49 = \)

e) \( \log_{11} 121 = \)

f) \( \log_{23} 523 = \)
**/** (3) Prove Your Identities

Read and understand the following short proof of a famous logarithm identity:

**Theorem:** \( \log_b (xz) = \log_b (x) + \log_b (z) \)

(For example, \( \log_2 16 = \log_2 (8 \cdot 2) = \log_2 8 + \log_2 2 = 3 + 1 = 4 \).)

**Proof:** Let \( n = \log_b (x) \). Let \( m = \log_b (z) \). We want to show \( n + m = \log_b (x \cdot z) \).

Converting between the two forms for logs, \( b^n = x \) and \( b^m = z \).

So \( x \cdot z = b^n \cdot b^m = b^{n+m} \)

Converting to the other form, \( \log_b (x \cdot z) = n + m \), which is what we wanted to show.

\[ \square \]

Now prove the following identities:

* a) \( \log_b (x / z) = \log_b (x) - \log_b (z) \) \hspace{1cm} \text{(Hint: Remember } x/z = x \cdot z^{-1} \text{)}

(continued)
**b) \[ \log_b (x^k) = k \cdot \log_b (x) \]** and thus \[ \log_b \left(\sqrt[1/k]{x}\right) = \frac{1}{k} \cdot \log_b (x) \]

**c) \[ \log_b a = \frac{1}{\log_a b} \]
** (4) I Don’t Care About Your Log Bases

a) Prove that

\[ \log_b x = \frac{\log_a x}{\log_a b} \]

for any a, b, and x.

b) Why does this mean that \( O(\log_b x) = O(\log_a x) \)? (In other words, why don’t we care about the base of the logarithm when working in order-log-n time?)