P AND NP PROBLEMS

* (1) I’ve Got No Direction

Let $UHAMPATH = \{<G, s, t> | G$ is an undirected graph and has a Hamiltonian path from $s$ to $t}\}$

Recall that a Hamiltonian path is one that travels through every node of the graph exactly once. (This is different from an Euler path, which travels through every edge exactly once.)

a) Give an example of some $G$, $s$, and $t$ such that $<G, s, t>$ is an element of $UHAMPATH$. (In other words, draw some undirected graph labeled $G$, including nodes labeled $s$ and $t$, that has a Hamiltonian path from $s$ to $t$.)

Now show that $UHAMPATH$ is in NP in two ways:

b) Describe a non-deterministic TM that decides $UHAMPATH$ quickly.

c) Describe a deterministic TM verifier that, given a free certificate (what is the certificate?) quickly verifies if the certificate proves that an input is in $UHAMPATH$. 

** (2) Double the Fun!

Let DOUBLE-SAT = { <φ> | φ has at least two satisfying assignments }

a) Give an example of some φ such that <φ> is a member of the language DOUBLE-SAT.

Now show that DOUBLE-SAT is in NP in two ways:

b) Describe a non-deterministic TM that decides DOUBLE-SAT quickly.

c) Describe a deterministic TM verifier that, given a free certificate (what is the certificate?) quickly verifies if the certificate proves that an input is in DOUBLE-SAT.
** (3) It’s All in the Family

Let \( \text{ALL}_{\text{DFA}} = \{<D> \mid D \text{ is a DFA and } L(D) = \Sigma^*\} \)

Show that \( \text{ALL}_{\text{DFA}} \) is in P. (In other words, give a TM that decides \( \text{ALL}_{\text{DFA}} \) in polynomial time.)

** (4) Closure Properties for NP

a) Show that NP is closed under union.
   (In other words, show that if for any two languages \( A \) and \( B \) in NP, \( A \cup B \) is also in NP.)

b) Show that NP is closed under concatenation.

c) Show that NP is closed under star.
(5) coNP Languages

We saw in class that CLIQUE and SUBSET-SUM are both in NP. If you are not sure why, stop here and figure out why on a separate piece of paper. (Don’t continue otherwise – it’ll just be confusing.)

We define \( \text{coNP} = \{L \mid \text{the complement of } L \text{ is in NP}\} \).

So CLIQUE and SUBSET-SUM are in coNP. No one knows whether coNP is equal to NP or not.

a) Write out the set definition of CLIQUE and SUBSET-SUM:

\[
\text{CLIQUE} = \{
\]

\[
\text{SUBSET-SUM} = \{
\]

b) Write out the set definition of their complements:

\[
\text{CLIQUE} = \{
\]

\[
\text{SUBSET-SUM} = \{
\]

c) Why can you not automatically say that CLIQUE and SUBSET-SUM are in NP by simply modifying the nondeterministic TM that decides CLIQUE and SUBSET-SUM to accept when it would reject, and reject when it would accept?

d) What can you say about the question of whether coNP = P?