**SET THEORY PROBLEMS**

**SOLUTIONS**

* (1) Formal as a Tux and Informal as Jeans

Describe the following sets in both formal and informal ways.

<table>
<thead>
<tr>
<th>Formal Set Notation Description</th>
<th>Informal English Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) {2, 4, 6, 8, 10, \ldots}</td>
<td>The set of all positive even integers</td>
</tr>
<tr>
<td>b) {\ldots, -3, -1, 1, 3,\ldots}</td>
<td>The set of all odd integers</td>
</tr>
<tr>
<td>c) {n \mid n = 2m \text{ for some } y \in \mathbb{N}}</td>
<td>The set of all positive even integers (using the convention that 0 is not a natural number)</td>
</tr>
<tr>
<td>d) {x \mid x=2n \text{ and } x=2k \text{ for some } n, k \in \mathbb{N}}</td>
<td>The set of all positive multiples of 6</td>
</tr>
<tr>
<td>e) {b \mid b \in \mathbb{Z} \text{ and } b=b+1}</td>
<td>\emptyset</td>
</tr>
<tr>
<td>f) {2, 20, 200}</td>
<td>The set containing the numbers 2, 20, and 200</td>
</tr>
<tr>
<td>g) {n \mid n \in \mathbb{Z} \text{ and } n &gt; 42}</td>
<td>The set containing all integers greater than 42</td>
</tr>
<tr>
<td>h) {n \mid n \in \mathbb{Z} \text{ and } n &lt; 42 \text{ and } n &gt; 0} = {n \mid n \in \mathbb{N} \text{ and } n &lt; 42}</td>
<td>The set containing all positive integers less than 42</td>
</tr>
<tr>
<td>i) {\text{hello}}</td>
<td>The set containing the string hello</td>
</tr>
<tr>
<td>j) {\text{bba, bab}}</td>
<td>The set containing the strings bba and bab</td>
</tr>
<tr>
<td>k) \emptyset = {}</td>
<td>The set containing nothing at all</td>
</tr>
<tr>
<td>l) {\varepsilon}</td>
<td>The set containing the empty string</td>
</tr>
</tbody>
</table>
* (2) You Want Me to Write What?

Fill in the blanks with \(\in, \notin, \subseteq, \supseteq, =,\) or \(\neq\).

Recall that \(\mathbb{Z}\) is the set of all integers and \(\emptyset\) is the empty set, \(\{\}\).

a) \(2 \quad \_\in\_ \quad \{2, 4, 6\}\)
b) \(\{2\} \quad \_\subseteq\_ \quad \{2, 4, 6\}\)
c) \(1.5 \quad \_\notin\_ \quad \mathbb{Z}\)
d) \(-1.5 \quad \_\notin\_ \quad \mathbb{Z}\)
e) \(15 \quad \_\in\_ \quad \mathbb{Z}\)
f) \(-15 \quad \_\subseteq\_ \quad \mathbb{Z}\)
g) \(\emptyset \quad \_\subseteq\_ \quad \mathbb{Z}\)
h) \(54 \quad \_\in\_ \quad \{6, 12, 18, \ldots\}\)
i) \(54 \quad \_\notin\_ \quad \{6, 12, 18\}\)
j) \(\{1, 3, 3, 5\} \quad \_\subseteq\_ \quad \{1, 3, 5\}\)
k) \(\{-3, 1, 5\} \quad \_\neq\_ \quad \{1, 3, 5\}\)
l) \(\{3, 1, 5\} \quad \_\subseteq\_ \quad \{1, 3, 5\}\)
(3) Set Operations

Let A = \{x, y\} and B = \{x, y, z\}.

a) Is A a subset of B? **YES**  

b) Is B a subset of A? **NO**  

c) \(A \cup B = \{x, y, z\} = B\)  

d) \(B \cup A = \{x, y, z\} = B\)  

e) \(A \cap B = \{x, y\} = A\)  

f) \(B \cap A = \{x, y\} = A\)  

g) \(A \times B = \{(x, x), (x, y), (x, z), (y, x), (y, y), (y, z)\}\)  

h) \(B \times A = \{(x, x), (x, y), (x, z), (y, x), (y, y), (z, x), (z, y)\}\)  

i) \(P(A) = \{\emptyset, \{x\}, \{x, y\}\}\)  

j) \(P(B) = \{\emptyset, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}\)  

(4) Does Order Matter?

Three important binary set operations are the union (\(\cup\)), intersection (\(\cap\)), and cross product (\(\times\)).

A binary operation is called **commutative** if the order of the things it operates on doesn’t matter.

For example, the addition (+) operator over the integers is commutative, because for all possible integers x and y, \(x + y = y + x\).

However, the division (÷) operator over the integers is **not** commutative, since \(x ÷ y \neq y ÷ x\) for all integers x and y. (Note it works for **some** integers x and y, specifically whenever \(x = y\), but not for **every** possible integers x and y.)

a) Is the union operation commutative? (Does \(A \cup B = B \cup A\) for all sets A and B?)  

**Yes.** \(A \cup B = \{x \mid x \in A \text{ or } x \in B\} = \{x \mid x \in B \text{ or } x \in A\} = B \cup A\)

b) Is the intersection operation commutative?  

**Yes.** \(A \cap B = \{x \mid x \in A \text{ and } x \in B\} = \{x \mid x \in B \text{ and } x \in A\} = B \cap A\)

c) Is the cross product operation commutative?  

**No.** **For a counterexample, see Problem (3g) and (3h) above.**  
**Recall that order matters for pairs.**
*** (5) Set Me Up

Consider the following sets: \( A = \{ \varnothing \} \), \( B = \{ A \} \), \( C = \{ B \} \), \( D = \{ A, \varnothing \} \)

True (T) or False (F)?

a) \( \varnothing \in A \)  
   b) \( \varnothing \subseteq A \)  
   c) \( \varnothing \in B \)  
   d) \( \varnothing \subseteq B \)  
   e) \( \varnothing \in C \)  
   f) \( \varnothing \subseteq C \)  
   g) \( \varnothing \in D \)  
   h) \( \varnothing \subseteq D \)

i) \( A \in B \)  
   j) \( A \subseteq B \)  
   k) \( A \in C \)  
   l) \( A \subseteq C \)  
   m) \( A \in D \)  
   n) \( A \subseteq D \)

o) \( B \in A \)  
   p) \( B \subseteq A \)  
   q) \( B \in C \)  
   r) \( B \subseteq C \)  
   s) \( B \in D \)  
   t) \( B \subseteq D \)

u) \( C \in A \)  
   v) \( C \subseteq A \)  
   w) \( C \in B \)  
   x) \( C \subseteq B \)  
   y) \( C \in D \)  
   z) \( C \subseteq D \)

Note: May be easier to think about the sets as:

\[
\begin{align*}
A &= \{ \{\} \} \\
B &= \{ \{\{\}\} \} \\
C &= \{ \{\{\{\}\}\} \} \\
D &= \{ \{\{\}\}, \{\} \}
\end{align*}
\]