PROOFS BY INDUCTION

Goal: Prove some statement P[n] is true for all integers $n \geq 1$

Step 1: State the base case P[1] and prove it.

Step 2: State the inductive hypothesis P[m].

Step 3: Prove the inductive case P[m+1],
***assuming* that the inductive hypothesis P[m] is true for some $m \geq n$**
It's often helpful to write P[m+1] in terms of something recognizable from P[m]

Example: Prove $1 + 2 + 3 + \ldots + n = \frac{1}{2} n \cdot (n+1)$ for all integers $n \geq 1$.

The statement P[n] is: "$1 + 2 + 3 + \ldots + n = \frac{1}{2} n \cdot (n+1)$"
We want to show this is true for any integer $n \geq 1$ using induction.

Step 1: State the base case P[1] and prove it

The base case P[1] is: "$1 = \frac{1}{2} \cdot 1 \cdot (1+1)$"
This is obviously true (proof by "duh").

Step 2: State the inductive hypothesis P[m].

P[m] is: "$1 + 2 + 3 + \ldots + m = \frac{1}{2} m \cdot (m+1)$"

Step 3: Assume the inductive hypothesis, and prove the inductive case:

Assume P[m]: $1 + 2 + 3 + \ldots + m = \frac{1}{2} m \cdot (m+1)$ is true for some $m \geq n$.

Now show P[m+1]: $1 + 2 + 3 + \ldots + (m+1) = \frac{1}{2} (m+1) \cdot ((m+1)+1)$ is true:

Rewrite P[m+1] in terms of something recognizable from P[m]:

$$1 + 2 + 3 + \ldots + (m+1) = 1 + 2 + 3 + \ldots + m + (m+1)$$
$$= \underline{(1 + 2 + 3 + \ldots + m)} + (m+1)$$

(Does the underlined first term look familiar?)

$$= (m(m+1)/2) + m + 1 \quad \text{by induction hypothesis}$$
$$= m(m+1)/2 + m + 1$$
$$= m(m+1)/2 + (m+1)(2/2)$$
$$= m(m+1)/2 + 2(m+1)/2$$

(continued)

(continued)

$$= (m+2)(m+1)/2$$
$$= (m+1+1)(m+1)/2$$
$$= ((m+1)+1)(m+1)/2$$
$$= (m+1)((m+1)+1)/2$$

So we've proven our base case P[1] (step 2) and our inductive case P[m+1] using the induction hypothesis P[m] (step 3).

Therefore, P[n] holds true for all $n \geq 1$. □