

REVIEW: COMPLEXITY THEORY

TIME($t(n)$) = {A | A is a language decided by a deterministic 1-tape TM in $O(t(n))$ time}

NTIME($t(n)$) = {A | A is a language decided by a non-deterministic 1-tape TM in $O(t(n))$ time}

P = $\bigcup \text{TIME}(n^k)$ where the union is over all natural numbers k

NP = $\bigcup \text{NTIME}(n^k)$ where the union is over all natural numbers k

“quickly” = “in polynomial time”

Polynomial Time

P = {L | L is a language that can be *decided* quickly}

Non-deterministic Polynomial Time (NOT “Not Polynomial”!)

NP = {L | L is a language that, given a certificate, can be *verified* quickly}

= {L | L is a language that can be decided quickly by a non-deterministic TM}

All languages in P are also in NP.

The \$1 million question: Are all languages in NP also in P?

A language L is **NP-complete** if:

- L is in NP
- Every language A in NP is polynomial-time reducible to L

If B is NP-complete and B is in P, then P=NP.

(In other words, if you can show any NP-complete problem is actually solvable in polynomial time, then all NP problems are also solvable in polynomial time.)