

COMBINATORICS PROBLEMS

Write answers as simply as you can, but don't necessary give a specific number – it's fine to leave answers in factorial or “n choose k” notation when needed.

* (1) Lanyards

You have a bag of 20 differently-colored lanyards. How many ways can you give them out to seven CTY students? (If you give out the same seven lanyards but to different students, that counts as a different way.)

* (2) Party Committee

Our class is trying to create a small committee for planning an end-of-session students. It needs to include one of the two instructors and two of the six students. How many possible committees are there?

* (3) Let's Play Ball

- a) A group of 20 friends are trying to form sports teams. They can either form a soccer team (of 11 players), a basketball team (of 5 players), or both. How many options are there?

- b) If the friends are also trying to pick the positions they're going to play (so that two teams with the same people are considered different if they have different positions assigned), how many options are there now?

**** (5) Frisbee**

A group of $n > 3$ campers are standing in a circle, throwing a frisbee. They throw the frisbee until everyone has thrown to everyone else, except that no one ever throws to either of the people standing next to them in the circle. How many throws were there?

**** (6) Prism**

A rectangular prism (a 3D rectangle) is divided into unit cubes. If the prism's dimensions are $a \times b \times c$, how many rectangular prisms (with integer-length sides) are there?

**** (7) Chess**

You have a chess board (8 x 8 squares) and eight rooks. You want to put all the rooks on the chessboard so that none of them can attack each other (i.e. no two rooks are in the same row or the same column). How many ways can you do this?

***** (9) Stones 'n Baskets**

If you have n identical stones and k numbered baskets, how many ways are there to put the stones into the baskets?

*****(10) Sum**

How many ways can you choose positive integers x , y , and z so that their sum is 100?

***** (11) Expand Away**

How many terms are there in the expansion of $(x + y + z)^{15}$? What are the coefficients of the $x^3y^7z^5$ and $x^6y^2z^7$ terms?

(12) Creativity

When you're done, think of a new combinatorics problem, solve it, and trade with a classmate who is also finished.