LOGARITHMS PROBLEMS

* (1) Convert (Logs, Not Religions)

Recall that $\log_b(x) = n \quad \leftrightarrow \quad b^n = x$

Convert between the two forms:

a) $\log_2 8 = 3$ $\leftrightarrow$

b) $\leftrightarrow$ $2^4 = 16$

c) $\log_{10} 1000 = 3$ $\leftrightarrow$

d) $\leftrightarrow$ $10^2 = 100$

e) $\log_3 27 = 3$ $\leftrightarrow$

** (2) Abraham Lincoln Was Born in a Log Cabin

Compute the values of the following logarithms:

a) $\log_2 8 =$

b) $\log_7 1 =$

c) $\log_{1432} 1 =$

d) $\log_7 49 =$

e) $\log_{11} 121 =$

f) $\log_{523} 523 =$

*/** (3) Prove Your Identities

Read and understand the following short proof of a famous logarithm identity:

Theorem: $\log_b(xz) = \log_b(x) + \log_b(z)$

(For example, $\log_2 16 = \log_2(8 \cdot 2) = \log_2 8 + \log_2 2 = 3 + 1 = 4$.)

Proof: Let $n = \log_b(x)$. Let $m = \log_b(z)$. We want to show $n + m = \log_b(x \cdot z)$.

Converting between the two forms for logs, $b^n = x$ and $b^m = z$.

So $x \cdot z = b^n \cdot b^m = b^{n+m}$

Converting to the other form, $\log_b(x \cdot z) = n + m$, which is what we wanted to show.

□

Now prove the following identities:

* a) $\log_b(x / z) = \log_b(x) - \log_b(z)$ (Hint: Remember $x/z = x \cdot z^{-1}$)

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(continued)

** b) $\log_b(x^k) = k \cdot \log_b(x)$ and thus $\log_b(\sqrt[k]{x}) = (1/k) \cdot \log_b(x)$

** c) $\log_b a = \dfrac{1}{\log_a b}$

(continued)

(continued)

** d) $b^{\log_b (x^y)} = x^y$

** (4) I Don't Care About Your Log Bases

a) Prove that $\log_b x = \frac{\log_a x}{\log_a b}$ for any a, b, and x.

b) Why does this mean that $O(\log_b x) = O(\log_a x)$? (In other words, why don't we care about the base of the logarithm when working in order-log-n time?)